Ministry of Higher Education and Scientific Research

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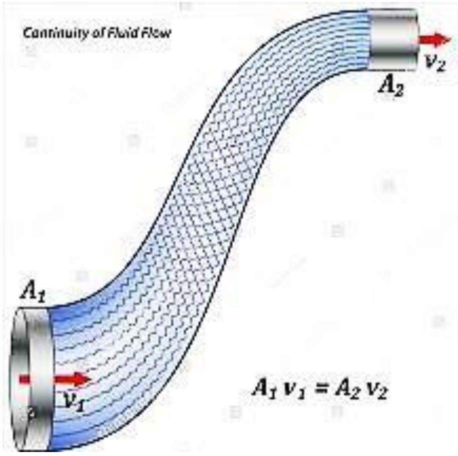
FLUID MECHANICS

Static Fluid

1.1 efinitions.



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References:

1- Fluid mechanics, Vector L. Streeter and E. Benjamin Wylie.

2- Fluid Mechanics and engineering application.

Robert L. Dogerti and Joshef B. Frinzieng

Chapter one Definitions

1- Fluid***: It is a substance that deform continuously under the action of the shear force. Its either gas or liquids.***

2- ***Shear stress*** $r = \frac{F}{A} = \frac{Shear\ force}{Suface\ area} = \frac{N}{m^2}$.

3- Shear force: It's the force components tangent to a surface of liquids.

4- Viscosity (μ): It's the properties of fluid by virtue of which it offers resistance to shear.

- Honey, Tar are example for high viscos liquids.
- Water and air have very small resistance.
- The viscosity of gases increase with temperature increasing.
- The viscosity of liquids decrease as temperature increasing.
- Units $\mu = \frac{N.s}{m^2}$ or $\frac{kg}{m.s}$
- $and\ commone\ units\ \ is\ Poise \rightarrow 1\ pois\ (\frac{g}{Cm.s}) = 0.1\ N.\frac{s}{m^2} = \frac{0.1kg}{m.s},\ \ 10\ p = \frac{1kg}{m.s}$

5- Kinematic Viscosity ($\mathcal{V}$): It's the ratio of viscosity to the mass density.

$\mathcal{V} = \frac{\mu}{\rho} = 1\frac{m^2}{s} = 100\frac{Cm^2}{s}\ (stocke)$.

6- ***Density (ρ): is the mass per unit volume*** $\boldsymbol{\rho = \frac{mass}{volume} = \frac{kg}{m^3}}$

For example $\rho_{water} = 1000\frac{kg}{m^3}$

7- Specific weight (γ): (unit gravity force) the force per unit volume. It change with location. $\boldsymbol{\gamma_{water} = \rho_{water}.g = 1000 * 9.81 = 9810\ \frac{N}{m^3}}$

8- ***Specific gravity (S): (relative density)*** $\boldsymbol{S = \frac{specific\ weight\ of\ substance}{specific\ weight\ of\ water} = \frac{\gamma_s}{\gamma_w}}$

9- Pressure (P):the normal force pushing against a plane area devised by area

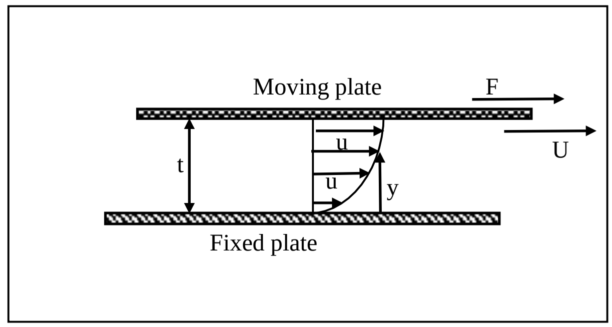
$$P = \frac{Force}{Area} = \frac{F}{A} = \frac{N}{m^2}\ pascal\ (pa\ or\ kpa)\ and \frac{kg}{Cm^2}\ bar\ .$$

10- Vapor pressure: The vapor molecules exert a partial pressure in the space known as vapor pressure.

11- Perfect gas: it is substance that satisfied the perfect das law $PV = mRT$.

Newtn law of viscosity:

$$F \text{ a } \frac{A\,U}{t}$$



A: is the area of the moving plate (m^2).

U: steady velocity of the moving plate (m/s).

t: the distance between the plates (m).

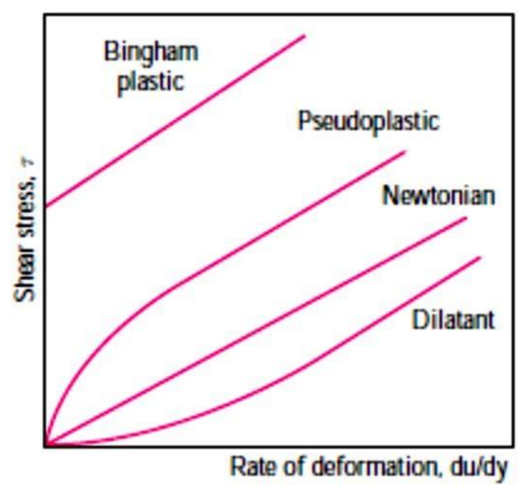
$$F = \mu \frac{A\,U}{t}$$

$$\because r = \frac{F}{A} = \mu \frac{U}{t}$$

$$\frac{U}{t} \quad defined\ as\ the\ total\ angler\ deformation\ of\ fluid$$

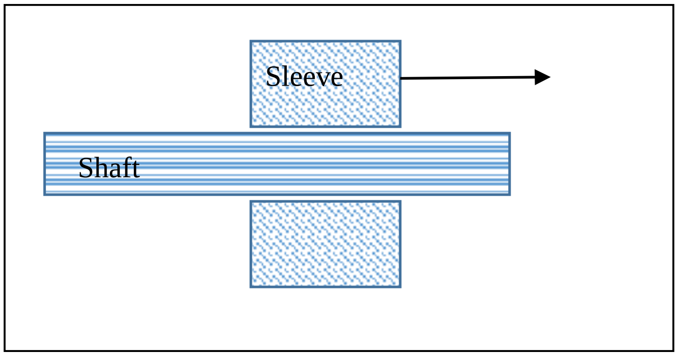
$$angler\ deformation\ of\ fluid\ \frac{du}{dy}$$

$$r = \mu \frac{du}{dy}$$ Newtonian fluid



Q1.5 A Newtonian fluid is n clearance between shaft and concentric sleeve. When a force of 500N is applied to the sleeve parallel to the shaft the sleeve attained a speed 1m/s. If a 1500N force is applied what speed will the sleeve attain? The temperature of the sleeve remain constant.

F_1=500N



U_1=1 m/s

$$F_2 = \mu \frac{A\,U}{t}$$

$$500 = \mu \frac{A\,1}{t}$$

$$\mu = \frac{500t}{A}$$

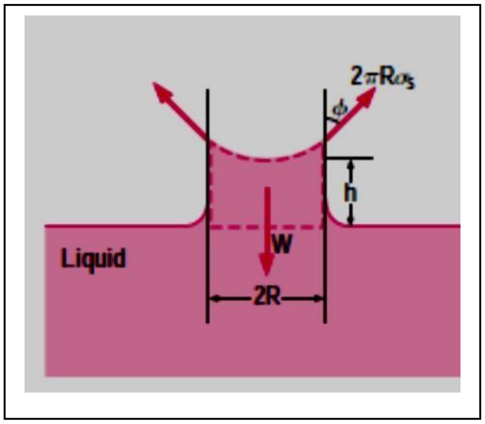
T= constant →μ = constant.

$$F_2 = \mu \frac{A\,U}{t} \rightarrow 1500 = \frac{500t}{A}\frac{A\,U}{t}$$

U=3m/s.

Specific volume (v_s): is the reciprocal of density. $v_s = \frac{1}{\rho} = \frac{m^3}{kg}$

Surface tension:



$$W = mg = \rho\, V\, g = \rho\, g\, (\pi R^2 h)$$

Equating the vertical components of the surface tension force to the weight gives:

$$W = F_{surface} \rightarrow \rho\, g\, (\pi R^2 h) = 2r\boldsymbol{R\sigma_s}\ \boldsymbol{cos(\phi)}$$

$$\therefore Capillary\ rise\ h = \frac{2\boldsymbol{\sigma_s}\ \boldsymbol{cos(\phi)}}{\rho\, g\, R}$$

Pressure at droplet $P = \frac{2\,\sigma}{r} = \frac{N}{m^2}$

14- Bulk modules of elasticity (K):

$$K = -\frac{dP}{\frac{dV}{V}} = \frac{N}{m^2} \quad or\ K = -\frac{\Delta P}{\frac{\Delta V}{V}} = -\frac{P_2 - P_1}{\frac{V_2 - V_1}{V_1}}$$

K: is the compressive stress per unit volume.

Ex: A liquid compressed in a cylinder has a volume of 1 liter (1000 cm³) at 1 MN/m² and volume of 995 cm³ at 2 MN/m². What is its bulk modulus of elasticity?

$$K = -\frac{\Delta P}{\Delta V/V} = \frac{(2-1)\ MN/m^2}{\frac{(995-1000)}{1000}} = 200\ MPa.$$

Q.1.42 $\sigma = 0.0736$ N/m

$$P = \frac{2\sigma}{r} = \frac{2 \times 0.0736}{\frac{0.05}{2}} \times 1000 = 5.89\ kPa.\ gage.$$

1.18 $d_s = 50$ mm $d_c = 50.1$ mm

$$\therefore t = \frac{50.1 - 50}{2} \qquad \therefore t = 0.05\ mm$$

μ at 0°C $= 1.6 \times 10^{-2}$ Pa.s

μ at 120°C $= 2 \times 10^{-3}$ Pa.s

$$F_1 = \mu \frac{AU}{t} = 1.6 \times 10^{-2} \frac{AU}{0.05 \times 10^{-3}} = 320\ AU$$

$$F_2 = 2 \times 10^{-3} \times \frac{1}{0.05 \times 10^{-3}} \times AU = 40\ AU$$

$$\therefore \frac{F_1}{F_2} = 8 \quad \& \quad \frac{F_1 - F_2}{F_1} = \frac{320AU - 40AU}{320\ AU} = 87\%$$

upper surface is in contact with air, which offers almost no resistance to the flow. Using Newton's law of viscosity, decide what the value of du/dy, y measured normal to the inclined plane, must be at the upper surface. Would a linear variation of u with y be expected?

1.4 What kinds of rheological materials are paint and grease?

1.5 A Newtonian fluid is in the clearance between a shaft and a concentric sleeve. When a force of 500 N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 1 m/s. If a 1500-N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

1.6 Determine the gravity force in newtons of 3 kg mass at a place where $g = 9.7\ m/s^2$.

1.7 When standard scale masses and a balance are used, a body is found to be equivalent in force of gravity to two of the 1-kg scale masses at a location where $g = 9.7\ m/s^2$. Calculate the gravity force on a correctly calibrated spring balance (for sea level) at this location.

1.8 Determine the unit gravity force γ for water at 25° C and $g = 9.75\ m/s^2$.

1.9 On another planet, where gravity is 3 m/s^2, find the force of gravity on 400 L of material $\rho = 800\ kg/m^3$.

1.10 A correctly calibrated spring scale records the gravity force of a 2-kg body as 17.0 N at a location away from the earth. What is the value of g at this location?

1.11 The gravity force on a bag of flour at sea level is 20 N. What is its mass at a location where $g = 9.6\ m/s^2$?

1.12 What is the kinematic viscosity of liquid of viscosity 0.002 Pa·s and a relative density of 0.8?

1.13 A shear stress of 4 mPa causes a Newtonian fluid to have an angular deformation of 1 rad/s. What is its viscosity?

1.14 A plate, 0.5 mm distant from a fixed plate, moves at 0.25 m/s and requires a force per unit area of 2 Pa to maintain this speed. Determine the viscosity of the substance between the plates.

1.15 Determine the viscosity of fluid between shaft and sleeve in Fig. 1.6.

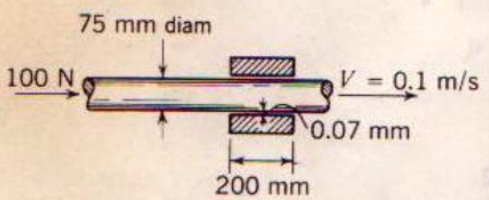


Figure 1.6 Problem 1.15.

1.16 A flywheel weighing 600 N has a radius of gyration of 300 mm. When it is rotating 600 rpm, its speed reduces 1 rpm/s owing to fluid viscosity between sleeve and shaft. The sleeve length is 50 mm; shaft diameter is 20 mm; and radial clearance is 0.05 mm. Determine the fluid viscosity.

1.17 A 25mm diameter steel cylinder 300 mm long falls, because of its own gravity force at a uniform rate of 0.1 m/s inside a tube of slightly larger diameter. A castor-oil film of constant thickness is between the cylinder and the tube. Determine the clearance between the tube and the cylinder. The temperature is 38°C. Relative density of steel = 7.85.

1.18 A piston of diameter 50.00 mm moves within a cylinder of 50.10 mm. Determine the percent decrease in force necessary to move the piston when the lubricant warms up from 0 to 120°C. Use crude-oil viscosity from Fig. C.1, Appendix C.

1.19 How much greater is the viscosity of water at 0°C than at 100°C? How much greater is its kinematic viscosity for the same temperature range?

1.20 A fluid has a viscosity of 0.6 Pa·s and a relative density of 0.7. Determine its kinematic viscosity.

1.21 A fluid has a relative density of 0.78. For a kinematic viscosity of 1.0×10^{-6} m^2/s determine the viscosity.

1.22 A body with gravity force of 500 N with a flat surface area of 0.2 m^2 slides down a lubricated inclined plane making a 30° angle with the horizontal. For viscosity of 0.1 Pa·s and body speed of 1 m/s determine the lubricant film thickness.

1.23 What is the viscosity of gasoline at 25°C?

1.24 Determine the kinematic viscosity of benzene at 27°C.

1.25 Calculate the value of the gas constant R for relative molecular mass of 44.

1.26 What is the specific volume of a substance of relative density 0.75?

1.27 What is the relation between specific volume and unit gravity force?

1.28 The density of a substance is 2900 kg/m^3. What is its (*a*) relative density, (*b*) specific volume, and (*c*) unit gravity force?

1.29 A force, expressed by $\mathbf{F} = 4\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$, acts upon a square area, 2 by 2 cm, in the xy plane. Resolve this force into a normal-force and a shear-force component. What are the pressure and the shear stress? Repeat the calculations for $\mathbf{F} = -4\mathbf{i} + 3\mathbf{j} - 9\mathbf{k}$.

1.30 A gas at 20°C and 0.2 MPa abs has a volume of 40 L and a gas constant $R = 210$ m·N/kg·K. Determine the density and mass of the gas.

1.31 What is the density of air at 400 kPa abs and 30°C?

1.32 What is the density of water vapor at 0.3 kPa abs and 30°C?

1.33 A gas with relative molecular mass 28 has a volume of 100 L and a pressure and temperature of 80 kPa abs and 330 K, respectively. What are its specific volume and density?

1.34 One kilogram of hydrogen is confined in a volume of 150 L at −40°C. What is the pressure?

1.35 Express the bulk modulus of elasticity in terms of density change rather than volume change.

1.36 For constant bulk modulus of elasticity, how does the density of a liquid vary with the pressure?

1.37 What is the bulk modulus of a liquid that has a density increase of 0.02 percent for a pressure increase of 0.6 MPa?

1.38 For $K = 2.2$ GPa for bulk modulus of elasticity of water what pressure is required to reduce its volume by 0.5 percent?

1.39 A steel container expands in volume 1 percent when the pressure within it is increased by 70 MPa. At standard pressure, P = 101.3 kPa it holds 450 kg water, $\rho = 1000$ kg/m^3. For $K = 2.06$ GPa when it is filled, how many kilograms mass water need be added to increase the pressure to 70 MPa?

1.40 What is the isothermal bulk modulus for air at 0.4 MPa abs?

1.41 At what pressure can cavitation be expected at the inlet of a pump that is handling water at 20°C?

1.42 What is the pressure within a droplet of water of 0.05 mm diameter at 20°C if the pressure outside the droplet is standard atmospheric pressure of 101.3 kPa?

1.43 A small circular jet of mercury 0.1 mm in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet when at 20°C?

1.44 Determine the capillary rise for distilled water at 40°C in a circular 6 mm diameter glass tube.

1.45 What diameter of glass tube is required if the capillary effects on the water within are not to exceed 0.5 mm?

1.46 Using the data given in Fig. 1.4, estimate the capillary rise of tap water between two parallel glass plates 5 mm apart.

1.47 A method of determining the surface tension of a liquid is to find the force needed to pull a

Lecture Two Solved Problem

Q1-

A reservoir of glycerin (glyc) has a mass of 1200 kg and a volume of 0.952 m³. Find the glycerin's weight (W), mass density (ρ), specific weight (γ), and specific gravity (s.g.).

$$F = W = ma = (1200)(9.81) = 11\,770\text{ N} \quad \text{or} \quad 11.77\text{ kN}$$
$$\rho = m/V = 1200/0.952 = 1261\text{ kg/m}^3$$
$$\gamma = W/V = 11.77/0.952 = 12.36\text{ kN/m}^3$$
$$\text{s.g.} = \gamma_{glyc}/\gamma_{H_2O\text{ at }4^\circ C} = 12.36/9.81 = 1.26$$

Q2-

A large plate moves with speed v_0 over a stationary plate on a layer of oil (see Fig. 1-5). If the velocity profile is that of a parabola, with the oil at the plates having the same velocity as the plates, what is the shear stress on the moving plate from the oil? If a linear profile is assumed, what is the shear stress on the upper plate?

For a parabolic profile, $v^2 = ay$. When $y = d$, $v = v_0$. Hence, $v_0^2 = ad$, $a = v_0^2/d$. Therefore,

$$v^2 = (v_0^2/d)(y) = (v_0^2)(y/d) \qquad v = v_0\sqrt{y/d} \qquad dv/dy = [(v_0)(1/\sqrt{d})(\tfrac{1}{2})(y^{-1/2})]$$
$$\tau = \mu\,(dv/dy) = \mu[(v_0)(1/\sqrt{d})(\tfrac{1}{2})(y^{-1/2})]$$

For $y = d$, $\tau = \mu[(v_0)(1/\sqrt{d})(\tfrac{1}{2})(d^{-1/2})] = \mu v_0/(2d)$. For a linear profile, $dv/dy = v_0/d$, $\tau = \mu(v_0/d)$.

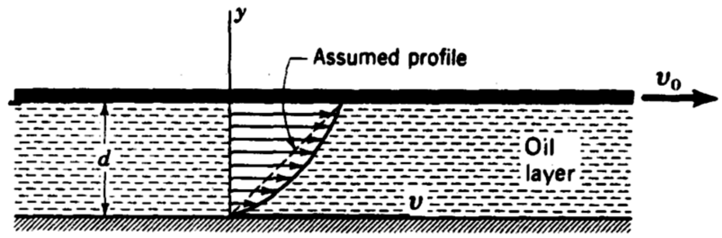


Fig. 1-5

Q2-

A square block weighing 1.1 kN and 250 mm on an edge slides down an incline on a film of oil 6.0 μm thick (see Fig. 1-6). Assuming a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7 mPa · s.

$$\tau = \mu\,(dv/dy) = (7\times10^{-3})[v_T/(6.0\times10^{-6})] = 1167v_T \qquad F_f = \tau A = (1167v_T)(0.250)^2 = 72.9v_T$$

At the terminal condition, equilibrium occurs. Hence, $1100\sin 20^\circ = 72.9v_T$, $v_T = 5.16$ m/s.

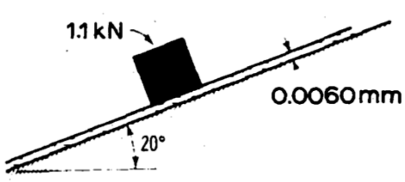


Fig. 1-6(a)

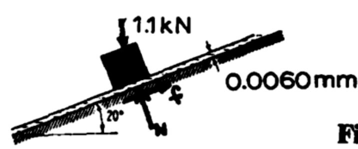


Fig. 1-6(b)

Q3-

A piston of weight 21 lb slides in a lubricated pipe, as shown in Fig. 1-7. The clearance between piston and pipe is 0.001 in. If the piston decelerates at 2.1 ft/s² when the speed is 21 ft/s, what is the viscosity of the oil?

$$\tau = \mu\,(dv/dy) = \mu[v/(0.001/12)] = 12\,000\mu v$$

$$F_f = \tau A = 12\,000\mu v[(\pi)(\tfrac{6}{12})(\tfrac{5}{12})] = 7854\mu v$$

$$\Sigma F = ma \qquad 21 - (7854)(\mu)(21) = (21/32.2)(-2.1) \qquad \mu = 1.36 \times 10^{-4}\ \text{lb}\cdot\text{s/ft}^2$$

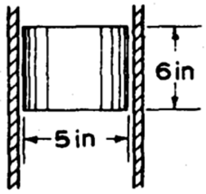


Fig. 1-7

Q4-

An 18-kg slab slides down a 15° inclined plane on a 3-mm-thick film of SAE 10 oil at 20 °C; the contact area is 0.3 m². Find the terminal velocity of the slab.

See Fig. 1-13.

$$\Sigma F_x = 0 \qquad W \sin\theta - \tau A_{\text{bottom}} = 0$$

$$\tau = \mu\,(dv/dy) = (8.14 \times 10^{-2})(v_T/0.003) = 27.1v_T$$

$$[(18)(9.81)](\sin 15°) - (27.1v_T)(0.3) = 0 \qquad v_T = 5.62\ \text{m/s}$$

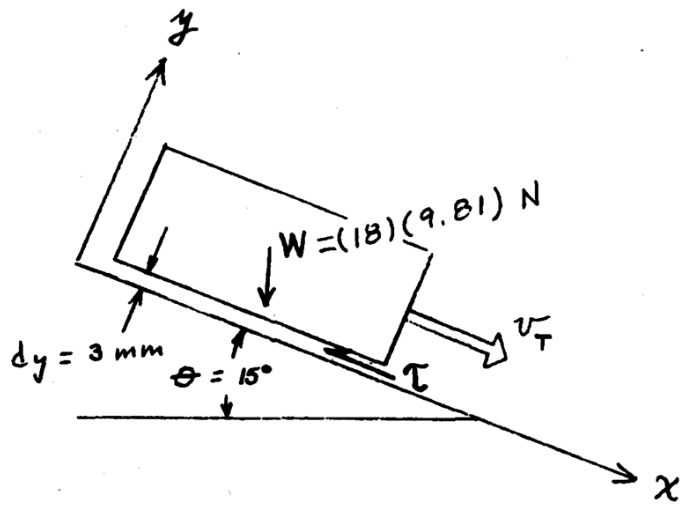


Fig. 1-13

Q5-

A shaft 70.0 mm in diameter is being pushed at a speed of 400 mm/s through a bearing sleeve 70.2 mm in diameter and 250 mm long. The clearance, assumed uniform, is filled with oil at 20 °C with $\nu = 0.005\ m^2/s$ and s.g. = 0.9. Find the force exerted by the oil on the shaft.

$$F = \tau A \qquad \tau = \mu\,(dv/dr) \qquad \mu = \rho\nu = [(0.9)(998)](0.005) = 4.49\ \text{kg/(m}\cdot\text{s)}$$

$$dr = (0.0702 - 0.0700)/2 = 0.0001\ \text{m} \qquad \tau = (4.49)(0.4/0.0001) = 17\,960\ \text{N/m}^2$$

$$A = (\pi)(7.00/100)(25/100) = 0.05498\ \text{m}^2 \qquad F = (17\,960)(0.05498) = 987\ \text{N}$$

Q6-

If the shaft in Prol Q5 is fixed axially and rotated inside the sleeve at 2000 rpm, determine the resisting torque exerted by the oil and the power required to rotate the shaft.

$$T = \tau A r \qquad \tau = \mu\,(dv/dr)$$

$$v = r\omega = [(7.00/2)/100][(2000)(2\pi/60)] = 7.330\ \text{m/s} \qquad dr = 0.0001\ \text{m}$$

$$\tau = (4.49)(7.330/0.0001) = 329.1\times 10^3\ \text{N/m}^2 \qquad A = (\pi)(7.00/100)(\tfrac{25}{100}) = 0.05498\ \text{m}^2$$

$$T = (329.1\times 10^3)(0.05498)[(7.00/2)/100] = 633\ \text{N}\cdot\text{m}$$

$$P = \omega T = [(2000)(2\pi/60)](633) = 132.6\times 10^3\ \text{W} \quad \text{or} \quad 132.6\ \text{kW}$$

Q7-

Air at 20 °C forms a boundary layer near a solid wall, in which the velocity profile is sinusoidal (see Fig. 1-14). The boundary-layer thickness is 7 mm and the peak velocity is 9 m/s. Compute the shear stress in the boundary layer at y equal to (a) 0, (b) 3.5 mm, and (c) 7 mm.

$$\tau = \mu\,(dv/dy) \qquad v = v_{max}\sin[\pi y/(2\delta)]$$

$$dv/dy = [\pi v_{max}/(2\delta)]\cos[\pi y/(2\delta)] = \{(\pi)(9)/[(2)(0.007)]\}\cos\{\pi y/[(2)(0.007)]\} = 2020\cos(224.4y)$$

Note: "$224.4y$" in the above equation is in radians.

$$\tau = (1.81\times 10^{-5})[2020\cos(224.4y)] = 0.03656\cos(224.4y)$$

(a) At $y = 0$, $\tau = 0.03656\cos[(224.4)(0)] = 0.0366$ Pa. **(b)** At $y = 0.0035$ m, $\tau = 0.03656\cos[(224.4)(0.0035)] = 0.0259$ Pa. **(c)** At $y = 0.007$ m, $\tau = 0.03656\cos[(224.4)(0.007)] = 0$.

