جامعة الفراهيدي

الكلية التقنية الهندسية

هندسة الطيران

المرحلة الثالثة

تصميم ميكانيكي

مادة الكورس

Dr. Ahmed Ali Akbar

الكورس الأول

السعر: 4000

التسلسل : 20 >>>

2021 / 12/ 28

الأسم :

Load and stress analysis:

* - Equilibrium and Free body diagrams:

Equilibrium conditions:

$\Sigma F_x = 0$

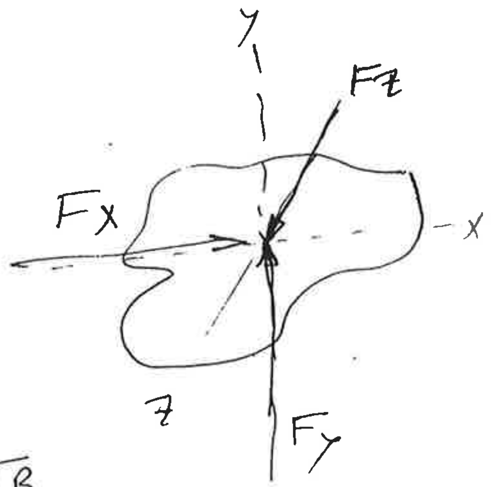
$\Sigma F_y = 0$

$\Sigma F_z = 0$

$\Sigma M_x = 0$

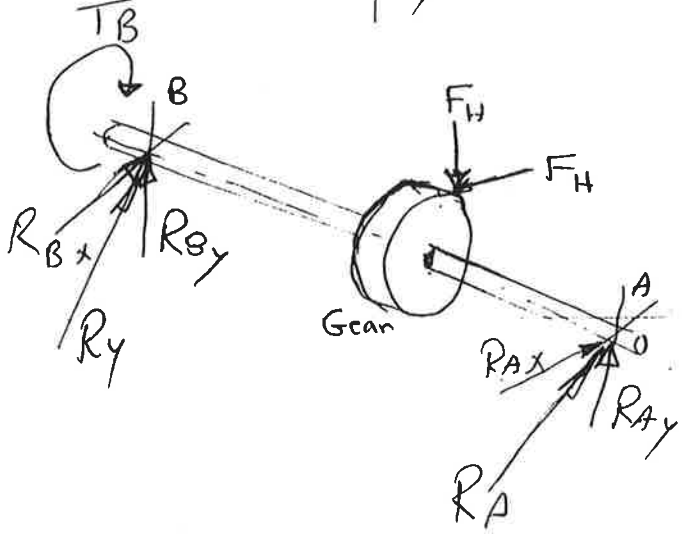
$\Sigma M_y = 0$

$\Sigma M_z = 0$



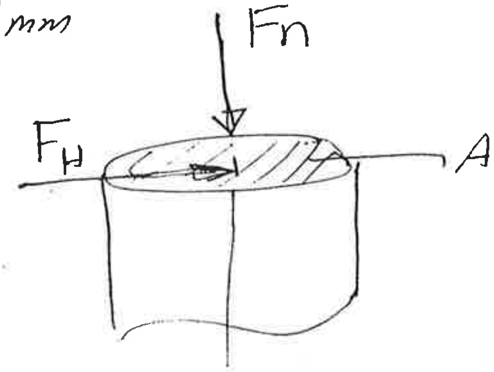
$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2}$