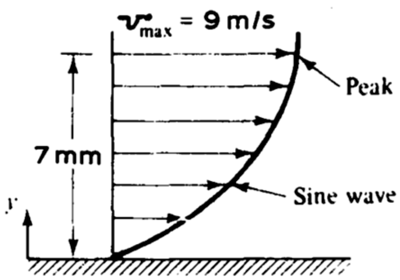


Fig. 1-14

Q9-

A disk of radius r_0 rotates at angular velocity ω inside an oil bath of viscosity μ, as shown in Fig. 1-15. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

$$\tau = \mu\,(dv/dy) = \mu(r\omega/h) \qquad \text{(on both sides)}$$

$$dT = (2)(r\tau\,dA) = (2)\{(r)[\mu(r\omega/h)](2\pi r\,dr)\} = (4\mu\omega\pi/h)(r^3\,dr)$$

$$T = \int_0^{r_0} \frac{4\mu\omega\pi}{h}(r^3\,dr) = \frac{4\mu\omega\pi}{h}\left[\frac{r^4}{4}\right]_0^{r_0} = \frac{\pi\mu\omega r_0^4}{h}$$

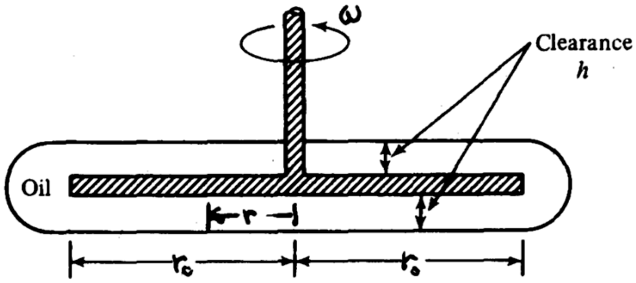


Fig. 1-15

Q10-

In Fig. 1-17*a*, oil of viscosity μ fills the small gap of thickness Y. Determine an expression for the torque T required to rotate the truncated cone at constant speed ω. Neglect fluid stress exerted on the circular bottom.

▌ See Fig. 1-17*b*. $\tau = \mu\,(dv/dy)$, $v = r\omega = (y \tan\alpha)(\omega)$, $dv/dy = (y\tan\alpha)(\omega)/Y$.

$$\tau = \mu\left[\frac{(y\tan\alpha)(\omega)}{Y}\right] = \frac{\mu y\omega\tan\alpha}{Y}$$

$$dA = 2\pi r\,ds = 2\pi(y\tan\alpha)(dy/\cos\alpha) = 2\pi y(\tan\alpha/\cos\alpha)(dy)$$

$$dF = \tau\,dA = \left(\frac{\mu y\omega\tan\alpha}{Y}\right)\left[2\pi y\left(\frac{\tan\alpha}{\cos\alpha}\right)(dy)\right] = \left(\frac{2\pi\mu\omega\tan^2\alpha}{Y\cos\alpha}\right)y^2\,dy$$

$$dT = r\,dF = (y\tan\alpha)\left(\frac{2\pi\mu\omega\tan^2\alpha}{Y\cos\alpha}\right)y^2\,dy = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)y^3\,dy$$

$$T = \int_a^{a+b}\left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)y^3\,dy = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)\left[\frac{y^4}{4}\right]_a^{a+b} = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)\left[\frac{(a+b)^4}{4} - \frac{a^4}{4}\right]$$

$$= \left(\frac{\pi\mu\omega\tan^3\alpha}{2Y\cos\alpha}\right)[(a+b)^4 - a^4]$$

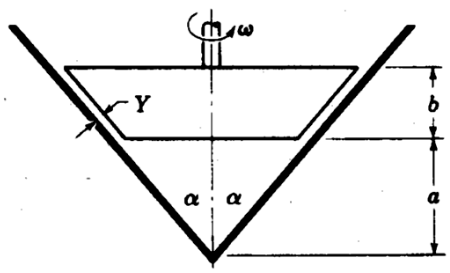


Fig. 1-17(*a*)

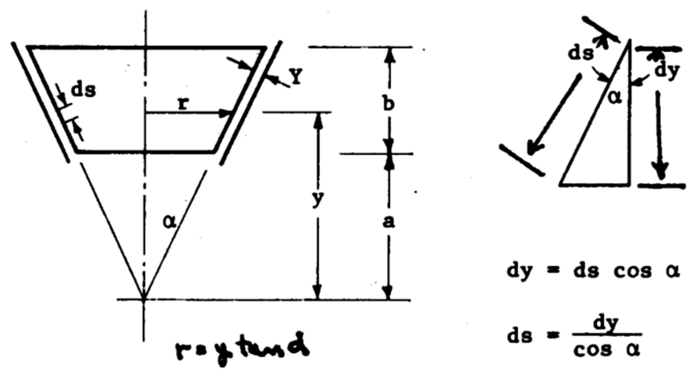


Fig. 1-17(*b*)

Q11-

A liquid compressed in a cylinder has a volume of 1000 cm³ at 1 MN/m² and a volume of 995 cm³ at 2 MN/m². What is its bulk modulus of elasticity (K)?

▌

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{2-1}{(995-1000)/1000} = 200\text{ MPa}$$

Q12-

A heavy tank contains oil (A) and water (B) subject to variable air pressure; the dimensions shown in Fig. 1-2 correspond to 1 atm. If air is slowly added from a pump to bring pressure p up to 1 MPa gage, what will be the total downward movement of the free surface of oil and air? Take average values of bulk moduli of elasticity of the liquids as 2050 MPa for oil and 2075 MPa for water. Assume the container does not change volume. Neglect hydrostatic pressures.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2050 = -\frac{1-0}{\Delta V_{oil}/[600\pi(300)^2/4]} \qquad \Delta V_{oil} = -20\,690\ \text{mm}^3$$

$$2075 = -\frac{1-0}{\Delta V_{H_2O}/[700\pi(300)^2/4]} \qquad \Delta V_{H_2O} = -23\,850\ \text{mm}^3$$

$$\Delta V_{total} = -44\,540\ \text{mm}^3$$

Let x = distance the upper free surface moves. $-44\,540 = -[\pi(300)^2/4]x$, $x = 0.630$ mm.

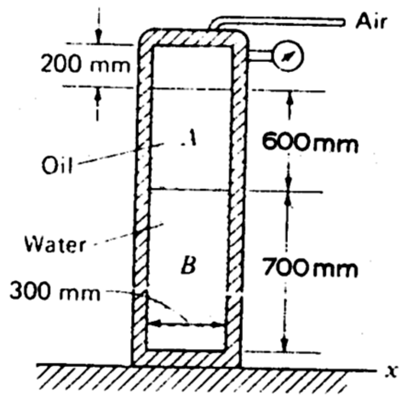


Fig. 1-2

Q13-

The tank of a leaky air compressor originally holds 90 L of air at 33 °C and 225 kPa. During a compression process, 4 grams of air is lost; the remaining air occupies 42 L at 550 kPa. What is the temperature of the remaining air?

$\rho_1 = p_1/RT_1 = (225 \times 10^3)/[(287)(33 + 273)] = 2.562\ \text{kg/m}^3 \qquad m = (2.562)(0.090) = 0.2306\ \text{kg}$

$\rho_2 = p_2/RT_2 \qquad (0.2306 - 0.004)/0.042 = (550 \times 10^3)/(287T_2) \qquad T_2 = 355\ \text{K}$ or $82\ °\text{C}$

Q14-

Two clean, parallel glass plates, separated by a distance $d = 1.5$ mm, are dipped in a bath of water. How far does the water rise due to capillary action, if $\sigma = 0.0730$ N/m?

Because the plates are clean, the angle of contact between water and glass is taken as zero. Consider the free-body diagram of a unit width of the raised water (Fig. 1-19). Summing forces in the vertical direction gives $(2)[(\sigma)(0.0015)] - (0.0015)^2(h)(\gamma) = 0$, $(2)[(0.0730)(0.0015)] - (0.0015)^2(h)(9790) = 0$, $h = 0.00994$ m, or 9.94 mm.

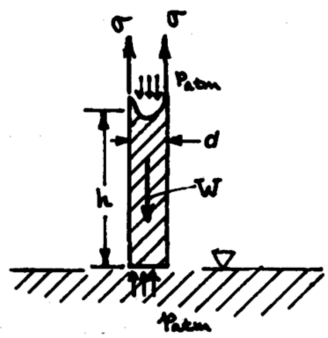


Fig. 1-19(c)

Q15-

A glass tube is inserted in mercury (Fig. 1-20); the common temperature is 20 °C. What is the upward force on the glass as a result of surface effects?

$F = (\sigma)(\pi d_o)(\cos 50°) + (\alpha)(\pi d_i)(\cos 50°) = (0.514)[(\pi)(0.035)](\cos 50°) + (0.514)[(\pi)(0.025)](\cos 50°) = 0.0623$ N

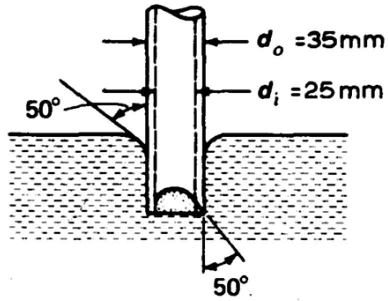


Fig. 1-20

Q16-

Find the capillary rise in the tube shown in Fig. 1-26 for a water–air–glass interface ($\theta = 0°$) if the tube radius is 1 mm and the temperature is 20 °C.

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.0728)(\cos 0°)}{(1000)(9.81)(\frac{1}{1000})} = 0.0148 \text{ m} \quad \text{or} \quad 14.8 \text{ mm}$$

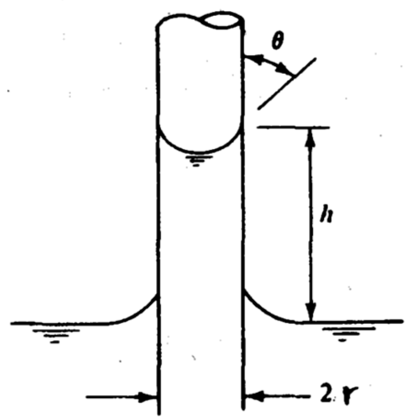


Fig. 1-26

Q17-

Find the capillary rise in the tube shown in Fig. 1-26 for a mercury–air–glass interface with $\theta = 130°$ if the tube radius is 1 mm and the temperature is 20 °C.

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.514)(\cos 130°)}{(13\,570)(9.81)(\frac{1}{1000})} = -0.0050 \text{ m} \quad \text{or} \quad -5.0 \text{ mm}$$

Q18-

If a bubble is equivalent to an air–water interface with $\sigma = 0.005$ lb/ft, what is the pressure difference between the inside and outside of a bubble of diameter 0.003 in?

$$p = 2\sigma/r = (2)(0.005)/[(0.003/2)/12] = 80.0 \text{ lb/ft}^2$$

Q19-

The surface tensions of mercury and water at 60 °C are 0.47 N/m and 0.0662 N/m, respectively. What capillary-height changes will occur in these two fluids when they are in contact with air in a glass tube of radius 0.30 mm? Use $\theta = 130°$ for mercury, and 0° for water; $\gamma = 132.3\ kN/m^3$ for mercury, and $9.650\ kN/m^3$ for water.

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

For mercury:

$$h = \frac{(2)(0.47)(\cos 130°)}{(132\,300)(0.30/1000)} = -0.0152\ \text{m} \quad \text{or} \quad -15.2\ \text{mm}$$

For water:

$$h = \frac{(2)(0.0662)(\cos 0°)}{(9650)(0.30/1000)} = 0.0457\ \text{m} \quad \text{or} \quad 45.7\ \text{mm}$$

Q20-

The glass tube in Fig. 1-28 is used to measure pressure p_1 in the water tank. The tube diameter is 1 mm and the water is at 30 °C. After correcting for surface tension, what is the true water height in the tube?

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.0712)(\cos 0°)}{(996)(9.81)[(\frac{1}{2})/1000]} = 0.029\ \text{m} \quad \text{or} \quad 2.9\ \text{cm}$$

True water height in the tube = 17 − 2.9 = 14.1 cm.

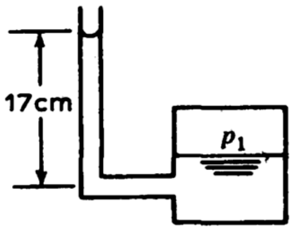


Fig. 1-28

Specific weight γ of pure water as a function of temperature and pressure for condition where $g = 32.2\ ft/s^2$ $(9.81\ m/s^2)$.

Lecture Three Fluid Static

Fluid Static

Pressure*: is defined as a normal force exerted by a fluid per unit area. It has the unit of Newton per square meter (N/m2), which is called a Pascal (Pa).*

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$1 \text{ kgf/cm}^2 = 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa}$$

$$= 0.9807 \text{ bar}$$

$$= 0.9679 \text{ atm}$$

The actual pressure at a given position is called the **absolute pressure,** and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, **are calibrated to read zero in the atmosphere** Figure (1).

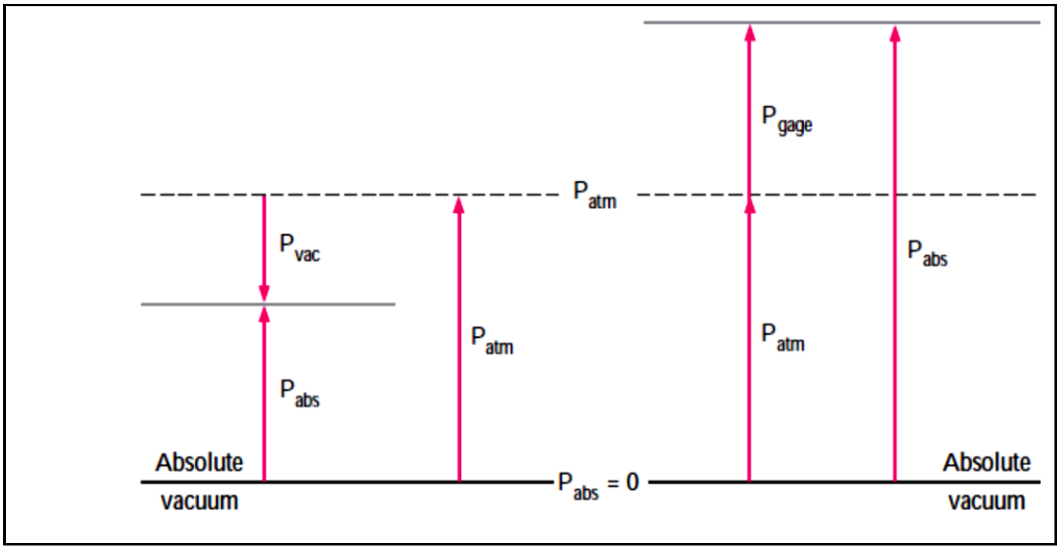
Figure (1): Bourdon pressure gage.

And so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the **gage pressure.** ***Pressures below atmospheric pressure are called vacuum pressures and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the***

absolute pressure. Absolute, gage, and vacuum pressures are all positive quantities and are related to each other by

$P_{abs} = P_{gage} + P_{atm}$ Eqn. (1)

$P_{abs} = P_{atm} - P_{vac}$ Eqn. (2)



Pressure at a Point:

Pressure is the *compressive force* per unit area, and it gives the impression of being a vector. However, ***pressure at any point in a fluid is the same <u>in all directions</u>. That is, it has magnitude but not a specific direction,*** and thus it is a scalar quantity. This can be demonstrated by considering a small wedge-shaped fluid element of unit length (into the page) in equilibrium, as shown in Figure (3). The mean pressures at the three surfaces are *P*1, *P*2, and *P*3, and the force acting on a surface is the product of mean pressure and the surface area. From Newton's second law, a force balance in the *x*- and *z*-directions gives

$$\sum F_x = ma_x = 0: \qquad P_1 \Delta z - P_3 l \sin\theta = 0 \qquad \text{Eqn. (3)}$$

$$\sum F_z = ma_z = 0: \qquad P_2 \Delta x - P_3 l \cos\theta - \frac{1}{2}\rho g \, \Delta x \, \Delta z = 0 \qquad \text{Eqn. (4)}$$

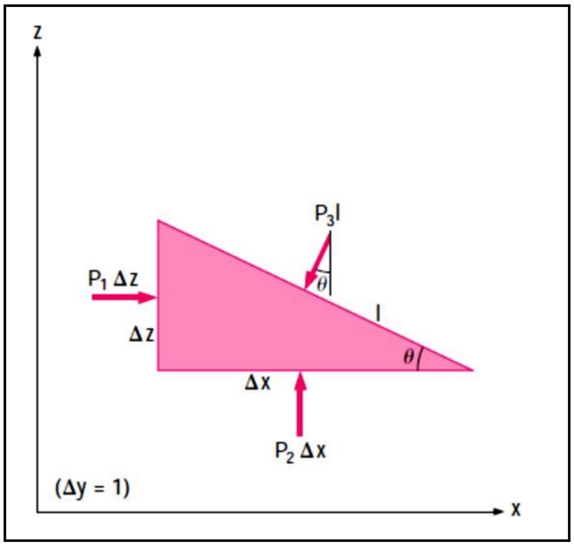


Figure (3): Forces acting on a wedge-shaped fluid element in equilibrium.

Where **ρ** is the density and $W = mg = \rho g\ .\Delta x.\ \Delta z\ /2$ is the weight of the fluid element. Noting that the wedge is a right triangle, we have $\Delta x = l \cos\theta$ and $\Delta z = l \sin\theta$. Substituting these geometric relations and dividing Eq. 3 by Δz and Eq. 4 by Δ*x* gives:

$$P_1 - P_3 = 0 \qquad \text{Eqn. (5)}$$

$$P_2 - P_3 - \frac{1}{2}\rho g\ \Delta z = 0 \qquad \text{Eqn. (6)}$$

The last term in Eqn. (6) drops out as $\Delta z \rightarrow 0$ and the wedge becomes infinitesimal, and thus the fluid element shrinks to a point. Then combining the results of these two relations gives:

$$P_1 = P_2 = P_3 = 0 \qquad \text{Eqn. (7)}$$

At a particular point, **P** has the following properties:

1. It is same in all directions.

2. It is ⊥ to any surface of the object.

Variation of Pressure with Depth:

It will come as no surprise to you that pressure in a fluid at rest does not change in the horizontal direction. This can be shown easily by considering a thin horizontal layer of fluid and doing a force balance in any horizontal direction. However, this is not the case in the vertical direction in a gravity field. Pressure in a fluid increases with depth because more fluid rests on deeper layers, and the effect of this "extra weight" on a deeper layer is balanced by an increase in pressure figure (4).

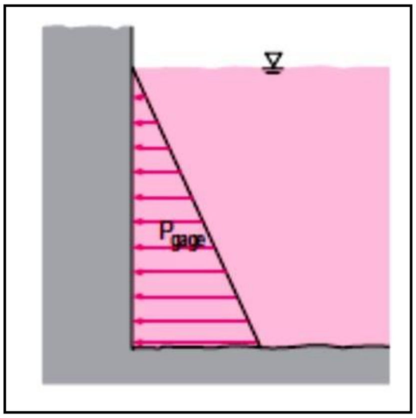


Figure (4): The pressure of a fluid at rest increases with depth (as a result of added weight).

To obtain a relation for the variation of pressure with depth, consider a rectangular fluid element of height Δz, length Δ*x*, and unit depth (into the page) in equilibrium, as shown in Figure (5). Assuming the density of the fluid r to be constant, a force balance in the vertical *z*-direction gives:

$$\sum F_z = ma_z = 0: \quad P_2\,\Delta x - P_1\,\Delta x - \rho\, g\, \Delta x\, \Delta z = 0 \qquad \text{Eqn. (8).}$$

Where $W = mg = g\ \Delta x\ \Delta z$ is the weight of the fluid element. Dividing by Δ*x* and rearranging gives:

$\Delta P = P_2 - P_1 = \boldsymbol{\rho} g\ \Delta z = \boldsymbol{\gamma}_s\ .\Delta z$ Eqn. (9).

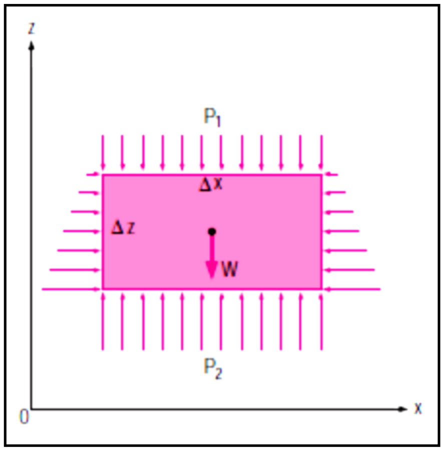


Figure (5): Free-body diagram of a rectangular fluid element in equilibrium

Where $\boldsymbol{\gamma}_s = \boldsymbol{\rho} g$ is the *specific weight* of the fluid. Thus, we conclude that the pressure difference between two points in a constant density fluid is proportional to the vertical distance Δz between the points and the density $\boldsymbol{\rho}$ of the fluid.

The gravitational acceleration g varies from 9.807 m/s^2 at sea level to 9.764 m/s^2 at an elevation of 14000 m where large passenger planes cruise. This is a change of just 0.4 percent in this extreme case. Therefore, g can be assumed to be constant with negligible error. For fluids whose density changes significantly with elevation, a relation for the variation of pressure with elevation can be obtained by dividing Eqn. (8) by $\Delta x\ \Delta z$, and taking the limit as $\Delta z \rightarrow 0$. It gives:

$$\frac{\partial p}{\partial z} = -\rho g \qquad \text{Eqn. (10)}$$

The negative sign is due to our taking the positive z direction to be upward so that dP is negative when dz is positive since pressure decreases in an upward direction. When the variation of density with elevation is known the pressure difference between points 1 and 2 can be determined by integration to be:

$$\Delta p = P_2 - P_1 = -\int \rho g dz \qquad \text{Eqn. (11)}$$

If we take point 1 to be at the free surface of a liquid open to the atmosphere figure (6), where the pressure is the atmospheric pressure Patm, then the pressure at a depth h from the free surface becomes:

$$P = P_{atm} + \boldsymbol{\rho} gh \qquad \text{Eqn. (12)}$$

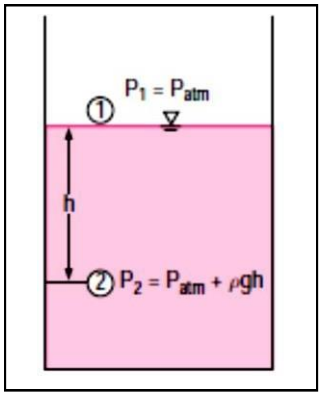


Figure (6): Pressure in a liquid at rest increases linearly with distance from the free surface.

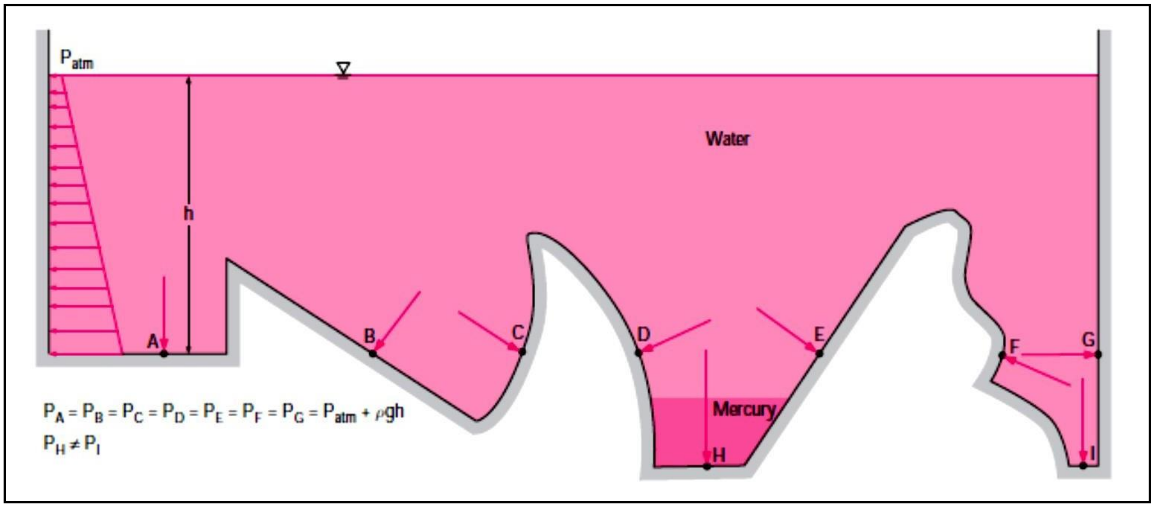


Figure (7): The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

<u>Pascal's Law:</u> Named for French scientist Blaise Pascal

- ***A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container***
- Car lift in a service station. See figure. A large output force can be applied by means of a small input force. Volume of liquid pushed down on left must equal volume pushed up on right.

Example (1): Circular cross section system. On left r_1 = 5 cm = 0.05 m. On right r_2 = 15 cm = 0.15 m. Car's weight mg = 13,300 N.

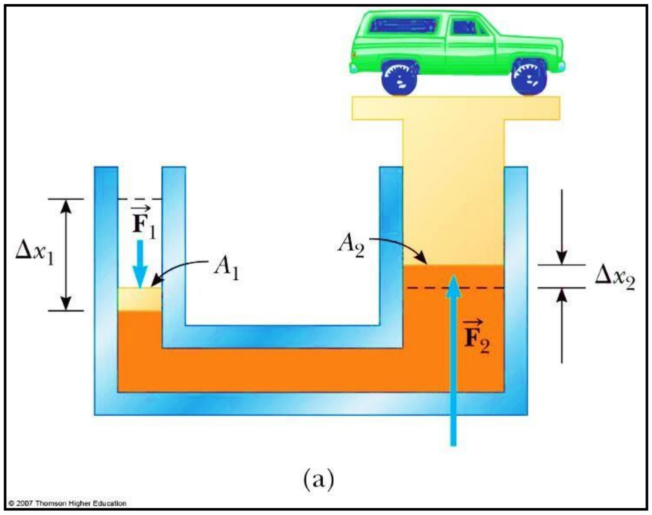
Newtonians 2nd Law on right: $\sum F_y = 0 = F_2 - mg$. Or F_2 = 13,300 N. Calculate minimum F_1 to lift the car & pressure P in the system.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\mathbf{F_1 = (A_1/A_2)F_2 = 1480 \text{ N}}$$

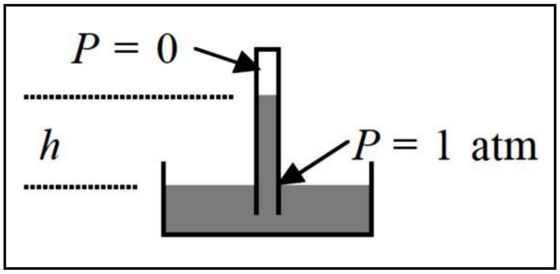
$$\mathbf{P = (F_1/A_1) = 1.88 \times 10^5 \text{ Pa}}$$



Measurement of pressure:

Atmospheric pressure measurements:

Pressure is measured using the two principles discussed above, that $P = \rho gh$, and Pascal's Principle. If a fluid, like mercury or water is put in a container which is open to the atmosphere at one end and closed at the other, with the closed end having zero pressure (it is a vacuum), then the following situation occurs.



E. Torricelli (1608)

$$P_{atm} = \rho gh \;\Rightarrow\; h = \frac{P_{atm}}{\rho g} \qquad \text{Eqn. (13)}$$

The pressure at any height is equal, so the pressure of the atmosphere, just equals the pressure of the liquid or ρgh. For different liquids with different densities, the height of the column at sea level will be different. For mercury it is 760 mm. For water, it is 10.3 m.

- **Gases are compressible. Thus, ρ varies!**

$$p_0 = p_{\text{sea level}} = 1.013\times10^5\,Pa$$

$$pV = nRT \;\Rightarrow\; \frac{p}{p_0} = \frac{\rho}{\rho_0}, \quad \text{for constant } T$$

$$\frac{dp}{dy} = -\rho g = -\rho_0 g\frac{p}{p_0}$$

$$\frac{dp}{dy} = -\rho_0 g\frac{p}{p_0} \;\Rightarrow\; \int_{p_0}^{p}\frac{dp'}{p'} = -\frac{\rho_0 g}{p_0}\int_{y_0}^{y} dy'$$

$$\ln p - \ln p_0 = -\frac{\rho_0 g}{p_0}(y - y_0)$$

$$p = p_0 e^{-(\rho_0 g/p_0)h} \xrightarrow{h\to 0} p = p_0 - \rho_0 gh$$

Where y = z = h= point elevation started from sea level.

Quantity	SI units	EE units	BG units
Density	kg/m^3	lb_m/ft^3	$slug/ft^3$
Pressure & shear stress	kPa = kN/m^2	1 psi = 1 lb_f/in^2 = 144 psf = 144 lb_f/ft^2	
Velocity	m/s	ft/s	
Viscosity	$N·s/m^2$ = $kg/m·s$	$lb_f·s/ft^2$ = 32.2 $lb_m/ft·s$	$lb_f·s/ft^2$ = $slug/ft·s$
Specific weight = ρg	N/m^3	lb_f/ft^3	
	Tabulated values at standard gravity		

<u>The Manometer:</u>

U-Tube Manometer: A manometer mainly consists of a glass or plastic U-tube containing one or more fluids such as mercury, water, alcohol, or oil. To keep the size of the manometer to a manageable level, heavy fluids such as mercury are used if large pressure differences are anticipated.

Consider the manometer shown in Figure (8) that is used to measure the pressure in the tank.

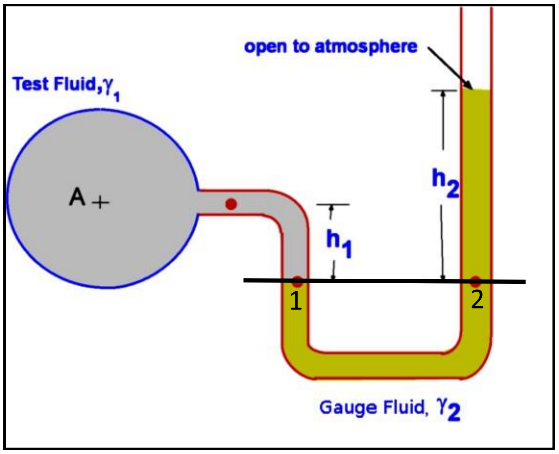


Figure (8) : U- Tube Manometer

Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value. Furthermore, since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, $P_2 = P_1$. The differential fluid column of height h is in static equilibrium, and it is open to the atmosphere. Then the pressure at point A is determined directly from Equation (14):

$$p_A + \rho_1 g h_1 = p_{atm} + \rho_2 g h_2 \qquad \text{Eqn. (14)}$$

Example (2):

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig 9. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h_1 = 0.1 m, h_2 = 0.2 m, and h_3 = 0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

SOLUTION The pressure in a pressurized water tank is measured by a multifluid manometer. The air pressure in the tank is to be determined.
Assumption The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air–water interface.
Properties The densities of water, oil, and mercury are given to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.
Analysis Starting with the pressure at point 1 at the air–water interface, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$$P_1 + \rho_{water} g h_1 + \rho_{oil} g h_2 - \rho_{mercury} g h_3 = P_{atm}$$

Solving for P_1 and substituting,

$$\begin{aligned} P_1 &= P_{atm} - \rho_{water} g h_1 - \rho_{oil} g h_2 + \rho_{mercury} g h_3 \\ &= P_{atm} + g(\rho_{mercury} h_3 - \rho_{water} h_1 - \rho_{oil} h_2) \\ &= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (850 \text{ kg/m}^3)(0.2 \text{ m})]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= 130 \text{ kPa} \end{aligned}$$

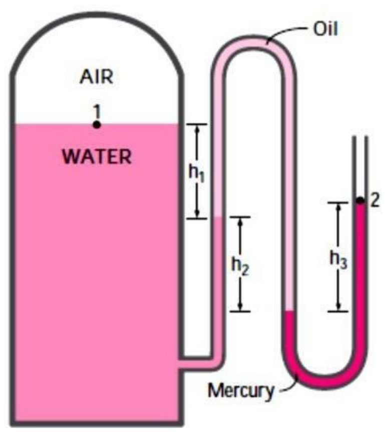


Figure (9):

Bourdon gage: named after the French engineer and inventor Eugene Bourdon (1808–1884), which consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle figure (10). When the tube is open to the atmosphere, the tube is undeflected, and the needle on the dial at this state is calibrated to read zero (gage pressure). When the fluid inside the tube is pressurized, the tube stretches and moves the needle in proportion to the pressure applied.

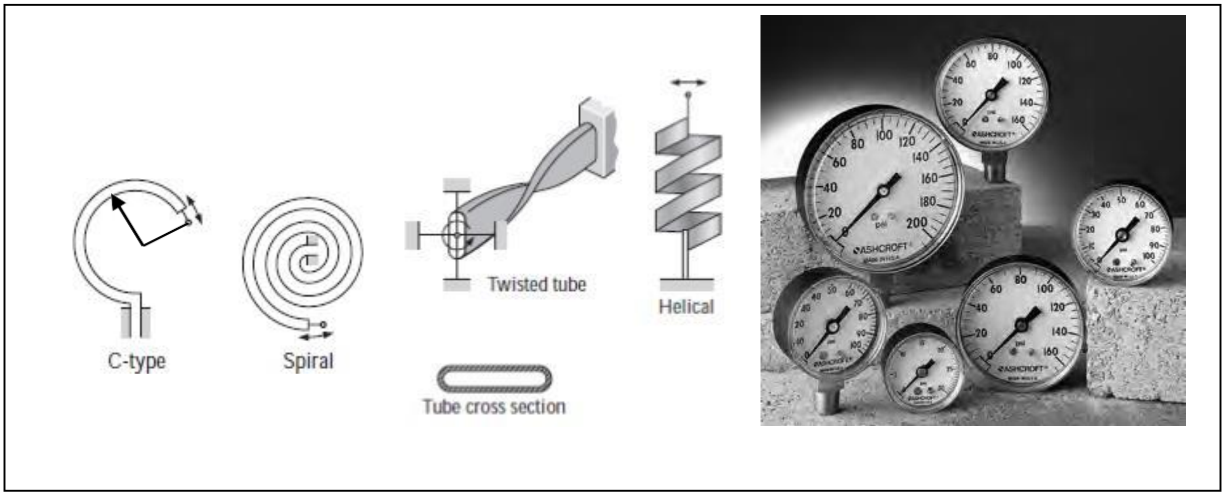


Figure (10): Various types of Bourdon tubes used to measure pressure.

Many engineering problems and some manometers involve multiple immiscible fluids of different densities stacked on top of each other. Such systems can be analyzed easily by remembering that (1) the pressure change across a fluid column of height *h* is $\Delta P = \boldsymbol{\rho} gh$, (2) pressure increases downward in a given fluid and decreases upward (i.e., $P_{bottom} > P_{top}$), and (3) two points at the same elevation in a continuous fluid at rest are at the same pressure.

The last principle, which is a result of ***Pascal's law***, allows us to "jump" from one fluid column to the next in manometers without worrying about pressure change as long as we don't jump over a different fluid, and the fluid is at rest. Then the pressure at any point can be determined by starting with a point of known pressure and adding or subtracting $\boldsymbol{\rho} gh$ terms as we advance toward the point of interest. For example, the pressure at the bottom of the tank in Figure (11) can be determined by starting at the free surface where the pressure is P_{atm}, moving downward until we reach point 1 at the bottom, and setting the result equal to $P1$. It gives

$P_{atm} + \boldsymbol{\rho}_1 gh_1 + \boldsymbol{\rho}_2 gh_2 + \boldsymbol{\rho}_3 gh_3 = P_1$

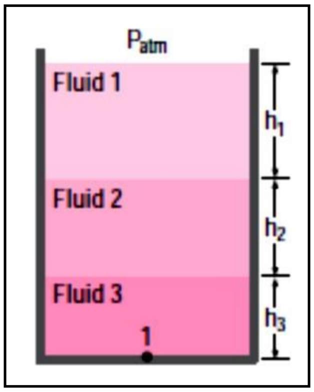


Figure (11):

HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES:

A plate exposed to a liquid, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface Figure (12).

Figure (12): Hoover Dome.

On a *plane* surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the *magnitude* of the force and its *point of application*, which is called the **center of pressure.** In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant. In such cases, it is convenient to subtract atmospheric pressure and work with the gage pressure only Figure (13). For example, $P_{gage} = \boldsymbol{\rho} gh$ at the bottom of the lake.

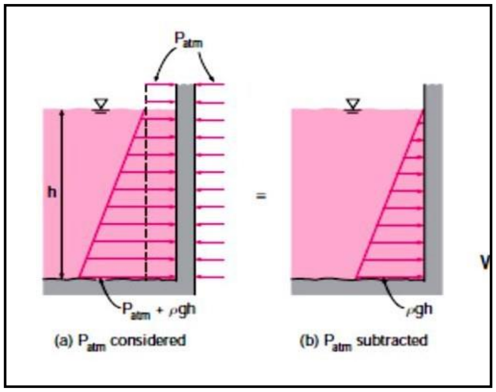


Figure (13)

Consider the top surface of a flat plate of arbitrary shape completely submerged in a liquid, as shown in Figure (14) together with its top view.

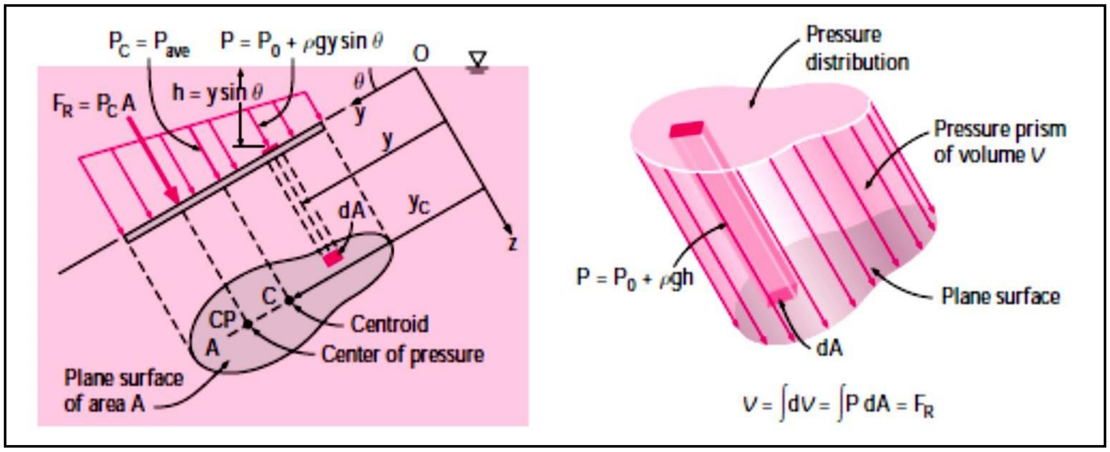


Figure (14): Hydrostatic force on an inclined plane surface completely submerged in a liquid.

The plane of this surface (normal to the page) intersects the horizontal free surface with an angle θ, and we take the line of intersection to be the *x*-axis. The absolute pressure above the liquid is P_0, which is the local atmospheric pressure *P*atm if the liquid is open to the atmosphere (but P_0 may be different than *P*atm if the space above the liquid is evacuated or pressurized). Then the absolute pressure at any point on the plate is

$P = P_o + \boldsymbol{\rho} gh = P_o + \boldsymbol{\rho} g\ y.\sin\theta$

Where *h* is the vertical distance of the point from the free surface and *y* is the distance of the point from the *x*-axis from point *O* in Figure (14). The resultant hydrostatic force F_R acting on the surface is determined by integrating the force (*P dA*) acting on a differential area *dA* over the entire surface area,

$$F_R = \int_A PdA = \int_A (P_o + \rho gy \sin\theta)dA = P_o A + \rho g\, sin\, \theta \int_A ydA$$

But the *first moment of area* $\int_A ydA$ is related to the *y*-coordinate of the centroid (or center) of the surface by

$$y_c = \frac{1}{A}\int_A y dA$$

Substituting:

$$F_R = (P_o + \rho g y_c \sin\theta)A = (P_o + \rho g h_c)A = P_c A = P_{ave} A$$

Where $P_C = P_0 + \rho g h_C$ is the pressure at the centroid of the surface, which is equivalent to the *average* pressure on the surface, and $h_C = y_C$ sin u is the *vertical distance* of the centroid from the free surface of the liquid Figure (15). Thus we conclude that:

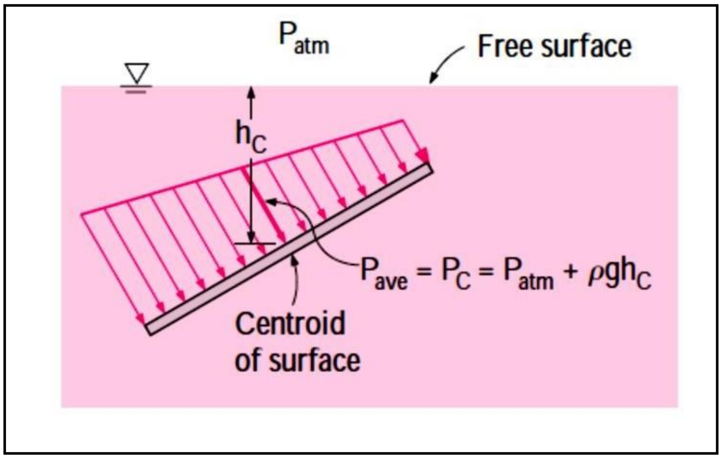


Figure (15): The pressure at the centroid of a surface is equivalent to the *average* pressure on the surface.

The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface (Figure (16).

The pressure P_0 is usually atmospheric pressure, which can be ignored in most cases since it acts on both sides of the plate. When this is not the case, a practical way of accounting for the contribution of P_0 to the resultant force is simply to add an equivalent depth $h_{equiv} = P_0 / \rho g$ to h_C ; that is, to assume the presence of an additional liquid layer of thickness h_{equiv} on top of the liquid with absolute vacuum above.

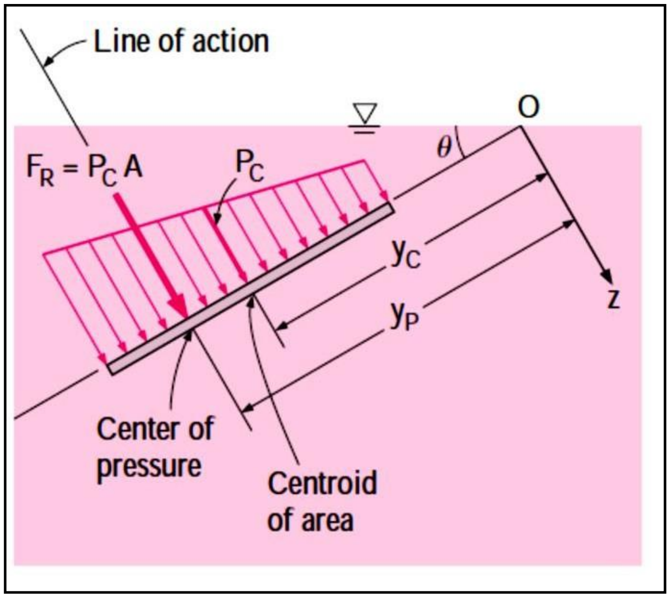


Figure (16): The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure.

Next we need to determine the line of action of the resultant force F_R. Two parallel force systems are equivalent if they have the same magnitude and the same moment about any point. The line of action of the resultant hydrostatic force, in general, does not pass through the centroid of the surface— it lies underneath where the pressure is higher. The point of intersection of the line of action of the resultant force and the surface is the **center of pressure.** The vertical location of the line of action is determined by equating the moment of the resultant force to the moment of the distributed pressure force about the *x*-axis. It gives

$$y_P F_R = \int_A yP\, dA = \int_A y(P_0 + \rho g y \sin\theta)\, dA = P_0 \int_A y\, dA + \rho g \sin\theta \int_A y^2\, dA$$

Or

$$y_P F_R = P_0 y_C A + \rho g \sin\theta \, I_{xx,O}$$

Where y_P is the distance of the center of pressure from the x-axis point O in Figure (16), and

$$I_{XX,o} = \int_A y^2 dA$$

is the *second moment of area* (also called the *area moment of inertia*) about the x-axis. The second moments of area are widely available for common shapes in engineering handbooks, but they are usually given about the axes passing through the centroid of the area. Fortunately, the second moments of area about two parallel axes are related to each other by the *parallel axis theorem,* which in this case is expressed as

$$I_{XX,o} = I_{XX,c} + y_c^2 A$$

where $I_{xx,C}$ is the second moment of area about the x-axis passing through the centroid of the area and y_C (the y-coordinate of the centroid) is the distance between the two parallel axes. Substituting the F_R relation from Eq. 3–19 and the $I_{xx,O}$ relation from Eq. 3–21 into Eq. 3–20 and solving for y_P gives

$$y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

For $P_0 = 0$, which is usually the case when the atmospheric pressure is ignored, it simplifies to

$$y_P = y_C + \frac{I_{xx,C}}{y_C A}$$

Knowing y_P, the vertical distance of the center of pressure from the free surface is determined from $h_P = y_P \sin\theta$. The $I_{xx,C}$ values for some common areas are given in Figure (17). For these and other areas that possess symmetry about the y-axis, the center of pressure lies on the *y*-axis directly below the centroid. The location of the center of pressure in such cases is simply the point on the surface of the vertical plane of symmetry at a distance h_P from the free surface.

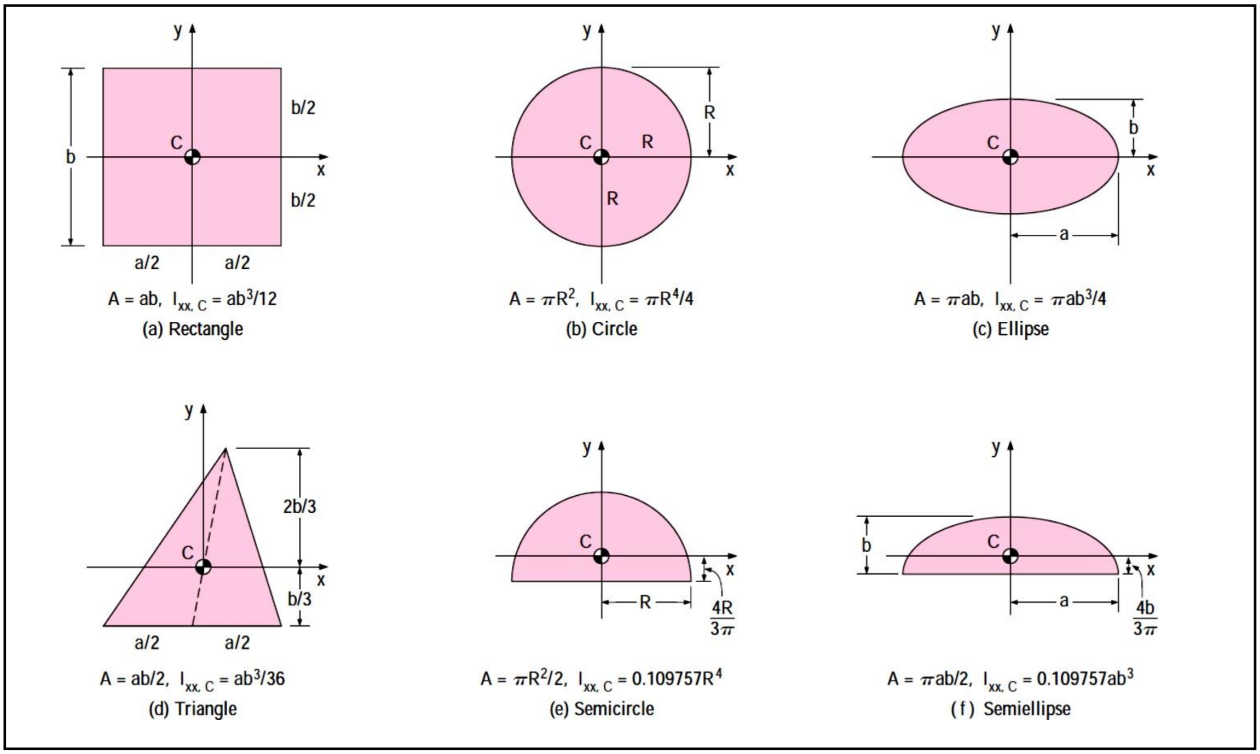


Figure (17): The centroid and the centroidal moments of inertia for some common geometries.

Whose height is the linearly varying pressure, as shown in Figure (18). This virtual **pressure prism** has an interesting physical interpretation: its *volume* is equal to the *magnitude* of the resultant hydrostatic force acting on the plate since $V = \int P\, dA$, and the line of action of this force passes through the *centroid* of this homogeneous prism. The projection of the centroid on the plate is the *pressure center*. Therefore, with the concept of pressure prism, the problem of describing the resultant hydrostatic force on a plane surface reduces to finding the volume and the two coordinates of the centroid of this pressure prism.

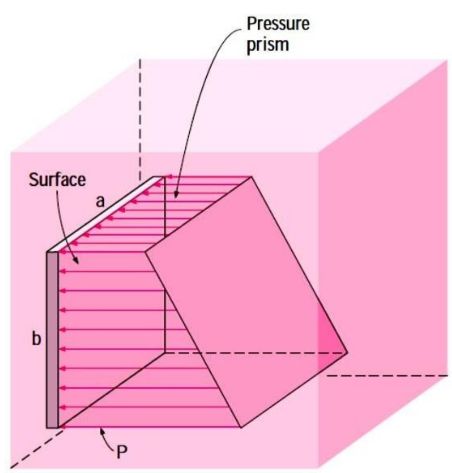


Figure (18):

The hydrostatic forces acting on a plane surface form a volume whose base (left face) is the surface and whose height is the pressure.

Special Case: Submerged Rectangular Plate

Consider a completely submerged rectangular flat plate of height *b* and width *a* tilted at an angle u from the horizontal and whose top edge is horizontal and is at a distance *s* from the free surface along the plane of the plate, as shown in Figure (19).

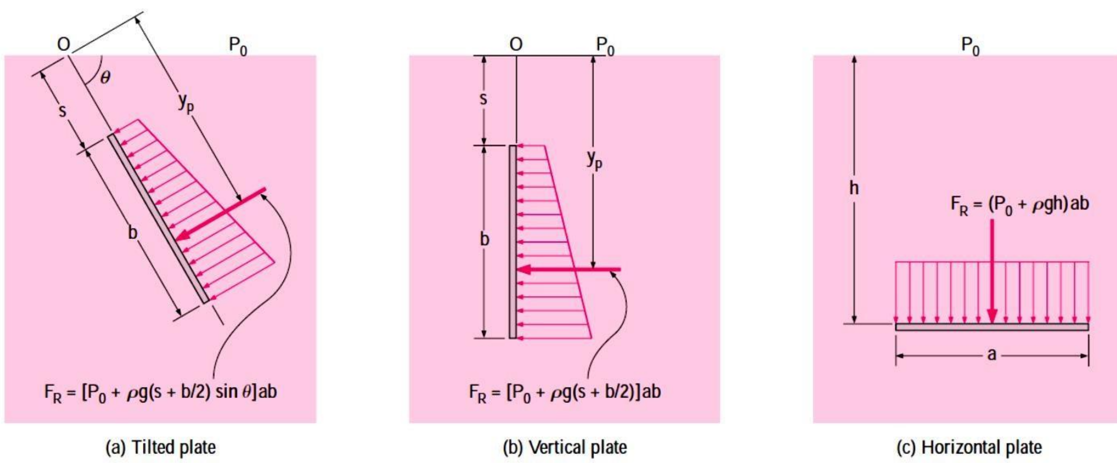


Figure (19): Hydrostatic force acting on the top surface of a submerged rectangular plate for tilted, vertical, and horizontal cases.

The resultant hydrostatic force on the upper surface is equal to the average pressure, which is the pressure at the midpoint of the surface, times the surface area *A*. That is.

Tilted rectangular plate: $F_R = P_C A = [P_0 + \rho g(s + b/2)\sin\theta]ab$

For a completely submerged *vertical* plate ($\theta = 90°$) whose top edge is horizontal, the hydrostatic force can be obtained by setting $\sin\theta = 1$ Figure (19-b)

Vertical rectangular plate: $F_R = [P_0 + \rho g(s + b/2)]ab$

Vertical rectangular plate ($s = 0$): $F_R = (P_0 + \rho gb/2)ab$

When the effect of P_0 is ignored since it acts on both sides of the plate, the hydrostatic force on a vertical rectangular surface of height b whose top edge is horizontal and at the free surface is $F_R = \theta gab^2/2$ acting at a distance of $2b/3$ from the free surface directly beneath the centroid of the plate. The pressure distribution on a submerged *horizontal* surface is uniform, and its magnitude is $P = P_0 + \rho gh$, where h is the distance of the surface from the free surface. Therefore, the hydrostatic force acting on a horizontal rectangular surface is

Horizontal rectangular plate: $F_R = (P_0 + \rho gh)ab$

and it acts through the midpoint of the plate Figure (19-*c*).

HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES:

For a submerged curved surface, the determination of the resultant hydrostatic force is more involved since it typically requires the integration of the pressure forces that change direction along the curved surface. The concept of the pressure prism in this case is not much help either because of the complicated shapes involved. The easiest way to determine the resultant hydrostatic force F_R acting on a two-dimensional curved surface is to determine the horizontal and vertical components F_H and F_V separately. This is done by considering the free-body diagram of the liquid block enclosed by the curved surface and the two plane surfaces (one horizontal and one vertical) passing through the two ends of the curved surface, as shown in Figure (20). Note that the vertical surface of the liquid block considered is simply the projection of the curved surface on a *vertical plane,* and the horizontal surface is the projection of the curved surface on a *horizontal plane.* The resultant force acting on the

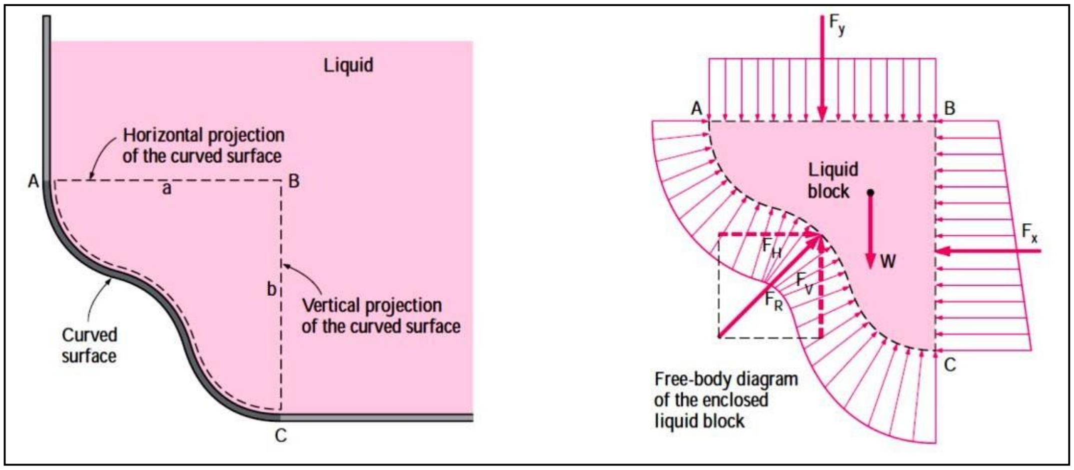


Figure (20): Determination of the hydrostatic force acting on a submerged curved surface.

curved solid surface is then equal and opposite to the force acting on the curved liquid surface (Newton’s third law). The force acting on the imaginary horizontal or vertical plane surface and its line of action can be determined. The weight of the enclosed liquid block of volume V is simply $W = \rho g V$, and it acts downward through the centroid of this volume. Noting that the fluid block is in static equilibrium, the force balances in the horizontal and vertical

directions give

Horizontal force component on curved surface: $F_H = F_x$

Vertical force component on curved surface: $F_V = F_y + W$

Where the summation $F_y + W$ is a vector addition (i.e., add magnitudes if both act in the same direction and subtract if they act in opposite directions). Thus, we conclude that

1. The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic force acting on the vertical projection of the curved surface.

2. The vertical component of the hydrostatic force acting on a curved surface is equal to the hydrostatic force acting on the horizontal projection of the curved surface, plus (minus, if acting in the opposite direction) the weight of the fluid block.

The magnitude of the resultant hydrostatic force acting on the curved surface is

$$F_R = \sqrt{F_H{}^2 + F_v{}^2}$$

and the tangent of the angle it makes with the horizontal is tan a $= F_V / F_H$. The exact location of the line of action of the resultant force (e.g., its distance from one of the end points of the curved surface) can be determined by taking a moment about an appropriate point. These discussions are valid for all curved surfaces regardless of whether they are above or below the liquid. Note that in the case of a *curved surface above a liquid*, the weight of the liquid is *subtracted* from the vertical component of the hydrostatic force since they act in opposite directions Figure (21).

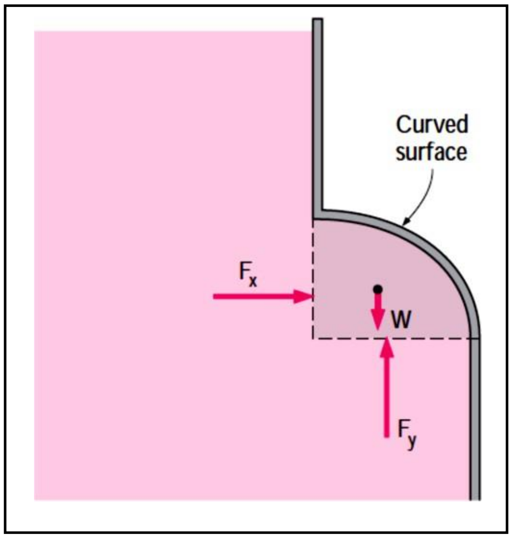


Figure (21): When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.

When the curved surface is a *circular arc* (full circle or any part of it), the resultant hydrostatic force acting on the surface always passes through the center of the circle. This is because the pressure forces are normal to the surface, and all lines normal to the surface of a circle pass through the center of the circle. Thus, the pressure forces form a concurrent force system at the center, which can be reduced to a single equivalent force at that point Figure (22).

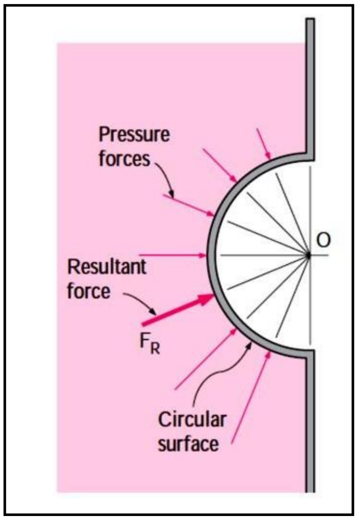


Figure (22): The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.

EXAMPLE 3–9 A Gravity-Controlled Cylindrical Gate

A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate, as shown in Fig. 3–36. When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per m length of the cylinder.

SOLUTION The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per m length are to be determined.

Assumptions **1** Friction at the hinge is negligible. **2** Atmospheric pressure acts on both sides of the gate, and thus it cancels out.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{36.1\ kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{bottom} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 39.2\ \text{kN}$$

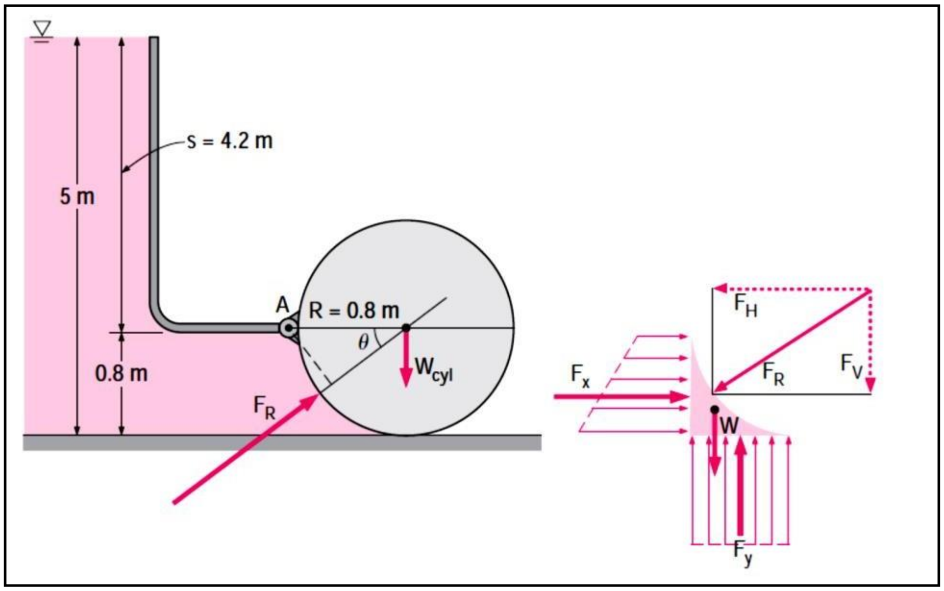


FIGURE 3–36 Schematic for Example 3–9 and the free-body diagram of the fluid underneath the cylinder.

Weight of fluid block per m length (downward):

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1\ \text{m})$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m})^2(1 - \pi/4)(1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 1.3\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3\ kN}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4^\circ$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.4° with the horizontal.

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R\sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R \sin\theta = (52.3\ \text{kN})\sin 46.4^\circ = \mathbf{37.9\ kN}$$

BUOYANCY AND STABILITY

It is a common experience that an object feels lighter and weighs less in a liquid than it does in air. This can be demonstrated easily by weighing a heavy object in water by a waterproof spring scale. Also, objects made of wood or other light materials float on water. These and other observations suggest that a fluid exerts an upward force on a body immersed in it. This force that tends to lift the body is called the **buoyant force** and is denoted by *FB*.

The buoyant force is caused by the increase of pressure in a fluid with depth. Consider, for example, a flat plate of thickness *h* submerged in a liquid of density $\boldsymbol{\rho}_f$ parallel to the free surface, as shown in Figure (22). The area of the top (and also bottom) surface of the plate is *A*, and its distance to the free surface is *s*. The pressures at the top and bottom surfaces of the plate are $\boldsymbol{\rho}_f\, gs$ and $\boldsymbol{\rho}_f\, g(s+h)$, respectively. Then the hydrostatic force $F_{top} = \boldsymbol{\rho}_f\, gsA$ acts downward on the top surface, and the larger force $F_{bottom} = \boldsymbol{\rho}_f\, g(s+h)A$ acts upward on the bottom surface of the plate. The difference between these two forces is a net upward force, which is the *buoyant force.*

$$F_B=F_{bottom}-F_{top}= \boldsymbol{\rho}_f\, g(s+h)A- \boldsymbol{\rho}_f\, gsA= \boldsymbol{\rho}_f\, ghA= \boldsymbol{\rho}_f gV$$

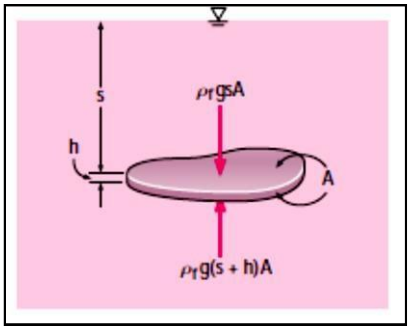


Figure (22). A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface.

where $V = hA$ is the volume of the plate. But the relation $\rho_f gV$ is simply the weight of the liquid whose volume is equal to the volume of the plate. Thus, we conclude that *the* ***buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate***. Note that the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body.

The relation in Eq. 3–32 is developed for a simple geometry, but it is valid for anybody regardless of its shape. This can be shown mathematically by a force balance, or simply by this argument: Consider an arbitrarily shaped solid body submerged in a fluid at rest and compare it to a body of fluid of the same shape indicated by dotted lines at the same distance from the free surface Figure (23).

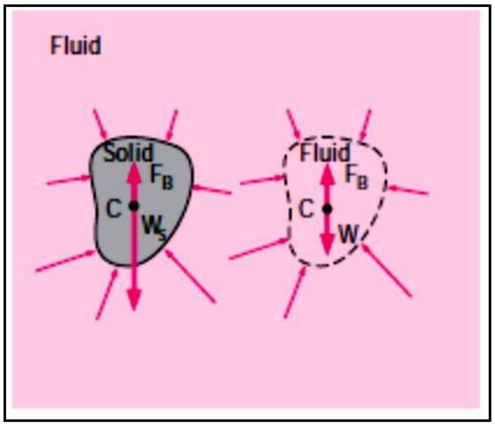


Figure (23).

The buoyant forces acting on these two bodies are the same since the pressure distributions, which depend only on depth, are the same at the boundaries of both. The imaginary fluid body is in static equilibrium, and thus the net force and net moment acting on it are zero. Therefore, the upward buoyant force must be equal to the weight of the imaginary fluid body whose volume is equal to the volume of the solid body. Further, the weight and the buoyant force must have the same line of action to have a zero moment. This is known as **Archimedes' principle,** after the Greek mathematician Archimedes (287–212 BC), and is expressed as:

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body. That is,

$$F_B = W \rightarrow \rho_f g V_s = \rho_{ave,body}\, g\, V_{total} \rightarrow V_{sub}/V_{total} = \rho_{ave,body}/\rho_f$$

Therefore, the submerged volume fraction of a floating body is equal to the ratio of the average density of the body to the density of the fluid. Note that when the density ratio is equal to or greater than one, the floating body becomes completely submerged.

It follows from these discussions that a body immersed in a fluid (1) remains at rest at any point in the fluid when its density is equal to the density of the fluid, (2) sinks to the bottom when its density is greater than the density of the fluid, and (3) rises to the surface of the fluid and floats when the density of the body is less than the density of the fluid (Fig. 3–39).

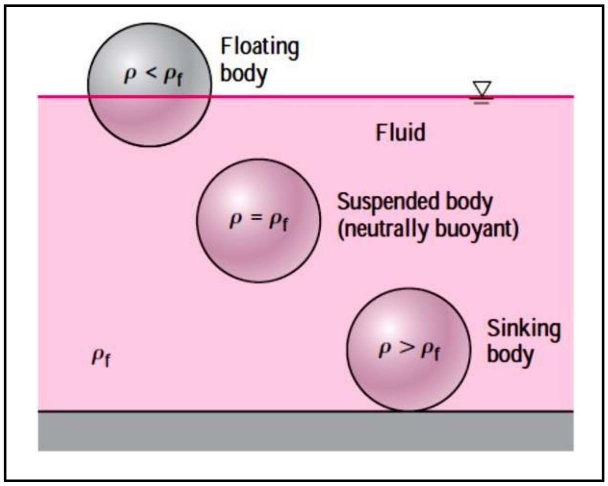


Fig 3-39

The buoyant force is proportional to the density of the fluid, and thus we might think that the buoyant force exerted by gases such as air is negligible. This is certainly the case in general, but there are significant exceptions. For example, the volume of a person is about 0.1 m3, and taking the density of air to be 1.2 kg/m3, the buoyant force exerted by air on the person is

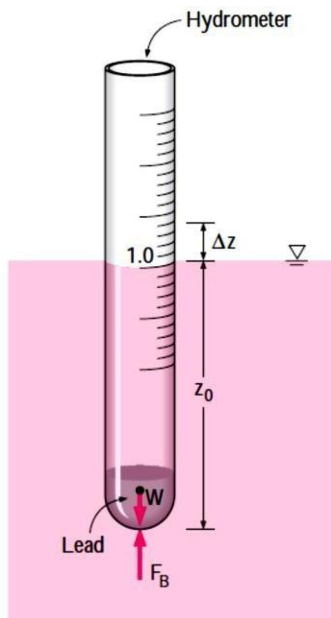
$$F_B = \rho_f g V = (1.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}^3) \cong 1.2\ \text{N}$$

The weight of an 80-kg person is 80 × 9.81 = 788 N. Therefore, ignoring the buoyancy in this case results in an error in weight of just 0.15 percent, which is negligible. But the buoyancy effects in gases dominate some important natural phenomena such as the rise of warm air in a cooler environment and thus the onset of natural convection currents, the rise of hot-air or helium balloons, and air movements in the atmosphere. A helium balloon, for example, rises as a result of the buoyancy effect until it reaches an altitude where the density of air (which decreases with altitude) equals the density of helium in the balloon—assuming the balloon does not burst by then, and ignoring the weight of the balloon's skin.

Archimedes' principle is also used in modern geology by considering the continents to be floating on a sea of magma.

EXAMPLE 3–10 Measuring Specific Gravity by a Hydrometer

If you have a seawater aquarium, you have probably used a small cylindrical glass tube with some lead-weight at its bottom to measure the salinity of the water by simply watching how deep the tube sinks. Such a device that floats in a vertical position and is used to measure the specific gravity of a liquid is called a *hydrometer* (Fig. 3–40). The top part of the hydrometer extends



above the liquid surface, and the divisions on it allow one to read the specific gravity directly. The hydrometer is calibrated such that in pure water it reads exactly 1.0 at the air–water interface. (*a*) Obtain a relation for the specific gravity of a liquid as a function of distance Δz from the mark corresponding to pure water and (*b*) determine the mass of lead that must be poured into a 1-cm-diameter, 20-cm-long hydrometer if it is to float halfway (the 10-cm mark) in pure water.

SOLUTION The specific gravity of a liquid is to be measured by a hydrometer. A relation between specific gravity and the vertical distance from the reference level is to be obtained, and the amount of lead that needs to be added into the tube for a certain hydrometer is to be determined.

Assumptions **1** The weight of the glass tube is negligible relative to the weight of the lead added. **2** The curvature of the tube bottom is disregarded.

Properties We take the density of pure water to be 1000 kg/m³.

Analysis (*a*) Noting that the hydrometer is in static equilibrium, the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer. In pure water, let the vertical distance between the bottom of the hydrometer and the free surface of water be z_0. Setting $F_B = W$ in this case gives

$$W_{hydro} = F_{B,\,w} = \rho_w g V_{sub} = \rho_w g A z_0 \qquad (1)$$

where A is the cross-sectional area of the tube, and ρ_w is the density of pure water.

In a fluid lighter than water ($\rho_f < \rho_w$), the hydrometer will sink deeper, and the liquid level will be a distance of Δz above z_0. Again setting $F_B = W$ gives

$$W_{hydro} = F_{B,\,f} = \rho_f g V_{sub} = \rho_f g A(z_0 + \Delta z) \qquad (2)$$

This relation is also valid for fluids heavier than water by taking the Δz below z_0 to be a negative quantity. Setting Eqs. (1) and (2) here equal to each other since the weight of the hydrometer is constant and rearranging gives

$$\rho_w g A z_0 = \rho_f g A(z_0 + \Delta z) \quad \rightarrow \quad SG_f = \frac{\rho_f}{\rho_w} = \frac{z_0}{z_0 + \Delta z}$$

which is the relation between the specific gravity of the fluid and Δz. Note that z_0 is constant for a given hydrometer and Δz is negative for fluids heavier than pure water.

(*b*) Disregarding the weight of the glass tube, the amount of lead that needs to be added to the tube is determined from the requirement that the weight of the lead be equal to the buoyant force. When the hydrometer is floating with half of it submerged in water, the buoyant force acting on it is

$$F_B = \rho_w g V_{sub}$$

Equating F_B to the weight of lead gives

$$W = mg = \rho_w g V_{sub}$$

Solving for m and substituting, the mass of lead is determined to be

$$m = \rho_w V_{sub} = \rho_w(\pi R^2 h_{sub}) = (1000\text{ kg/m}^3)[\pi(0.005\text{ m})^2(0.1\text{ m})] = \mathbf{0.00785\ kg}$$

Discussion Note that if the hydrometer were required to sink only 5 cm in water, the required mass of lead would be one-half of this amount. Also, the assumption that the weight of the glass tube is negligible needs to be checked since the mass of lead is only 7.85 g.

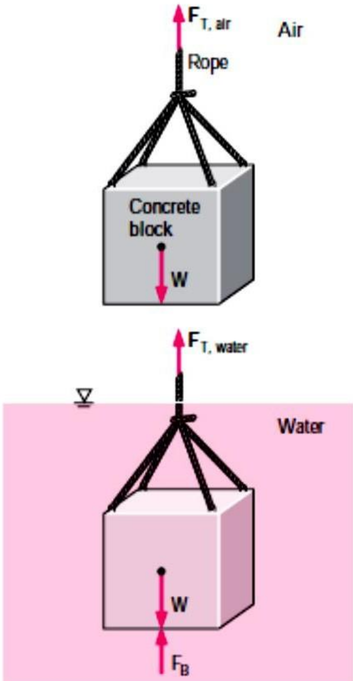


FIGURE 3–41
Schematic for Example 3–11.

EXAMPLE 3–11 Weight Loss of an Object in Seawater

A crane is used to lower weights into the sea (density = 1025 kg/m³) for an underwater construction project (Fig. 3–41). Determine the tension in the rope of the crane due to a rectangular 0.4-m × 0.4-m × 3-m concrete block (density = 2300 kg/m³) when it is (*a*) suspended in the air and (*b*) completely immersed in water.

SOLUTION A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is in water.

Assumptions **1** The buoyancy of air is negligible. **2** The weight of the ropes is negligible.

Properties The densities are given to be 1025 kg/m³ for seawater and 2300 kg/m³ for concrete.

Analysis (*a*) Consider the free-body diagram of the concrete block. The forces acting on the concrete block in air are its weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$V = (0.4\text{ m})(0.4\text{ m})(3\text{ m}) = 0.48\text{ m}^3$$

$$F_{T,\,air} = W = \rho_{concrete}\, gV$$

$$= (2300\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.48\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{10.8\text{ kN}}$$

(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upward. The force balance in this case gives

$$F_B = \rho_f\, gV = (1025\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.48\text{ m}^3)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{4.8\text{ kN}}$$

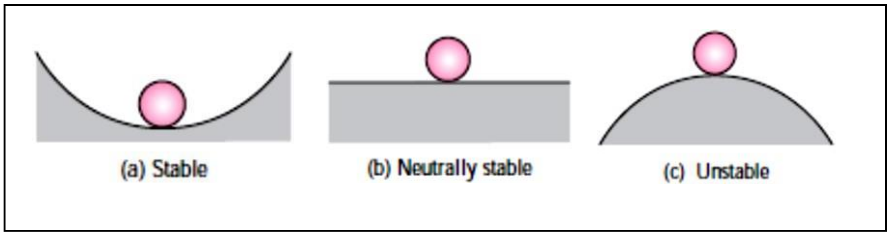
$$F_{T,\,water} = W - F_B = 10.8 - 4.8 = \mathbf{6.0\text{ kN}}$$

Discussion Note that the weight of the concrete block, and thus the tension of the rope, decreases by (10.8 − 6.0)/10.8 = 55 percent in water.

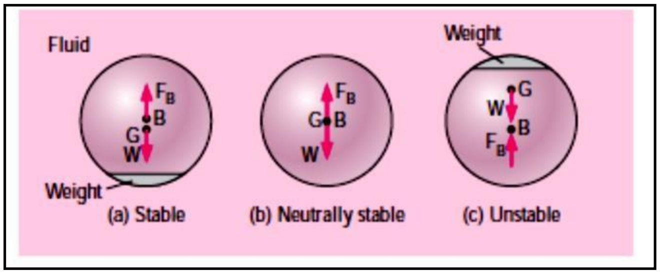
Stability of Immersed and Floating Bodies

An important application of the buoyancy concept is the assessment of the stability of immersed and floating bodies with no external attachments. This topic is of great importance in the design of ships and submarines (Fig. 3–42). Here we provide some general qualitative discussions on vertical and rotational stability.

We use the “ball on the floor” analogy to explain the fundamental concepts of stability and instability. Shown in Fig. 3–43 are three balls at rest on the floor. Case (*a*) ***is stable since any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position. Case (b) is neutrally stable because if someone moves the ball to the right or left, it would stay put at its new location. It has no tendency to move back to its original location, nor does it continue to move away. Case (c) is a situation in which the ball may be at rest at the moment, but any disturbance, even an infinitesimal one, causes the ball to roll off the hill—it does not return to its original position; rather it diverges from it. This situation is unstable.*** What about a case where the ball is on an *inclined* floor? It is not really appropriate to discuss stability for this case since the ball is not in a state of equilibrium. In other words, it cannot be at rest and would roll down the hill even without any disturbance.

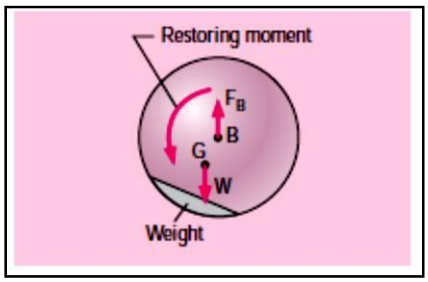


For an immersed or floating body in static equilibrium, the weight and the buoyant force acting on the body balance each other, and such bodies are inherently stable in the *vertical direction.* If an immersed neutrally buoyant body is raised or lowered to a different depth, the body will remain in equilibrium at that location. If a floating body is raised or lowered somewhat by a vertical force, the body will return to its original position as soon as the external effect is removed. Therefore, a floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance. The *rotational stability* of an *immersed body* depends on the relative locations of the *center of gravity G* of the body and the *center of buoyancy B*, which is the centroid of the displaced volume. An immersed body is *stable* if the body is bottom-heavy and thus point *G* is directly below point *B* (Fig. 3–44).

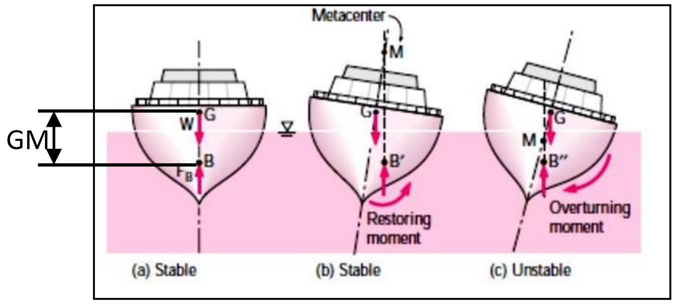


A rotational disturbance of the body in such cases produces a *restoring moment* to return the body to its original stable position. Thus, a stable design for a submarine calls for the engines and the cabins for the crew to be located at the lower half in order to shift the weight to the bottom as much as possible. Hot-air or helium balloons (which can be viewed as being immersed in air) are also stable since the cage that carries the load is at the bottom. An immersed body whose center of gravity *G* is directly above point *B* is *unstable,* and any disturbance will cause this body to turn upside down. A body for which *G* and *B* coincide is *neutrally stable.* This is the case for bodies whose density is constant throughout. For such bodies, there is no tendency to overturn or right themselves.

What about a case where the center of gravity is not vertically aligned with the center of buoyancy (Fig. 3–45)? It is not really appropriate to discuss stability for this case since the body is not in a state of equilibrium.



In other words, it cannot be at rest and would rotate toward its stable state even without any disturbance. The restoring moment in the case shown in Fig. 3–45 is counterclockwise and causes the body to rotate counterclockwise so as to align point *G* vertically with point *B*. Note that there may be some oscillation, but eventually the body settles down at its stable equilibrium state [case (*a*) of Fig. 3–44]. The stability of the body of Fig. 3–45 is analogous to that of the ball on an inclined floor. Can you predict what would happen if the weight in the body of Fig. 3–45 were on the opposite side of the body? The rotational stability criteria are similar for *floating bodies.* Again, if the floating body is bottom-heavy and thus the center of gravity *G* is directly below the center of buoyancy *B*, the body is always stable. But unlike immersed bodies, a floating body may still be stable when *G* is directly above *B* (Fig. 3–46).



This is because the centroid of the displaced volume shifts to the side to a point *B*+ during a rotational disturbance while the center of gravity *G* of the body remains unchanged. If point *B*+ is sufficiently far, these two forces create a restoring moment and return the body to the original position. A measure of stability for floating bodies is the **metacentric height (*GM*)**, ***which is the distance between the center of gravity G and the metacenter M—the intersection point of the lines of action of the buoyant force through the body before and after rotation***. The metacenter may be considered to be a fixed point for most hull shapes for small rolling angles up to about 20°. A floating body is stable if point *M* is above point *G*, and thus *GM* is positive, and unstable if point *M* is below point *G*, and thus *GM* is negative. In the latter case, the weight and the buoyant force acting on the tilted body generate an overturning moment instead of a restoring moment,

causing the body to capsize. The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

As already discussed, a boat can tilt to some maximum angle without capsizing, but beyond that angle it overturns (and sinks). We make a final analogy between the stability of floating objects and the stability of a ball rolling along the floor. Namely, imagine the ball in a trough between two hills (Fig. 3–47). The ball returns to its stable equilibrium position after being perturbed—up to a limit. If the perturbation amplitude is too great, the ball rolls down the opposite side of the hill and does not return to its equilibrium position. This situation is described as stable up to some limiting level of disturbance, but unstable beyond.

Solved problem

Q (1):

A pressure gage 7.0 m above the bottom of a tank containing a liquid reads 64.94 kPa; another gage at height 4.0 m reads 87.53 kPa. Compute the specific weight and mass density of the fluid.

$$\gamma = \Delta p/\Delta h = (87.53 - 64.94)/(7.0 - 4.0) = 7.53\ \text{kN/m}^3 \quad \text{or} \quad 7530\ \text{N/m}^3$$

$$\rho = \gamma/g = 7530/9.81 = 786\ \text{kg/m}^3$$

Q (2):

A pressure gage 19.0 ft above the bottom of a tank containing a liquid reads 13.19 psi; another gage at height 14.0 ft reads 15.12 psi. Compute the specific weight, mass density, and specific gravity of the liquid.

$$\Delta p = \gamma(\Delta h) \qquad (15.12 - 13.19)(144) = (\gamma)(19.0 - 14.0) \qquad \gamma = 55.6\ \text{lb/ft}^3$$

$$\rho = \gamma/g = 55.6/32.2 = 1.73\ \text{slug/ft}^3 \qquad \text{s.g.} = 55.6/62.4 = 0.891$$

Q (3):

If the weight density of mud is given by $\gamma = 65.0 + 0.2h$, where γ is in lb/ft³ and depth h is in ft, determine the pressure, in psi, at a depth of 17 ft.

$dp = \gamma\, dh = (65.0 + 0.2h)\, dh$. Integrating both sides: $p = 65.0h + 0.1h^2$. For $h = 17$ ft: $p = (65.0)(17)/144 + (0.1)(17)^2/144 = 7.87$ psi.

Q (4):

If the atmospheric pressure is 0.900 bar abs and a gage attached to a tank reads 390 mmHg vacuum, what is the absolute pressure within the tank?

$$p = \gamma h \qquad p_{atm} = 0.900 \times 100 = 90.0\ \text{kPa}$$

$$p_{gage} = [(13.6)(9.79)]\left(\tfrac{390}{1000}\right) = 51.9\ \text{kPa vacuum} \quad \text{or} \quad -51.9\ \text{kPa}$$

$$p_{abs} = 90.0 + (-51.9) = 38.1\ \text{kPa}$$

Q (5):

The closed tank in Fig. 2-3 is at 20 °C. If the pressure at point *A* is 98 kPa abs, what is the absolute pressure at point *B*? What percent error results from neglecting the specific weight of the air?

$p_A + \gamma_{air} h_{AC} - \gamma_{H_2O} h_{DC} - \gamma_{air} h_{DB} = p_B$, $98 + (0.0118)(5) - (9.790)(5-3) - (0.0118)(3) = p_B = 78.444$ kPa. Neglecting air, $p_B = 98 - (9.790)(5-3) = 78.420$ kPa; error $= (78.444 - 78.420)/78.444 = 0.00031$, or 0.031%.

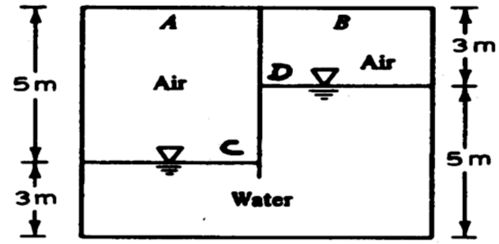


Fig. 2-3

Q (6):

For a gage reading at *A* of −2.50 psi, determine the (*a*) elevations of the liquids in the open piezometer columns *E*, *F*, and *G* and (*b*) deflection of the mercury in the U-tube gage in Fig. 2-14. Neglect the weight of the air.

(*a*) The liquid between the air and the water would rise to elevation 49.00 ft in piezometer column *E* as a result of its weight. The actual liquid level in the piezometer will be lower, however, because of the vacuum in the air above the liquid. The amount the liquid level will be lowered (*h* in Fig. 2-14) can be determined by

$(-2.50)(144) + [(0.700)(62.4)](h) = 0$, $h = 8.24$ ft. Elevation at $L = 49.00 - 8.24 = 40.76$ ft; $(-2.50)(144) + [(0.700)(62.4)][49.00 - 38.00] = p_M$, $p_M = 120.5\ \text{lb/ft}^2$. Hence, pressure head at $M = 120.5/62.4 = 1.93$ ft of water. Elevation at $N = 38.00 + 1.93 = 39.93$ ft; $120.5 + (62.4)(38.00 - 26.00) = p_O$, $p_O = 869.3\ \text{lb/ft}^2$. Hence, pressure head at $O = 869.3/[(1.600)(62.4)] = 8.71$ ft (of the liquid with s.g. = 1.600). Elevation at $Q = 26.00 + 8.71 = 34.71$ ft. **(b)** $869.3 + (62.4)(26.00 - 14.00) - [(13.6)(62.4)](h_1) = 0$, $h_1 = 1.91$ ft.

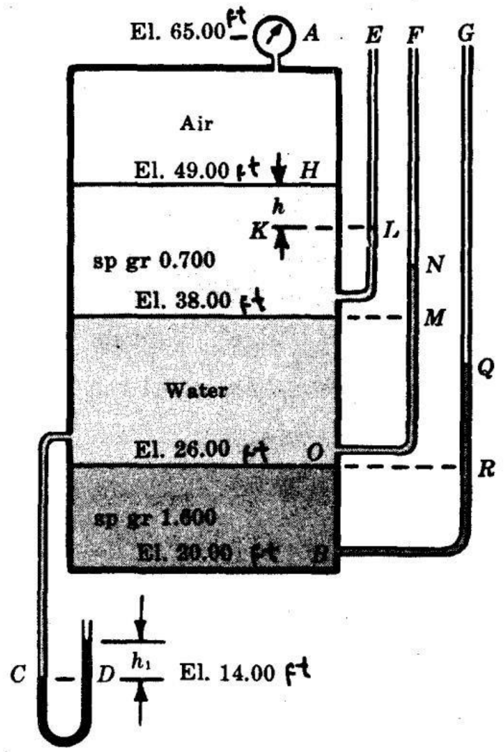


Fig. 2-14

Q (7):

In Fig. 2-23, s.g.$_1$ = 0.84, s.g.$_2$ = 1.0, h_2 = 96 mm, and h_1 = 159 mm. Find p_A in mmHg gage. If the barometer reading is 729 mmHg, what is p_A in mmH$_2$O absolute?

$$p_A + (0.84)(96) - (1.0)(159) = 0$$

$$p_A = 78.4 \text{ mmH}_2\text{O gage} = 78.4/13.6 = 5.76 \text{ mmHg gage}$$
$$= 78.4 + (13.6)(729) = 9993 \text{ mmH}_2\text{O absolute}$$

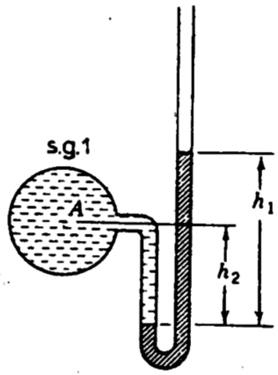


Fig. 2-23

Q (8):

A differential manometer is shown in Fig. 2-33. Calculate the pressure difference between points *A* and *B*.

$$p_A + [(0.92)(62.4)][(x+12)/12] - [(13.6)(62.4)](\tfrac{12}{12}) - [(0.92)(62.4)][(x+24)/12] = p_B$$
$$p_A - p_B = 906 \text{ lb/ft}^2$$

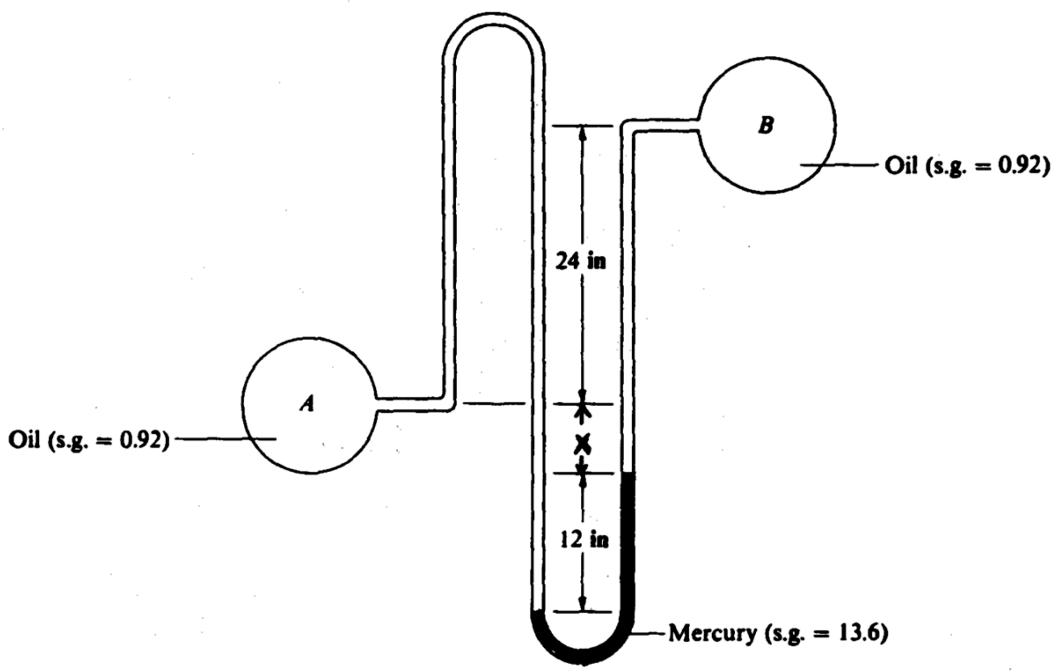


Fig. 2-33

Q (9):

A differential manometer is attached to a pipe, as shown in Fig. 2-35. Calculate the pressure difference between points *A* and B.

$$p_A - [(0.91)(62.4)](y/12) - [(13.6)(62.4)](\tfrac{4}{12}) + [(0.91)(62.4)][(y+4)/12] = p_B$$

$$p_A - p_B = 264\ \text{lb/ft}^2$$

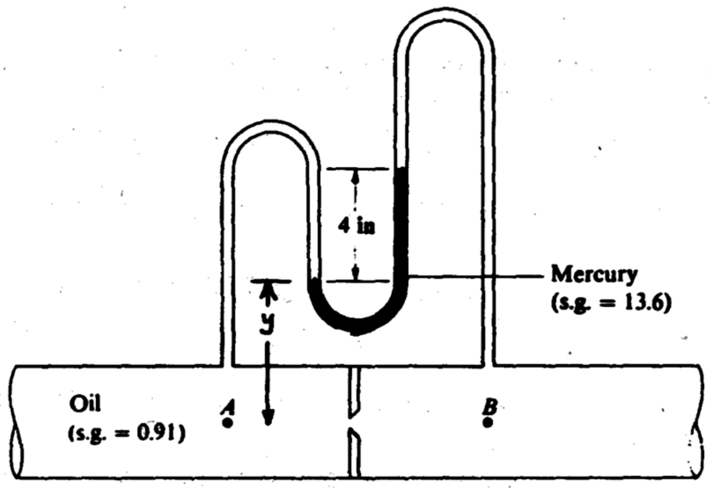


Fig. 2-35

Q (10):

Calculate the pressure difference between A and B for the setup shown in Fig. 2-42.

$$p_A + (62.4)(66.6/12) - [(13.6)(62.4)](40.3/12) + (62.4)(22.2/12) - [(13.6)(62.4)](30.0/12) - (62.4)(10.0/12) = p_B$$

$$p_A - p_B = 4562\ \text{lb/ft}^2 \quad \text{or} \quad 31.7\ \text{lb/in}^2$$

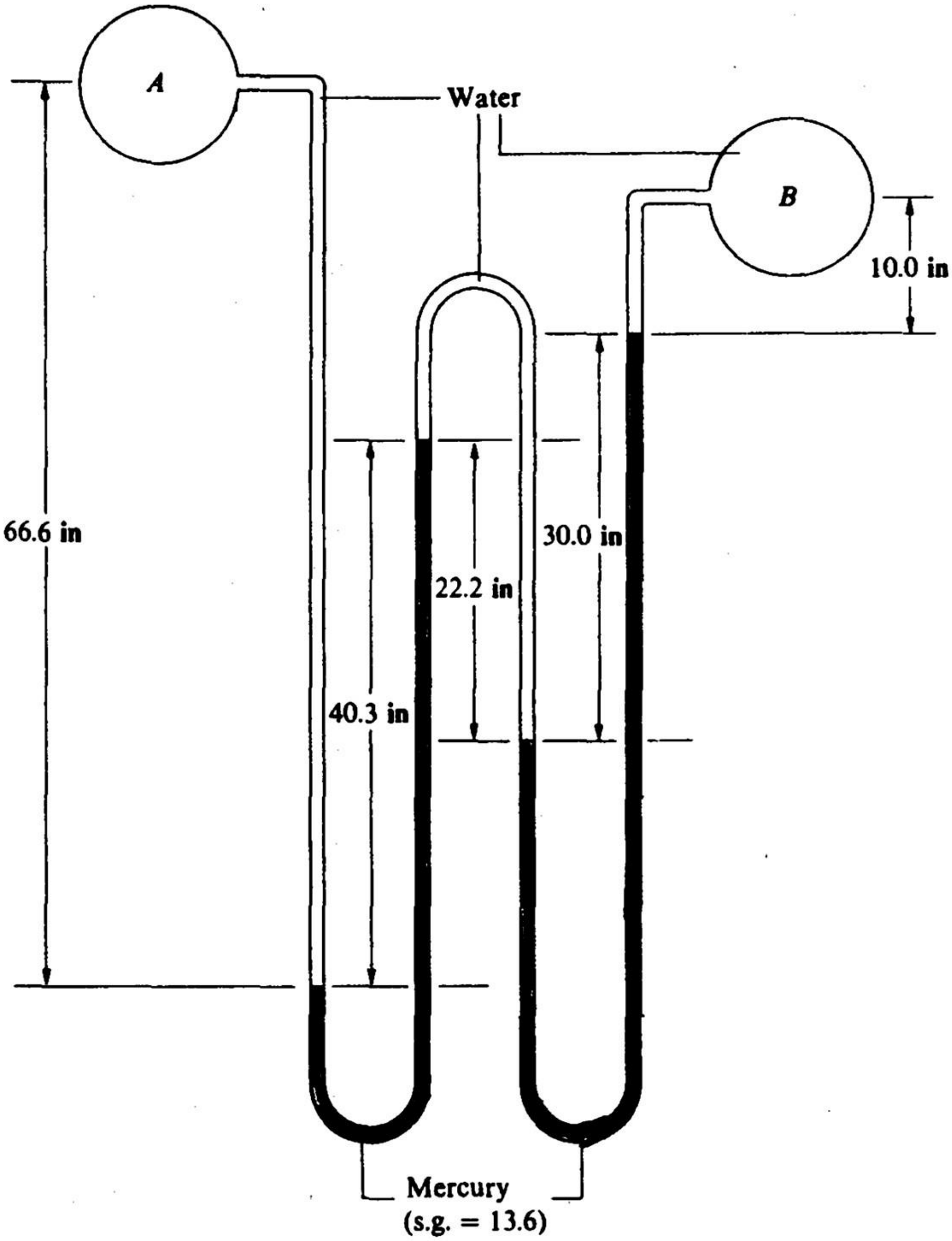


Fig. 2-42

Q (11):

Calculate the pressure difference between *A* and *B* for the setup shown in Fig. 2-43.

$$p_A - (9.79)x - [(0.8)(9.79)](0.70) + (9.79)(x - 0.80) = p_B \qquad p_A - p_B = 13.3\ \text{kN/m}^2$$

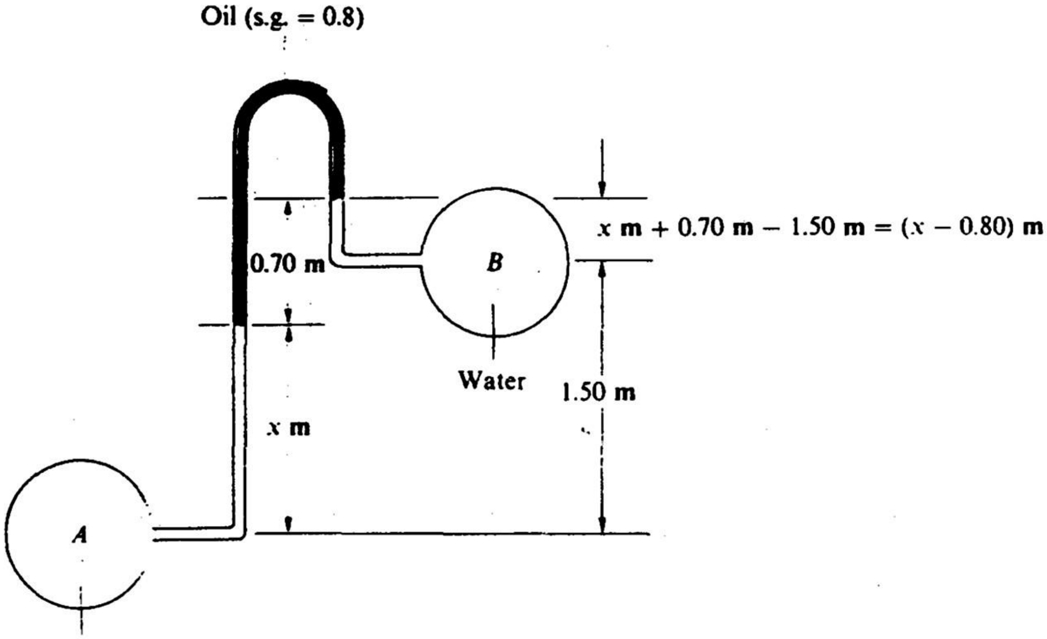


Fig. 2-43

Q (12):

The inclined manometer in Fig. 2-54*a* contains Meriam red manometer oil (s.g. = 0.827). Assume the reservoir is very large. What should the angle θ be if each inch along the scale is to represent a change of 0.8 lb/ft² in gage pressure p_A?

From Fig. 2-54*b*, $\Delta p = \gamma \Delta z$, or

$$0.8\ \text{lb/ft}^2 = [(0.827)(62.4\ \text{lb/ft}^3)](\tfrac{1}{12}\ \text{ft})(\sin\theta)$$

from which $\theta = 10.72°$.

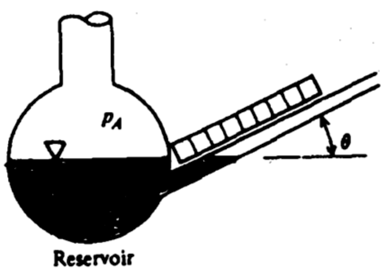


Fig. 2-54(*a*)

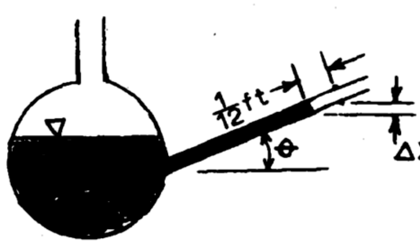


Fig. 2-54(*b*)

Q (13):

Water flows downward in a pipe at 35°, as shown in Fig. 2-57. The pressure drop $p_1 - p_2$ is partly due to gravity and partly due to friction. The mercury manometer reads a 5-in height difference. What is the total pressure drop $p_1 - p_2$? What is the pressure drop due to friction only between 1 and 2? Does the manometer reading correspond only to friction drop?

$$p_1 + (62.4)(6 \sin 35° + x/12 + \tfrac{5}{12}) - [(13.6)(62.4)](\tfrac{5}{12}) - (62.4)(x/12) = p_2$$

$$p_1 - p_2 = 112.9 \text{ lb/ft}^2 \qquad \text{(total pressure drop)}$$

$$\text{Pressure drop due to friction only} = [(13.6)(62.4) - 62.4](\tfrac{5}{12}) = 327.6 \text{ lb/ft}^2$$

Manometer reads only the friction loss.

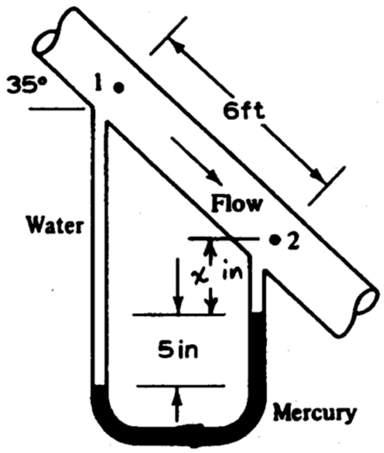


Fig. 2-57

Q (14):

In Fig. 2-60*a* the inclined manometer measures the excess pressure at *A* over that at *B*. The reservoir diameter is 2.5 in and that of the inclined tube is $\frac{1}{4}$ in. For $\theta = 32°$ and gage fluid with s.g. = 0.832, calibrate the scale in psi per ft.

$$p_A = \gamma(\Delta h + \Delta y) + p_B \qquad \text{(see Fig. 2-60}b) \qquad p_A - p_B = \gamma(\Delta h + \Delta y)$$

From Fig. 2-60*b*, $(A_A)(\Delta y) = (A_B)(R)$ or $\Delta y = A_B R/A_A$, $\Delta h = R \sin\theta$, $p_A - p_B = \gamma(R \sin\theta + A_B R/A_A) = \gamma R(\sin\theta + A_B/A_A)$, $A_B/A_A = [\pi(\frac{1}{4})^2/4]/[\pi(2.5)^2/4] = \frac{1}{100}$; $p_A - p_B = [(0.832)(62.4)](R)(\sin 32° + \frac{1}{100})/144 = 0.1947R$. The scale factor is thus 0.1947 psi/ft.

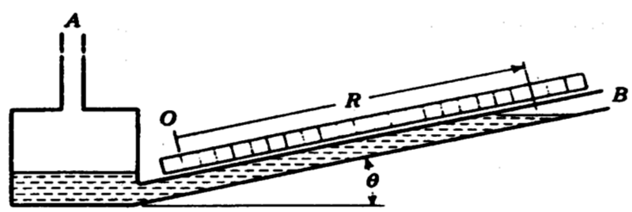


Fig. 2-60(*a*)

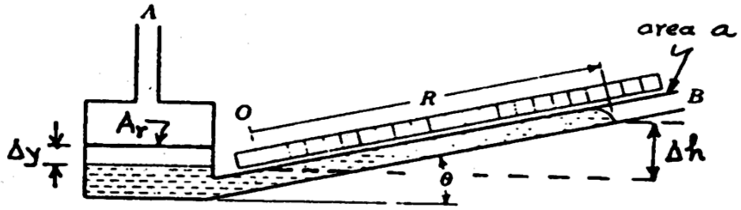


Fig. 2-60(*b*)

Q (15):

Find the difference in pressure between tanks A and B in Fig. 2-64 if $d_1 = 330$ mm, $d_2 = 160$ mm, $d_3 = 480$ mm, and $d_4 = 230$ mm.

$$p_A + (9.79)(0.330) - [(13.6)(9.79)](0.480 + 0.230 \sin 45°) = p_B \qquad p_A - p_B = 82.33 \text{ kPa}$$

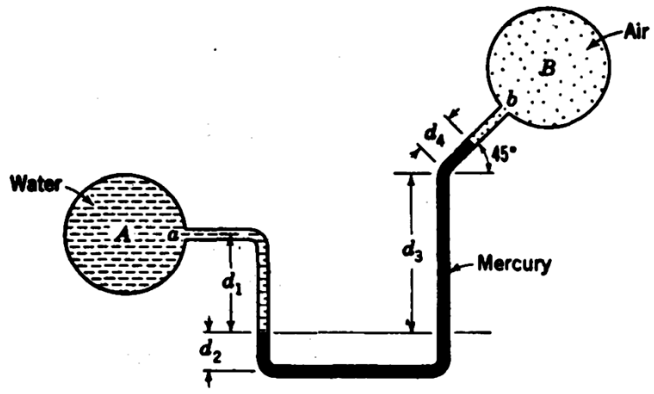


Fig. 2-64

EXAMPLE 2.2 A manometer connects an oil pipeline and a water pipeline as shown in Fig. 2.4. Determine the difference in pressure between the two pipelines using the readings on the manometer. Use $S_{oil} = 0.86$ and $S_{Hg} = 13.6$.

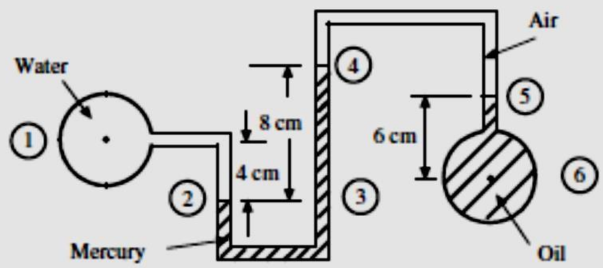


Figure 2.4

Solution: The points of interest have been positioned on the manometer in Fig. 2.4. The pressure at point 2 is equal to the pressure at point 3:

$$p_2 = p_3$$
$$p_{water} + \gamma_{water} \times 0.04 = p_4 + \gamma_{Hg} \times 0.08$$

Note that the heights must be in meters. The pressure at point 4 is essentially the same as that at point 5, since the specific weight of air is negligible compared with that of the oil. So,

$$p_4 = p_5$$
$$= p_{oil} - \gamma_{oil} \times 0.06$$

Finally,

$$p_{water} - p_{oil} = -\gamma_{water} \times 0.04 + \gamma_{Hg} \times 0.08 - \gamma_{oil} \times 0.06$$
$$= -9800 \times 0.04 + (13.6 \times 9800)0.08 - (0.86 \times 9800)0.06 = 10\,780 \text{ Pa}$$

Q (15):

If a triangle of height *d* and base *b* is vertical and submerged in liquid with its vertex at the liquid surface (see Fig. 3-1), derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{2d}{3} + \frac{bd^3/36}{(2d/3)(bd/2)} = \frac{3d}{4}$$

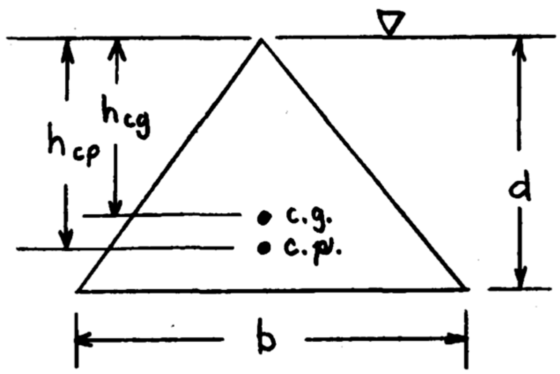
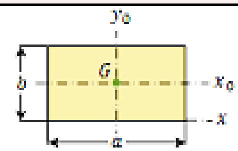
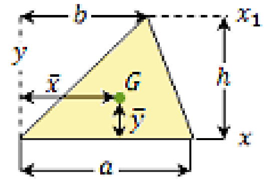
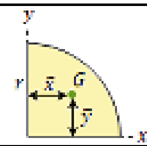
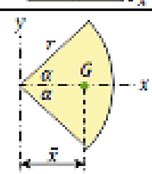
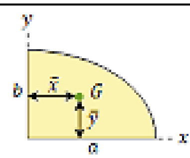
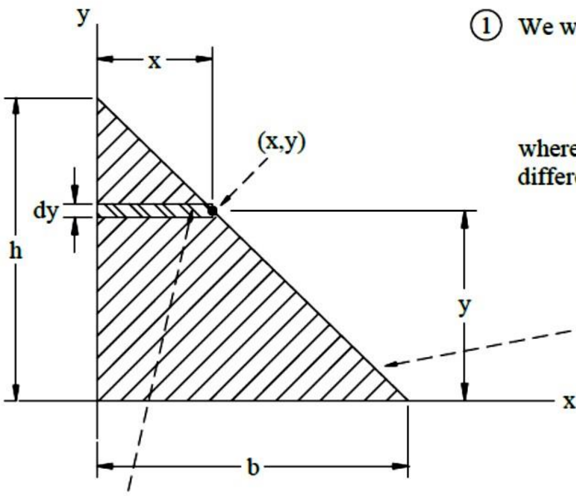


Fig. 3-1

Properties of Plane Figures

Shape	*Figure*	*Centroid*	*Area Moment of Inertia*	
Rectangular Area		–	$I_x = \frac{ab^3}{3}$ $I_g = \frac{ab^3}{12}$	$J = \frac{ab}{12}(a^2 + b^2)$
Triangular Area		$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{ah^3}{12}$ $I_g = \frac{ah^3}{36}$ $I_{x_1} = \frac{ah^3}{4}$	
Quarter Circular Area		$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$	$J = \frac{\pi r^4}{8}$
Area of Circular Sector		$\bar{x} = \frac{2}{3}\frac{r\sin\alpha}{\alpha}$	$I_x = \frac{r^4}{4}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)$ $I_y = \frac{r^4}{4}\left(\alpha + \frac{1}{2}\sin 2\alpha\right)$	$J = \frac{1}{2}r^4\alpha$
Area of Elliptical Quadrant		$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}$ $I_y = \frac{\pi a^3 b}{16}$	$J = \frac{\pi ab}{16}(a^2 + b^2)$

10.1 Moments of Inertia by Integration Example 3, page 2 of 3



① We want to evaluate

$$I_x = \int y^2 \, dA$$

where y has the same value throughout differential element dA.

② dA = area of rectangle

$= x \times dy$

③ Since we will integrate with respect to y, we must replace x by a function of y.

④ Equation of line

$$y = \text{slope} \times x + \text{intercept}$$

$$= -\left(\frac{h}{b}\right) x + h \qquad (1)$$

⑤ Solve for x to get

$$x = \left(-\frac{b}{h}\right) y + b$$

so,

$$dA = x \times dy$$

$$= \left(-\frac{b}{h} y + b\right) dy$$

10.1 Moments of Inertia by Integration Example 3, page 3 of 3

⑥ $I_x = \int y^2 \, dA$

$$= \int_0^h y^2 \left(-\frac{b}{h} y + b\right) dy$$

$$= b \int_0^h \left(-\frac{y^3}{h} + y^2\right) dy$$

$$= b\left[-\frac{y^4}{4h} + \frac{y^3}{3}\right]_0^h$$

$$= bh^3\left[-\frac{1}{4} + \frac{1}{3}\right]$$

$$= \frac{bh^3}{12} \qquad \leftarrow \text{Ans.}$$

⑦ Limits of integration are from bottom of triangle to top.

Q (16):

If a triangle of height d and base b is vertical and submerged in liquid with its vertex a distance a below the liquid surface (see Fig. 3-2), derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(a + \frac{2d}{3}\right) + \frac{bd^3/36}{(a+2d/3)(bd/2)} = \left(a + \frac{2d}{3}\right) + \frac{d^2}{18(a+2d/3)}$$

$$= \frac{18(a^2 + 4ad/3 + 4d^2/9) + d^2}{18(a+2d/3)} = \frac{6a^2 + 8ad + 3d^2}{6(a+2d/3)}$$

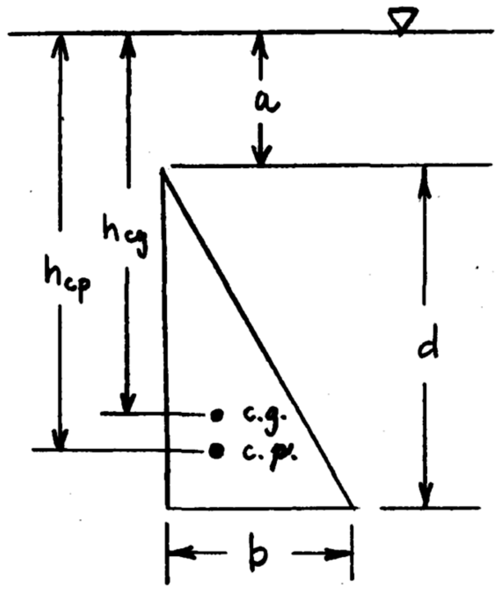


Fig. 3-2

Q (17):

If a triangle of height d and base b is vertical and submerged in liquid with its base at the liquid surface (see Fig. 3-3), derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{3} + \frac{bd^3/36}{(d/3)(bd/2)} = \frac{d}{3} + \frac{d}{6} = \frac{d}{2}$$

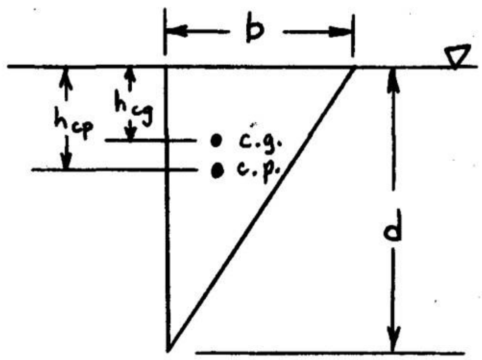


Fig. 3-3

Q (18):

A circular area of diameter *d* is vertical and submerged in a liquid. Its upper edge is coincident with the liquid surface (see Fig. 3-4). Derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{2} + \frac{\pi d^4/64}{(d/2)(\pi d^2/4)} = \frac{d}{2} + \frac{d}{8} = \frac{5d}{8}$$

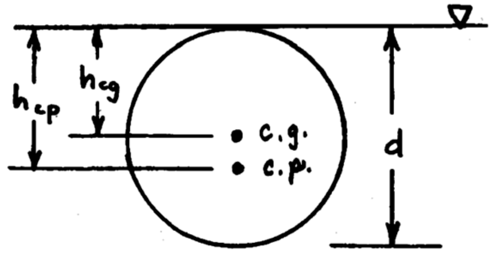


Fig. 3-4

Q (19):

A vertical semicircular area of diameter *d* and radius *r* is submerged and has its diameter in a liquid surface (see Fig. 3-5). Derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} \qquad h_{cg} = \frac{4r}{3\pi} \qquad I_x = \frac{1}{2}\left(\frac{\pi d^4}{64}\right) = \frac{1}{2}\left[\frac{\pi(2r)^4}{64}\right] = \frac{\pi r^4}{8}$$

$$I_{cg} = \frac{\pi r^4}{8} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(r^4) \qquad h_{cp} = \frac{4r}{3\pi} + \frac{[\pi/8 - 8/(9\pi)](r^4)}{[4r/(3\pi)][(\pi r^2/2)]} = 0.589r$$

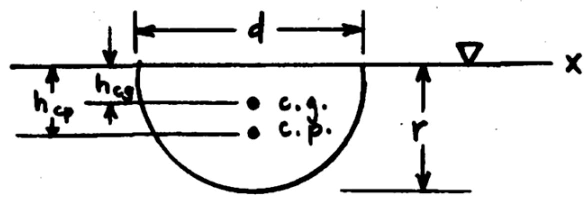


Fig. 3-5

Q (20):

A vertical, rectangular gate with water on one side is shown in Fig. 3-7. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)(3 + 1.2/2)[(2)(1.2)] = 84.59 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(3 + \frac{1.2}{2}\right) + \frac{(2)(1.2)^3/12}{(3 + 1.2/2)[(2)(1.2)]} = 3.633 \text{ m}$$

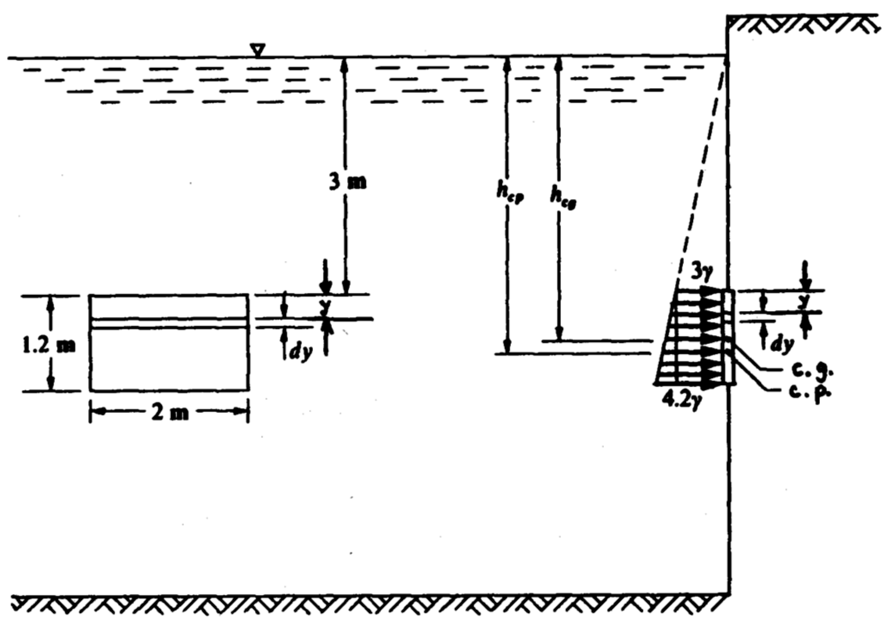


Fig. 3-7

Q (21):

3.11 An inclined, rectangular gate with water on one side is shown in Fig. 3-9. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (62.4)[8 + \tfrac{1}{2}(4\cos 60^\circ)][(4)(5)] = 11\,230\text{ lb}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left(\frac{8}{\cos 60^\circ} + \frac{4}{2}\right) + \frac{(5)(4)^3/12}{(8/\cos 60^\circ + \frac{4}{2})[(4)(5)]} = 18.07\text{ ft}$$

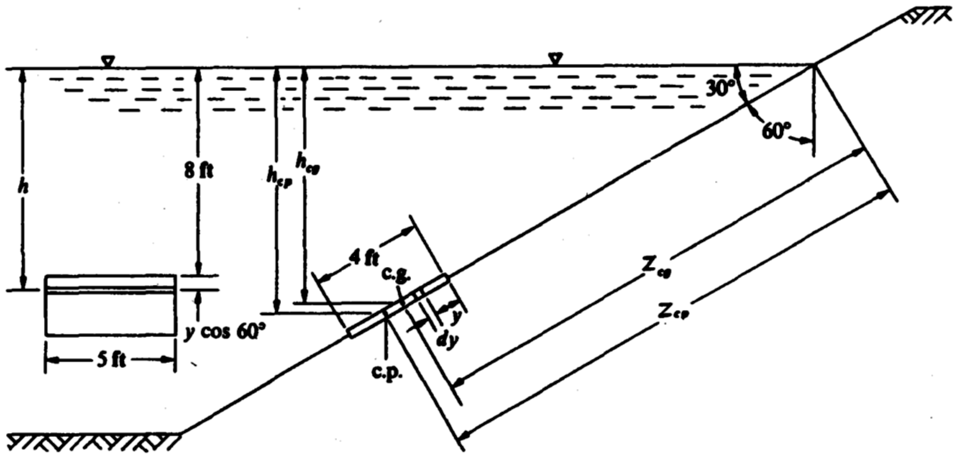


Fig. 3-9

3.12 Solve Prob. 3.11 by the integration method.

$$F = \int \gamma h\, dA = \int_0^4 (62.4)(8 + y\cos 60^\circ)(5\,dy) = (312)\left[8y + \frac{y^2}{4}\right]_0^4 = 11\,230\text{ lb}$$

$$h_{cp} = \frac{\int \gamma h^2\, dA}{F} = \frac{\int_0^4 (62.4)(8 + y\cos 60^\circ)^2(5\,dy)}{11\,230} = \frac{\int_0^4 (312)(64 + 8y + y^2/4)\,dy}{11\,230}$$

$$= \frac{(312)[64y + 4y^2 + y^3/12]_0^4}{11\,230} = 9.04\text{ ft}$$

Q (22):

An inclined, circular gate with water on one side is shown in Fig. 3-10. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)[1.5 + \tfrac{1}{2}(1.0 \sin 60°)][\pi(1.0)^2/4] = 14.86 \text{ kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left[\frac{1.5}{\sin 60°} + \frac{1}{2}(1.0)\right] + \frac{\pi(1.0)^4/64}{[1.5/\sin 60° + \frac{1}{2}(1.0)][\pi(1.0)^2/4]} = 2.260 \text{ m}$$

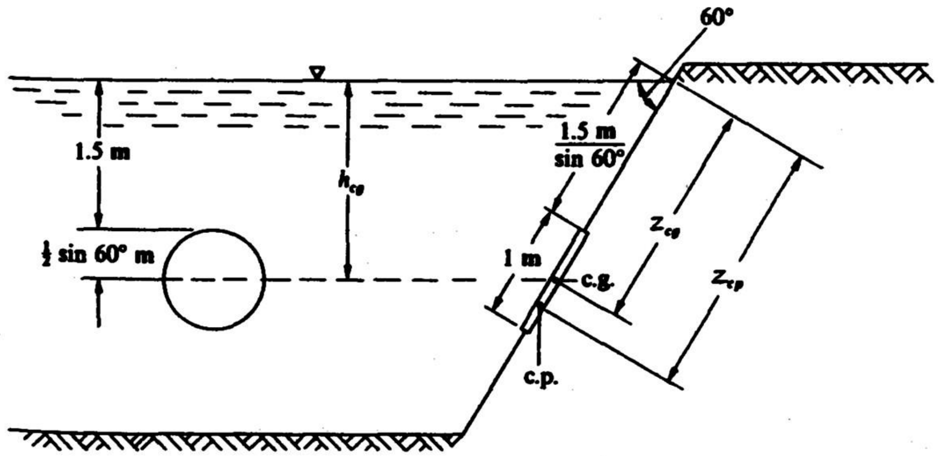


Fig. 3-10

Q (23):

A vertical, triangular gate with water on one side is shown in Fig. 3-11. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)[3 + \tfrac{2}{3}(1)][(1.2)(1)/2] = 21.54 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = [3 + (\tfrac{2}{3})(1)] + \frac{(1.2)(1)^3/36}{[3 + \tfrac{2}{3}(1)][(1.2)(1)/2]} = 3.68 \text{ m}$$

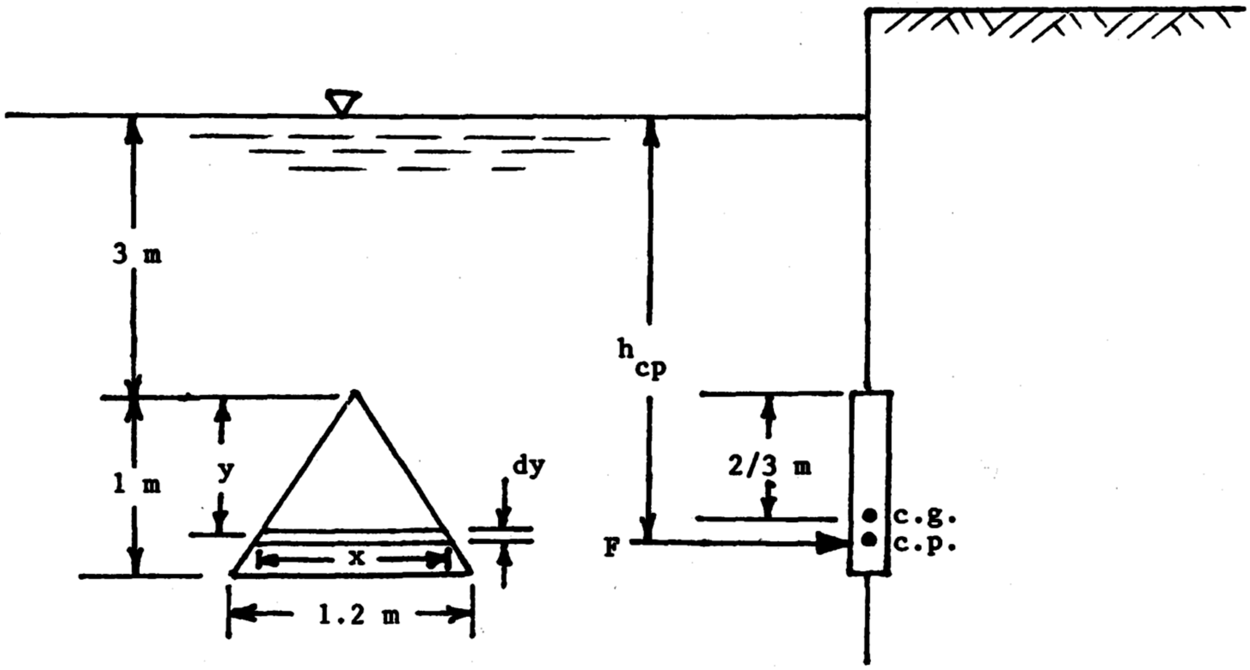


Fig. 3-11

Q (24):

The gate in Fig. 3-13 is 4 ft wide, is hinged at point *B*, and rests against a smooth wall at *A*. Compute (*a*) the force on the gate due to seawater pressure, (*b*) the (horizontal) force *P* exerted by the wall at point *A*, and (*c*) the reaction at hinge *B*.

(*a*)

$$F = \gamma h_{cg} A = (64)(17 - \tfrac{7.2}{2})[(4)(12)] = 30\,106 \text{ lb}$$

(*b*)

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(4)(12)^3/12](\frac{7.2}{12})}{(17 - \frac{7.2}{2})[(4)(12)]} = -0.537 \text{ ft}$$

$$\sum M_B = 0 \qquad (P)(7.2) - (30\,106)(12 - 6 - 0.537) = 0 \qquad P = 22\,843 \text{ lb}$$

(*c*)

$$\sum F_x = 0 \qquad B_x + (30\,106)(\tfrac{7.2}{12}) - 22\,843 = 0 \qquad B_x = 4779 \text{ lb}$$

$$\sum F_y = 0 \qquad B_y - (30\,106)(\tfrac{9.6}{12}) = 0 \qquad B_y = 24\,085 \text{ lb}$$

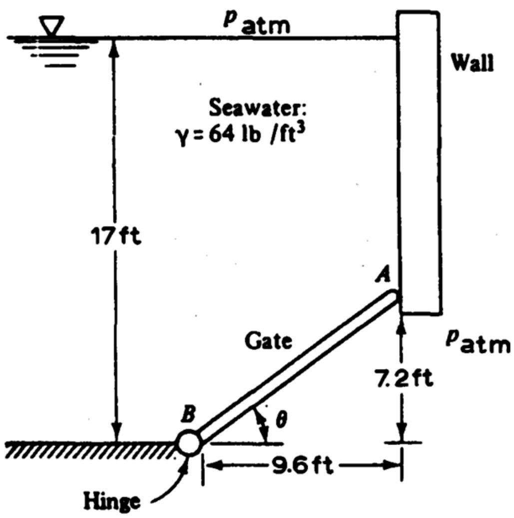


Fig. 3-13(*a*)

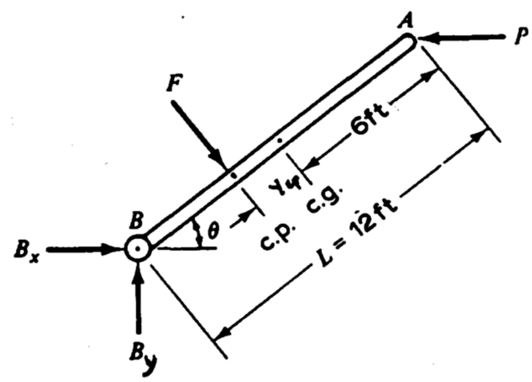


Fig. 3-13(*b*)

Q (25):

The tank in Fig. 5-26*a* is 3 m wide into the paper. Neglecting atmospheric pressure, compute the hydrostatic horizontal, vertical, and resultant force on quarter-circle panel *BC*.

▮ $F_H = \gamma h_{cg} A = (9.79)(4 + \frac{5}{2})[(5)(3)] = 954.5$ kN, F_V = weight of water above panel $BC = (9.79)[(3)(5)(4)] + (9.79)[(3)(\pi)(5)^2/4] = 1164$ kN, $F_{resultant} = \sqrt{954.5^2 + 1164^2} = 1505$ kN. As seen in Fig. 5-26*b*, $F_{resultant}$ passes through point *O* and acts down and to the right at an angle of arctan (1164/954.5), or 50.6°.

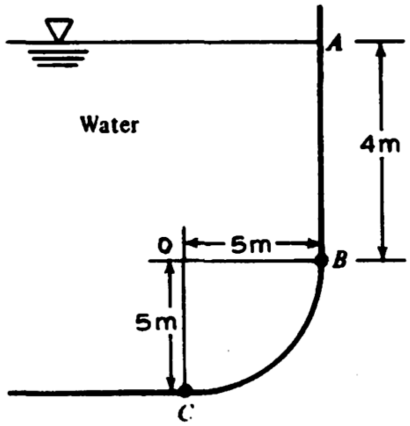


Fig. 5-26(*a*)

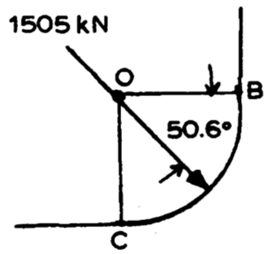


Fig. 5-26(*b*)

Gate *AB* in Fig. 5-27*a* is a quarter circle 7 ft wide, hinged at *B* and resting against a smooth wall at *A*. Compute the reaction forces at *A* and *B*.

$$F_H = \gamma h_{cg} A = (64)(11 - \tfrac{6}{2})[(7)(6)] = 21\,504 \text{ lb} \qquad y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(7)(6)^3/12](\sin 90^\circ)}{(11 - \frac{6}{2})[(7)(6)]} = -0.375 \text{ ft}$$

Thus, F_H acts at $\frac{6}{2} - 0.375$, or 2.625 ft above point *B*. F_V = weight of seawater above gate AB = $(64)(7)[(11)(6)] - (64)(7)[(\pi)(6)^2/4] = 29\,568 - 12\,667 = 16\,901$ lb. The location of F_V can be determined by taking moments about point *A* in Fig. 5-27*b*. $(29\,568)(\frac{6}{2}) - (12\,667)[(4)(6)/(3\pi)] = 16\,901x$, $x = 3.340$ ft. The forces acting on the gate are shown in Fig. 5-27*c*.

$$\sum M_B = 0 \qquad (21\,504)(2.625) + (16\,901)(6 - 3.340) - 6A_x = 0 \qquad A_x = 16\,901 \text{ lb}$$

$$\sum F_x = 0 \qquad 21\,504 - B_x - 16\,901 = 0 \qquad B_x = 4603 \text{ lb}$$

$$\sum F_y = 0 \qquad B_y - 16\,901 = 0 \qquad B_y = 16\,901 \text{ lb}$$

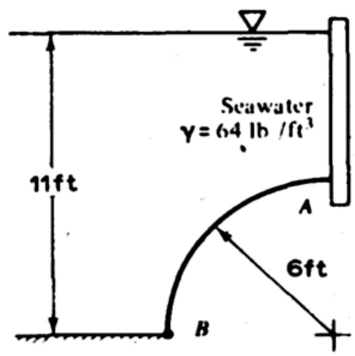


Fig. 5-27(*a*)

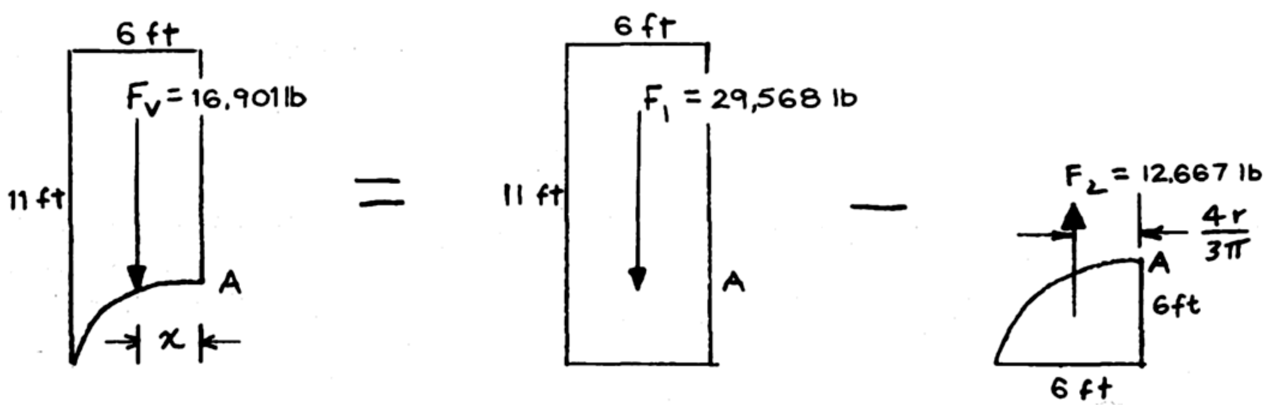


Fig. 5-27(*b*)

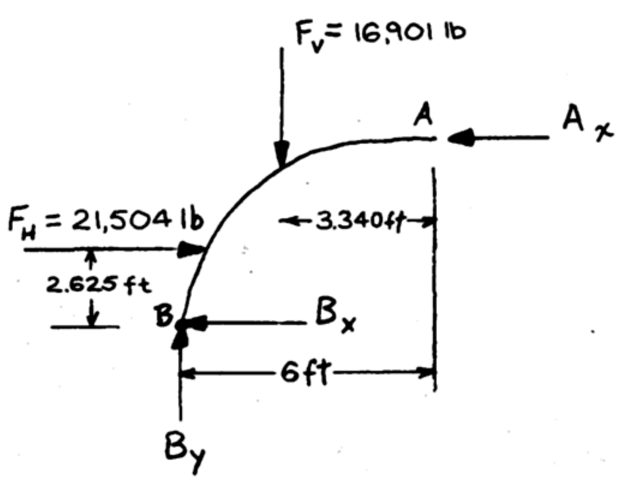


Fig. 5-27(c)

Q (26)

5.43 In the cross section shown in Fig. 5-40, *BC* is a quarter-circle. If the tank contains water to a depth of 6 ft, determine the magnitude and location of the horizontal and vertical components on wall *ABC* per 1 ft width.

$$F_H = \gamma \bar{h} A = (62.4)[(0+6)/2][(1)(6)] = 1123 \text{ lb} \qquad h_{cp} = (\tfrac{2}{3})(6) = 4.00 \text{ ft}$$

$$F_V = \text{weight of water above surface } BC = (62.4)(1)[(6)(5)] - (62.4)(1)[(\pi)(5)^2/4] = 1872 - 1225 = 647 \text{ lb}$$

The location of F_V can be determined by taking moments about point *B*. $(1872)(\tfrac{5}{2}) - (1225)[(4)(5)/(3\pi)] = 647x_{cp}$, $x_{cp} = 3.22$ ft.

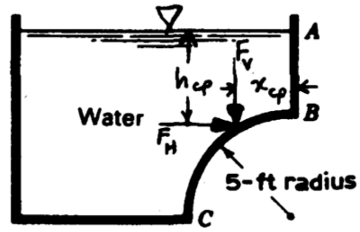


Fig. 5-40

5.44 Rework Prob. 5.43 where the tank is closed and contains gas at a pressure of 10 psi.

$$F_H = pA_V = [(10)(144)][(1)(6)] = 8640 \text{ lb} \qquad h_{cp} = \tfrac{6}{2} = 3.00 \text{ ft}$$

$$F_V = pA_H = [(10)(144)][(1)(5)] = 7200 \text{ lb} \qquad x_{cp} = \tfrac{5}{2} = 2.50 \text{ ft}$$

Buoyancy force

Q (27)

A stone weighs 105 lb in air. When submerged in water, it weighs 67.0 lb. Find the volume and specific gravity of the stone.

Buoyant force (F_b) = weight of water displaced by stone (W) = 105 − 67.0 = 38.0 lb

$$W = \gamma V = 62.4V \qquad 38.0 = 62.4V \qquad V = 0.609\ \text{ft}^3$$

$$\text{s.g.} = \frac{\text{weight of stone in air}}{\text{weight of equal volume of water}} = \frac{105}{(0.609)(62.4)} = 2.76$$

Q (28)

A piece of irregularly shaped metal weighs 300.0 N in air. When the metal is completely submerged in water, it weighs 232.5 N. Find the volume of the metal.

$$F_b = W \qquad 300.0 - 232.5 = [(9.79)(1000)](V) \qquad V = 0.00689\ \text{m}^3$$

Q (29)

A cube of timber 1.25 ft on each side floats in water as shown in Fig. 6-1. The specific gravity of the timber is 0.60. Find the submerged depth of the cube.

$$F_b = W \qquad 62.4[(1.25)(1.25)(D)] = [(0.60)(62.4)][(1.25)(1.25)(1.25)] \qquad D = 0.750\ \text{ft}$$

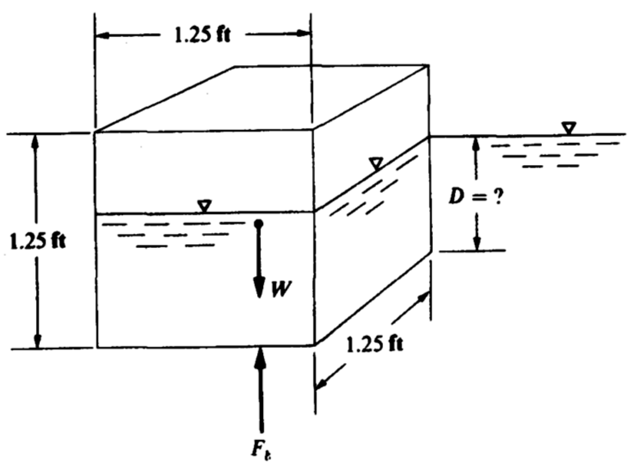


Fig. 6-1

Special Case 1: Fluids at Rest

For fluids at rest or moving on a straight path at constant velocity, all components of acceleration are zero, and the relations in Eqs. 3–43 reduce to

$$\frac{\partial P}{\partial x} = 0, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{dP}{dz} = -\rho g \qquad \textbf{(3–44)}$$

Which confirm that, in fluids at rest, the pressure remains constant in any horizontal direction (*P* is independent of *x* and *y*) and varies only in the vertical direction as a result of gravity and thus $P = P(z)$. These relations are applicable for both compressible and incompressible fluids.

Special Case 2: Free Fall of a Fluid Body

A freely falling body accelerates under the influence of gravity. When the air resistance is negligible, the acceleration of the body equals the gravitational acceleration, and acceleration in any horizontal direction is zero. Therefore, $a_x = a_y = 0$ and $a_z = -g$. Then the equations of motion for accelerating fluids (Eqs. 3–43) reduce to

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0 \quad \rightarrow \quad P = \text{constant} \qquad \textbf{(3–45)}$$

Therefore, in a frame of reference moving with the fluid, it behaves like it is in an environment with zero gravity. Also, the gage pressure in a drop of liquid in free fall is zero throughout. (Actually, the gage pressure is slightly above zero due to surface tension, which holds the drop intact.) When the direction of motion is reversed and the fluid is forced to accelerate vertically with $az = +g$ by placing the fluid container in an elevator or a space vehicle propelled upward by a rocket engine, the pressure gradient in the z-direction is dP/dz =+2rg. Therefore, the pressure difference across a fluid layer now doubles relative to the stationary fluid case (Fig. 3–49).

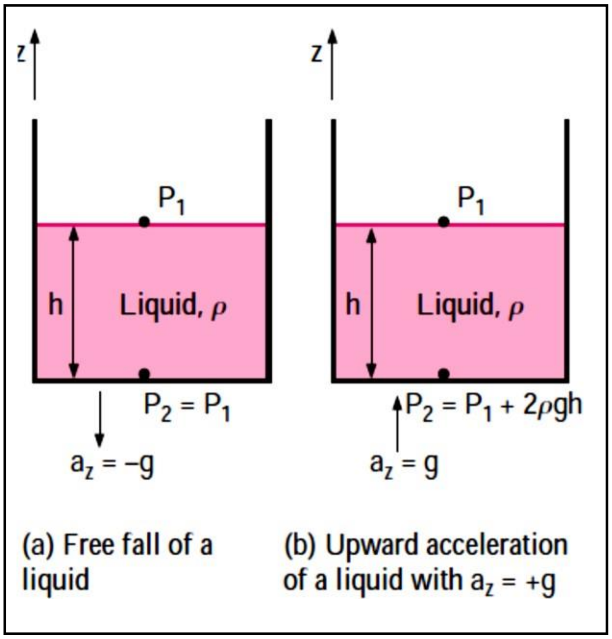


Figure (3-49) The effect of acceleration on the pressure of a liquid during free fall and upward acceleration.

Therefore, in a frame of reference moving with the fluid, it behaves like it is in an environment with zero gravity. Also, the gage pressure in a drop of liquid in free fall is zero throughout. (Actually, the gage pressure is slightly above zero due to surface tension, which holds the drop intact.) When the direction of motion is reversed and the fluid is forced to accelerate vertically with a_z =+g by placing the fluid container in an elevator or a space vehicle propelled upward by a rocket engine, the pressure gradient in the z-direction is dP/dz =-2rg. Therefore, the pressure difference across a fluid layer now doubles relative to the stationary fluid case (Fig. 3–49).

Acceleration on a Straight Path

Consider a container partially filled with a liquid. The container is movingon a straight path with a constant acceleration. We take the projection of the path of motion on the horizontal plane to be the *x*-axis, and the projection on the vertical plane to be the *z*-axis, as shown in Fig. 3–50.

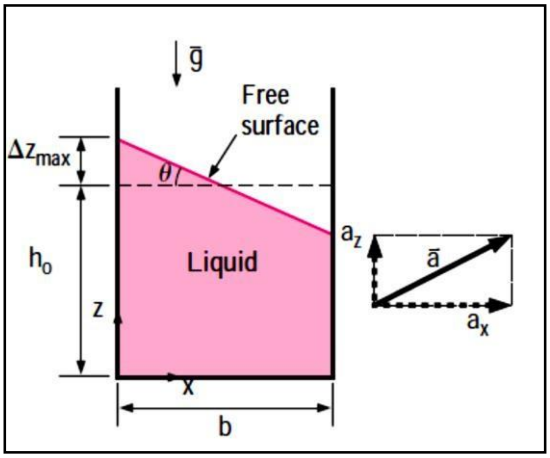


Figure 3-50 Rigid-body motion of a liquid in a linearly accelerating tank.

The *x*- and *z*components of acceleration are a_x and a_z. There is no movement in the *y*direction, and thus the acceleration in that direction is zero, $a_y = 0$. Then the equations of motion for accelerating fluids (Eqs. 3–43) reduce to

$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z) \qquad \textbf{(3–46)}$$

Therefore, pressure is independent of y. Then the total differential of $P = P(x, z)$, which is $(\partial P/\partial x)\,dx + (\partial P/\partial z)\,dz$, becomes

$$dP = -\rho a_x\,dx - \rho(g + a_z)\,dz \qquad \textbf{(3–47)}$$

For ρ = constant, the pressure difference between two points 1 and 2 in the fluid is determined by integration to be

$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \qquad \textbf{(3–48)}$$

Taking point 1 to be the origin ($x = 0$, $z = 0$) where the pressure is P_0 and point 2 to be any point in the fluid (no subscript), the pressure distribution can be expressed as:

$$P = P_0 - \rho a_x x - \rho(g + a_z)z \qquad (3\text{–}49)$$

The vertical rise (or drop) of the free surface at point 2 relative to point 1 can be determined by choosing both 1 and 2 on the free surface (so that $P1= P2$), and solving Eq. 3–48 for $z_2 - z_1$ (Fig. 3–51),

$$\text{Vertical rise of surface:} \qquad \Delta z_s = z_{s2} - z_{s1} = -\frac{a_x}{g + a_z}(x_2 - x_1) \qquad (3\text{–}50)$$

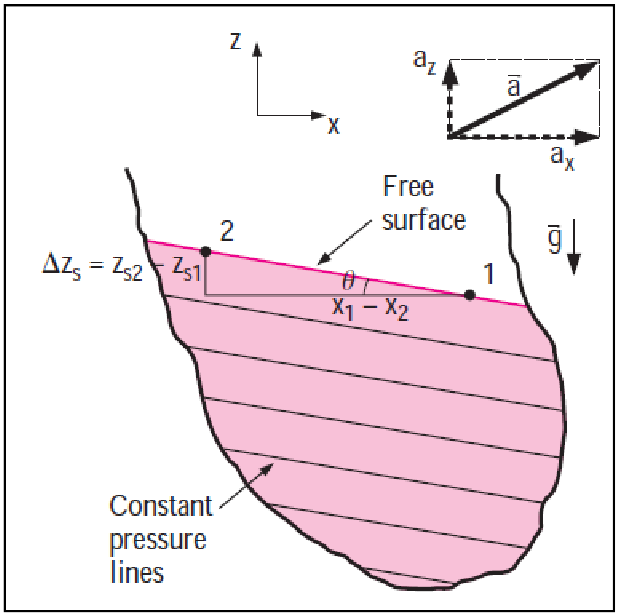


FIGURE 3–51 Lines of constant pressure (whichare the projections of the surfaces of Constant pressure on the *xz*-plane) in a linearly accelerating liquid, and the vertical rise.

Where z_s is the *z*-coordinate of the liquid's free surface. The equation for surfaces of constant pressure, called **isobars,** is obtained from Eq. 3–47 by setting $dP = 0$ and replacing *z* by zisobar, which is the *z*-coordinate (the vertical distance) of the surface as a function of *x*. It gives

Surfaces of constant pressure: $$\frac{dz_{isobar}}{dx} = -\frac{a_x}{g + a_z} = \text{constant} \qquad (3\text{–}51)$$

Thus we conclude that the isobars (including the free surface) in an incompressible fluid with constant acceleration in linear motion are parallel surfaces whose slope in the *xz*-plane is

Slope of isobars: $$\text{Slope} = \frac{dz_{isobar}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta \qquad (3\text{–}52)$$

Obviously, the free surface of such a fluid is a *plane* surface, and it is inclined unless $a_x = 0$ (the acceleration is in the vertical direction only). Also, the conservation of mass together with the assumption of incompressibility (**ρ**= *constant*) requires that the volume of the fluid remain constant before and during acceleration. Therefore, the rise of fluid level on one side must be balanced by a drop of fluid level on the other side.

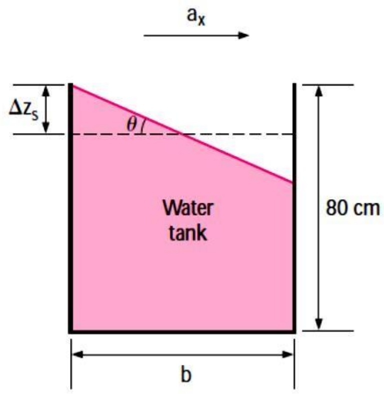


FIGURE 3–52
Schematic for Example 3–12.

EXAMPLE 3–12 Overflow from a Water Tank During Acceleration

An 80-cm-high fish tank of cross section 2 m × 0.6 m that is initially filled with water is to be transported on the back of a truck (Fig. 3–52). The truck accelerates from 0 to 90 km/h in 10 s. If it is desired that no water spills during acceleration, determine the allowable initial water height in the tank. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion?

SOLUTION A fish tank is to be transported on a truck. The allowable water height to avoid spill of water during acceleration and the proper orientation are to be determined.

Assumptions **1** The road is horizontal during acceleration so that acceleration has no vertical component ($a_z = 0$). **2** Effects of splashing, braking, driving over bumps, and climbing hills are assumed to be secondary and are not considered. **3** The acceleration remains constant.

Analysis We take the *x*-axis to be the direction of motion, the *z*-axis to be the upward vertical direction, and the origin to be the lower left corner of the tank. Noting that the truck goes from 0 to 90 km/h in 10 s, the acceleration of the truck is

$$a_x = \frac{\Delta V}{\Delta t} = \frac{(90 - 0)\ \text{km/h}}{10\ \text{s}}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 2.5\ \text{m/s}^2$$

The tangent of the angle the free surface makes with the horizontal is

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{2.5}{9.81 + 0} = 0.255 \qquad \text{(and thus } \theta = 14.3°\text{)}$$

The maximum vertical rise of the free surface occurs at the back of the tank, and the vertical midplane experiences no rise or drop during acceleration since it is a plane of symmetry. Then the vertical rise at the back of the tank relative to the midplane for the two possible orientations becomes

Case 1: The long side is parallel to the direction of motion:

$$\Delta z_{s1} = (b_1/2)\tan\theta = [(2\ \text{m})/2] \times 0.255 = 0.255\ \text{m} = \mathbf{25.5\ cm}$$

Case 2: The short side is parallel to the direction of motion:

$$\Delta z_{s2} = (b_2/2)\tan\theta = [(0.6\ \text{m})/2] \times 0.255 = 0.076\ \text{m} = \mathbf{7.6\ cm}$$

Rotation in a Cylindrical Container

We know from experience that when a glass filled with water is rotated about its axis, the fluid is forced outward as a result of the so-called centrifugal force, and the free surface of the liquid becomes concave. This is known as the forced vortex motion.

Consider a vertical cylindrical container partially filled with a liquid. The container is now rotated about its axis at a constant angular velocity of ω, as shown in Fig. 3–53. After initial transients, the liquid will move as a rigid body together with the container. There is no deformation, and thus there can be no shear stress, and every fluid particle in the container moves with the same angular velocity.

This problem is best analyzed in cylindrical coordinates (r, θ, z), with z taken along the centerline of the container directed from the bottom toward the free surface, since the shape of the container is a cylinder, and the fluid particles undergo a circular motion. The centripetal acceleration of a fluid particle rotating with a constant angular velocity of ω at a distance r from the axis of rotation is $r\omega^2$ and is directed radially toward the axis of rotation (negative r-direction). That is, $a_r = -r\omega^2$. There is symmetry about the z-axis, which is the axis of rotation, and thus there is no θ dependence. Then $P = P(r, z)$ and $a_\theta = 0$. Also, $a_z = 0$ since there is no motion in the z-direction.

Then the equations of motion for rotating fluids (Eqs. 3–43) reduce to

$$\frac{\partial P}{\partial r} = \rho r\omega^2, \qquad \frac{\partial P}{\partial \theta} = 0, \qquad \text{and} \qquad \frac{\partial P}{\partial z} = -\rho g \tag{3–53}$$

Then the total differential of $P = P(r, z)$, which is $dP = (\partial P/\partial r)dr + (\partial P/\partial z)dz$, becomes

$$dP = \rho r\omega^2\, dr - \rho g\, dz \tag{3–54}$$

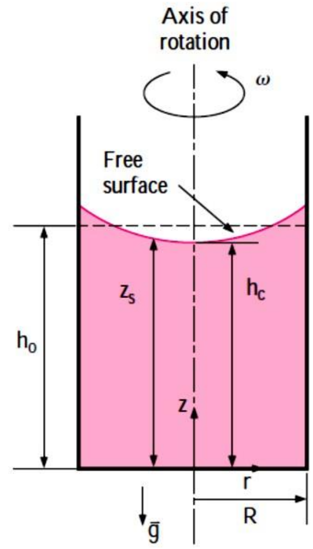


FIGURE 3–53
Rigid-body motion of a liquid in a rotating vertical cylindrical container.

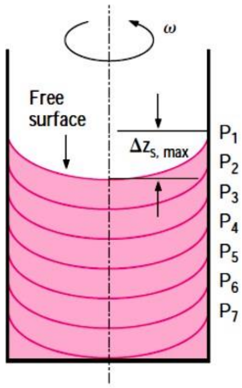


FIGURE 3–54
Surfaces of constant pressure in a rotating liquid.

The equation for surfaces of constant pressure is obtained by setting $dP = 0$ and replacing z by z_{isobar}, which is the z-value (the vertical distance) of the surface as a function of r. It gives

$$\frac{dz_{isobar}}{dr} = \frac{r\omega^2}{g} \quad \textbf{(3–55)}$$

Integrating, the equation for the surfaces of constant pressure is determined to be

Surfaces of constant pressure: $$z_{isobar} = \frac{\omega^2}{2g}r^2 + C_1 \quad \textbf{(3–56)}$$

which is the equation of a parabola. Thus we conclude that the surfaces of constant pressure, including the free surface, are paraboloids of revolution (Fig. 3–54).

The value of the integration constant C_1 is different for different paraboloids of constant pressure (i.e., for different isobars). For the free surface, setting $r = 0$ in Eq. 3–56 gives $z_{isobar}(0) = C_1 = h_c$, where h_c is the distance of the free surface from the bottom of the container along the axis of rotation (Fig. 3–53). Then the equation for the free surface becomes

$$z_s = \frac{\omega^2}{2g}r^2 + h_c \quad \textbf{(3–57)}$$

where z_s is the distance of the free surface from the bottom of the container at radius r. The underlying assumption in this analysis is that there is sufficient liquid in the container so that the entire bottom surface remains covered with liquid.

The volume of a cylindrical shell element of radius r, height z_s, and thickness dr is $dV = 2\pi r z_s\, dr$. Then the volume of the paraboloid formed by the free surface is

$$V = \int_{r=0}^{R} 2\pi z_s r\, dr = 2\pi \int_{r=0}^{R} \left(\frac{\omega^2}{2g}r^2 + h_c\right) r\, dr = \pi R^2\left(\frac{\omega^2 R^2}{4g} + h_c\right) \quad \textbf{(3–58)}$$

Since mass is conserved and density is constant, this volume must be equal to the original volume of the fluid in the container, which is

$$V = \pi R^2 h_0 \quad \textbf{(3–59)}$$

where h_0 is the original height of the fluid in the container with no rotation. Setting these two volumes equal to each other, the height of the fluid along the centerline of the cylindrical container becomes

$$h_c = h_0 - \frac{\omega^2 R^2}{4g} \quad \textbf{(3–60)}$$

Then the equation of the free surface becomes

Free surface: $$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2) \quad \textbf{(3–61)}$$

The maximum vertical height occurs at the edge where $r = R$, and the maximum height difference between the edge and the center of the free surface is determined by evaluating z_s at $r = R$ and also at $r = 0$, and taking their difference,

Maximum height difference: $$\Delta z_{s,\,max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g} R^2 \quad \textbf{(3–62)}$$

When ρ = constant, the pressure difference between two points 1 and 2 in the fluid is determined by integrating $dP = \rho r \omega^2\, dr - \rho g\, dz$. This yields

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1) \quad \textbf{(3–63)}$$

Taking point 1 to be the origin ($r = 0$, $z = 0$) where the pressure is P_0 and point 2 to be any point in the fluid (no subscript), the pressure distribution can be expressed as

Pressure variation: $$P = P_0 + \frac{\rho\omega^2}{2} r^2 - \rho g z \quad \textbf{(3–64)}$$

Note that at a fixed radius, the pressure varies hydrostatically in the vertical direction, as in a fluid at rest. For a fixed vertical distance z, the pressure varies with the square of the radial distance r, increasing from the centerline toward the outer edge. In any horizontal plane, the pressure difference between the center and edge of the container of radius R is $\Delta P = \rho\omega^2 R^2/2$.

EXAMPLE 3–13 Rising of a Liquid During Rotation

A 20-cm-diameter, 60-cm-high vertical cylindrical container, shown in Fig. 3–55, is partially filled with 50-cm-high liquid whose density is 850 kg/m^3. Now the cylinder is rotated at a constant speed. Determine the rotational speed at which the liquid will start spilling from the edges of the container.

SOLUTION A vertical cylindrical container partially filled with a liquid is rotated. The angular speed at which the liquid will start spilling is to be determined.

Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** The bottom surface of the container remains covered with liquid during rotation (no dry spots).

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin ($r = 0$, $z = 0$), the equation for the free surface of the liquid is given as

$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

Then the vertical height of the liquid at the edge of the container where $r = R$ becomes

$$z_s(R) = h_0 + \frac{\omega^2 R^2}{4g}$$

where $h_0 = 0.5$ m is the original height of the liquid before rotation. Just before the liquid starts spilling, the height of the liquid at the edge of the container equals the height of the container, and thus $z_s(R) = 0.6$ m. Solving the last equation for ω and substituting, the maximum rotational speed of the container is determined to be

$$\omega = \sqrt{\frac{4g[z_s(R) - h_0]}{R^2}} = \sqrt{\frac{4(9.81 \text{ m/s}^2)[(0.6 - 0.5) \text{ m}]}{(0.1 \text{ m})^2}} = \mathbf{19.8 \ rad/s}$$

Noting that one complete revolution corresponds to 2π rad, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

$$\dot{n} = \frac{\omega}{2\pi} = \frac{19.8 \text{ rad/s}}{2\pi \text{ rad/rev}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \mathbf{189 \ rpm}$$

Therefore, the rotational speed of this container should be limited to 189 rpm to avoid any spill of liquid as a result of the centrifugal effect.

Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. We should also verify that our assumption of no dry spots is valid. The liquid height at the center is

$$z_s(0) = h_0 - \frac{\omega^2 R^2}{4g} = 0.4 \text{ m}$$

Since $z_s(0)$ is positive, our assumption is validated.

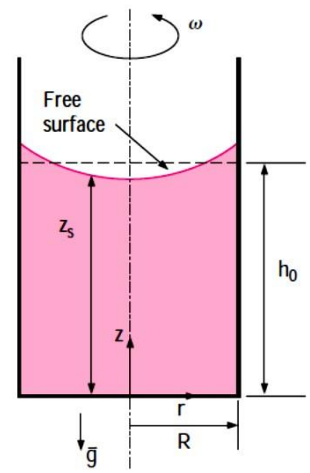


FIGURE 3–55
Schematic for Example 3–13.

Q (30)

3–108E An 8-ft-long tank open to the atmosphere initially contains 3-ft-high water. It is being towed by a truck on a level road. The truck driver applies the brakes and the water level at the front rises 0.5 ft above the initial level. Determine the deceleration of the truck. *Answer:* 4.08 ft/s²

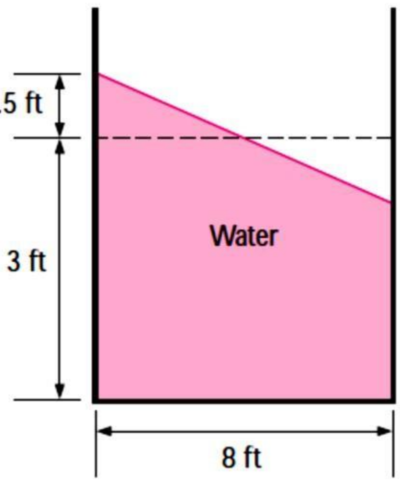


FIGURE P3–108E

3-108E

Solution A water tank partially filled with water is being towed by a truck on a level road. The maximum acceleration (or deceleration) of the truck to avoid spilling is to be determined.

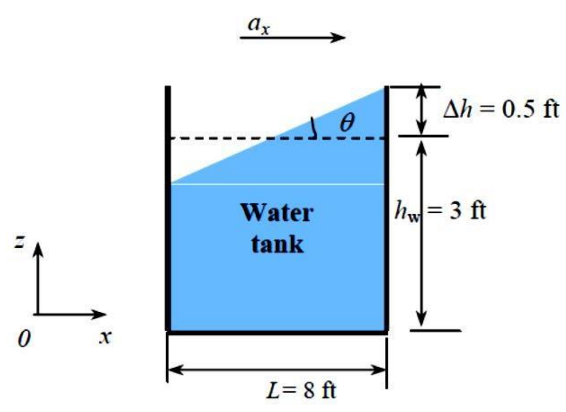
Assumptions **1** The road is horizontal so that deceleration has no vertical component ($a_z = 0$). **2** Effects of splashing and driving over bumps are assumed to be secondary, and are not considered. **3** The deceleration remains constant.

Analysis We take the x-axis to be the direction of motion, the z-axis to be the upward vertical direction. The shape of the free surface just before spilling is shown in figure. The tangent of the angle the free surface makes with the horizontal is given by

$$\tan\theta = \frac{-a_x}{g + a_z} \quad \rightarrow \quad a_x = -g\tan\theta$$

where $a_z = 0$ and, from geometric considerations, $\tan\theta$ is

$$\tan\theta = \frac{\Delta h}{L/2}$$



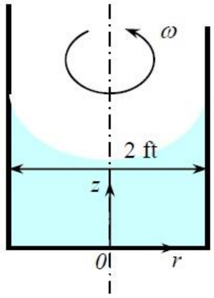
Substituting,

$$a_x = -g\tan\theta = -g\frac{\Delta h}{L/2} = -(32.2\ \text{ft/s}^2)\frac{0.5\ \text{ft}}{(8\ \text{ft})/2} = \mathbf{-4.08\ ft/s^2}$$

3–97E A 2-ft-diameter vertical cylindrical tank open to the atmosphere contains 1-ft-high water. The tank is now rotated about the centerline, and the water level drops at the center while it rises at the edges. Determine the angular velocity at which the bottom of the tank will first be exposed. Also determine the maximum water height at this moment.

3-97E

Solution A vertical cylindrical tank open to the atmosphere is rotated about the centerline. The angular velocity at which the bottom of the tank will first be exposed, and the maximum water height at this moment are to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Water is an incompressible fluid.

Analysis Taking the center of the bottom surface of the rotating vertical cylinder as the origin (r = 0, z = 0), the equation for the free surface of the liquid is given as

$$z_s(r) = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

where h_0 = 1 ft is the original height of the liquid before rotation. Just before dry spot appear at the center of bottom surface, the height of the liquid at the center equals zero, and thus $z_s(0)$ = 0. Solving the equation above for ω and substituting,

$$\omega = \sqrt{\frac{4gh_0}{R^2}} = \sqrt{\frac{4(32.2 \text{ ft/s}^2)(1 \text{ ft})}{(1 \text{ ft})^2}} = 11.35 \text{ rad/s} \cong \mathbf{11.4 \text{ rad/s}}$$

Noting that one complete revolution corresponds to 2π radians, the rotational speed of the container can also be expressed in terms of revolutions per minute (rpm) as

$$\dot{n} = \frac{\omega}{2\pi} = \frac{11.35 \text{ rad/s}}{2\pi \text{ rad/rev}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \mathbf{108 \text{ rpm}}$$

Therefore, the rotational speed of this container should be limited to 108 rpm to avoid any dry spots at the bottom surface of the tank.

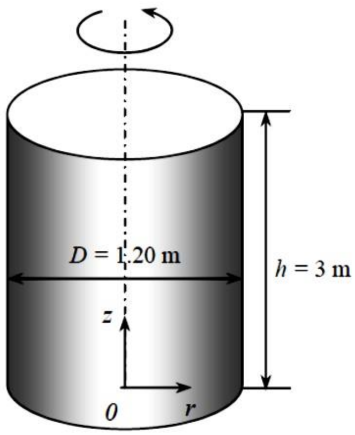
The maximum vertical height of the liquid occurs a the edges of the tank ($r = R$ = 1 ft), and it is

$$z_s(R) = h_0 + \frac{\omega^2 R^2}{4g} = (1 \text{ ft}) + \frac{(11.35 \text{ rad/s})^2 (1 \text{ ft})^2}{4(32.2 \text{ ft/s}^2)} = \mathbf{2.00 \text{ ft}}$$

3–105 A 1.2-m-diameter, 3-m-high sealed vertical cylinder is completely filled with gasoline whose density is 740 kg/m3. The tank is now rotated about its vertical axis at a rate of 70 rpm. Determine (*a*) the difference between the pressuresat the centers of the bottom and top surfaces and (*b*) the difference between the pressures at the center and the edge of the bottom surface.

3-105

Solution A vertical cylindrical tank is completely filled with gasoline, and the tank is rotated about its vertical axis at a specified rate. The pressures difference between the centers of the bottom and top surfaces, and the pressures difference between the center and the edge of the bottom surface are to be determined.



Assumptions **1** The increase in the rotational speed is very slow so that the liquid in the container always acts as a rigid body. **2** Gasoline is an incompressible substance.

Properties The density of the gasoline is given to be 740 kg/m³.

Analysis The pressure difference between two points 1 and 2 in an incompressible fluid rotating in rigid body motion is given by

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

where R = 0.60 m is the radius, and

$$\omega = 2\pi \dot{n} = 2\pi(70 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 7.330 \text{ rad/s}$$

(*a*) Taking points 1 and 2 to be the centers of the bottom and top surfaces, respectively, we have $r_1 = r_2 = 0$ and $z_2 - z_1 = h = 3\text{ m}$. Then,

$$P_{\text{center, top}} - P_{\text{center, bottom}} = 0 - \rho g(z_2 - z_1) = -\rho g h$$

$$= -(740 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 21.8 \text{ kN/m}^2 = \mathbf{21.8 \text{ kPa}}$$

(*b*) Taking points 1 and 2 to be the center and edge of the bottom surface, respectively, we have $r_1 = 0$, $r_2 = R$, and $z_2 = z_1 = 0$. Then,

$$P_{\text{edge, bottom}} - P_{\text{center, bottom}} = \frac{\rho\omega^2}{2}(R_2^2 - 0) - 0 = \frac{\rho\omega^2 R^2}{2}$$

$$= \frac{(740 \text{ kg/m}^3)(7.33 \text{ rad/s})^2(0.60 \text{ m})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = 7.16 \text{ kN/m}^2 = \mathbf{7.16 \text{ kPa}}$$

EXAMPLE 2.5 A 120-cm-long tank contains 80 cm of water and 20 cm of air maintained at 60 kPa above the water. The 60-cm-wide tank is accelerated at 10 m/s^2. After equilibrium is established, find the force acting on the bottom of the tank.

Solution: First, sketch the tank using the information given in the problem statement. It appears as in Fig. 2.12. The distance x can be related to y by using Eq. (2.26):

$$\tan\alpha = \frac{a_x}{g} = \frac{10}{9.81} = \frac{y}{x} \qquad \therefore\ y = 1.019x$$

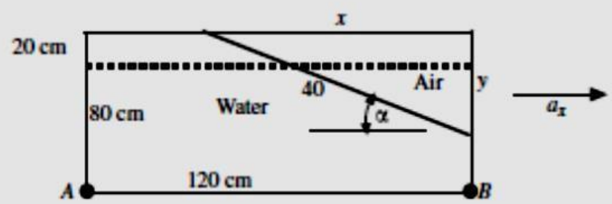


Figure 2.12

Equate the area of the air before and after to find either x or y:

$$120 \times 20 = \frac{1}{2}xy = \frac{1.019}{2}x^2 \qquad \therefore\ x = 68.63\text{ cm} \quad \text{and} \quad y = 69.94\text{ cm}$$

The pressure will remain unchanged in the air above the water since the air volume does not change. The pressures at A and B are then (use Eq. (2.25))

$$p_A = 60000 + 1000 \times 10 \times (1.20 - 0.6863) + 9810 \times 1.0\text{ m} = 74900\text{ Pa}$$
$$p_B = 60\,000 + 9810 \times (1.00 - 0.6994) = 62900\text{ Pa}$$

The average pressure on the bottom is $(p_A + p_B)/2$. Multiply the average pressure by the area to find the force acting on the bottom:

$$F = \frac{p_A + p_B}{2}A = \frac{74\,900 + 62\,900}{2}(1.2 \times 0.6) = 49610\text{ N}$$

EXAMPLE 2.6 The cylinder in Fig. 2.13 is rotated about the center axis as shown. What rotational speed is required so that the water just touches point A. Also, find the force on the bottom of the cylinder.

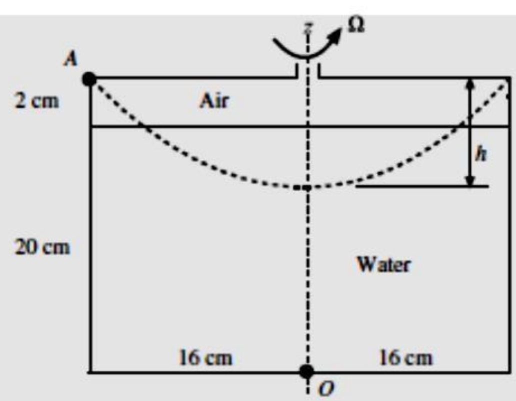


Figure 2.13

Solution: The volume of the air before and after must be the same. Recognizing that the volume of a paraboloid of revolution is half of the volume of a circular cylinder of the same radius and height, the height of the paraboloid of revolution is found:

$$\pi \times 0.16^2 \times 0.02 = \frac{1}{2}\pi \times 0.16^2 h \qquad \therefore\ h = 0.04\text{ m}$$

Use Eq. (2.30) to find Ω:

$$0.04 = \frac{\Omega^2 \times 0.16^2}{2 \times 9.81} \qquad \Omega = 5.54\,\text{rad/s}$$

The pressure on the bottom as a function of the radius r is $p(r)$, given by

$$p - p_0 = \frac{\rho\Omega^2}{2}(r^2 - r_1^2)$$

where $p_0 = 9810 \times (0.20 - 0.04) = 1570$ Pa. So,

$$p = \frac{1000 \times 5.54^2}{2} r^2 + 1570 = 15\,346 r^2 + 1570$$

The pressure is integrated over the area to find the force to be

$$\int_0^{0.16} (15\,346 r^2 + 1570) 2\pi r \, dr = 142.1\,\text{N}$$