$\tan\theta = \frac{R_y}{R_x}$ direction



Stress $\sigma = \frac{F_n}{A}$ = normal stress N/mm^2

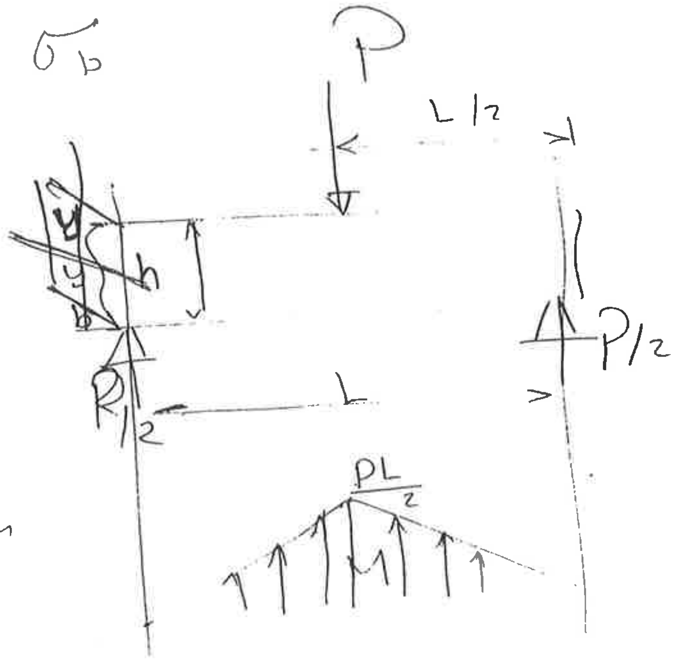
shear stress $= \frac{F_H}{A}$ N/mm^2



Bending stresses σ_b

Max. Bending Moment

$$M = \frac{PL}{2} \quad \text{---} \; ①$$

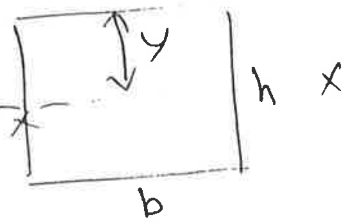


$$\sigma_b = \frac{M \cdot Y}{I} \quad \text{---} \; ②$$

where y = distance from neutral axis

$$y = \frac{h}{2}$$

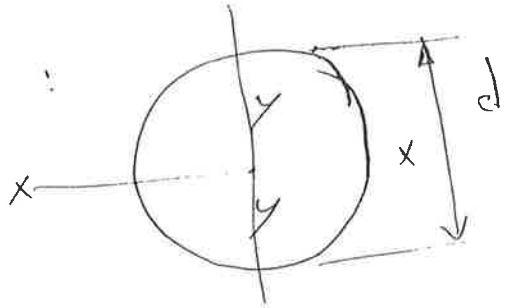
I for rectangular sections



$$I = \frac{bh^3}{12}$$

$$\sigma_b = \frac{M \cdot \frac{h}{2}}{\frac{bh^3}{12}} = \frac{6M}{bh^2} \quad \text{---} \; N/mm^2$$

For Circular Sections :



$$y = \frac{d}{2} = r$$

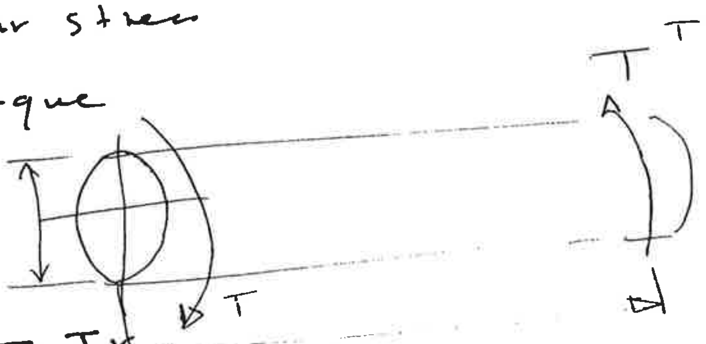
$$I = \frac{\pi}{64} d^4$$

$$\sigma_b = \frac{M \cdot \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32M}{\pi d^3} \quad N/mm^2$$

* A shaft Subjected to Torsion.

$\tau = G\theta \cdot r$ shear stress

$T = GJ \cdot \theta$ — Torque



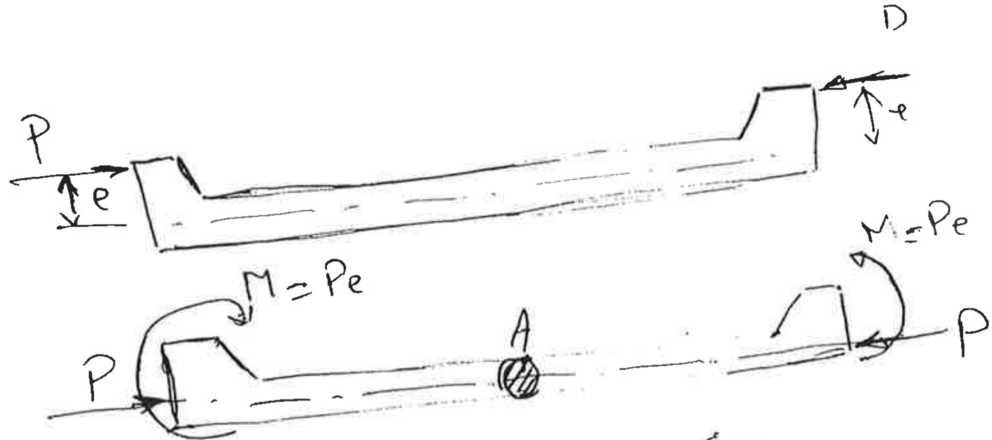
$\frac{T}{\tau} = \frac{GJ\theta}{G\theta \cdot r}$

$\frac{T}{\tau} = \frac{J}{r} \Rightarrow \tau = \frac{T \cdot r}{J}$

The shear stress resulted from the Torsional Moment T

$$\tau = \frac{T \cdot d/2}{\frac{\pi}{32} d^4} = \frac{16T}{\pi d^3} \quad \text{———} \quad *$$

* Combined axial load and bending load:



Combined stress in the member is:

$$\sigma_{comb} = \frac{P}{A} + \frac{P \cdot e}{\frac{I}{y}} \quad \text{———} \quad *$$

* Stresses due to Combined Torsion and Bending loading on a shaft.

$$\tau = \frac{T \cdot d/2}{J} = \frac{16T}{\pi d^3}$$

$$J = \frac{\pi}{32} d^4$$

$$\sigma_x = \frac{My}{I} = \frac{32M}{\pi d^3}$$

$$\sigma_{max} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \longrightarrow *$$

For designing the shaft diameter. the ~~larger~~ greater is selected.

$$M_{equivalent} = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) \longrightarrow *$$

Equivalent Torque $T = \sqrt{M^2 + T^2} \longrightarrow *$

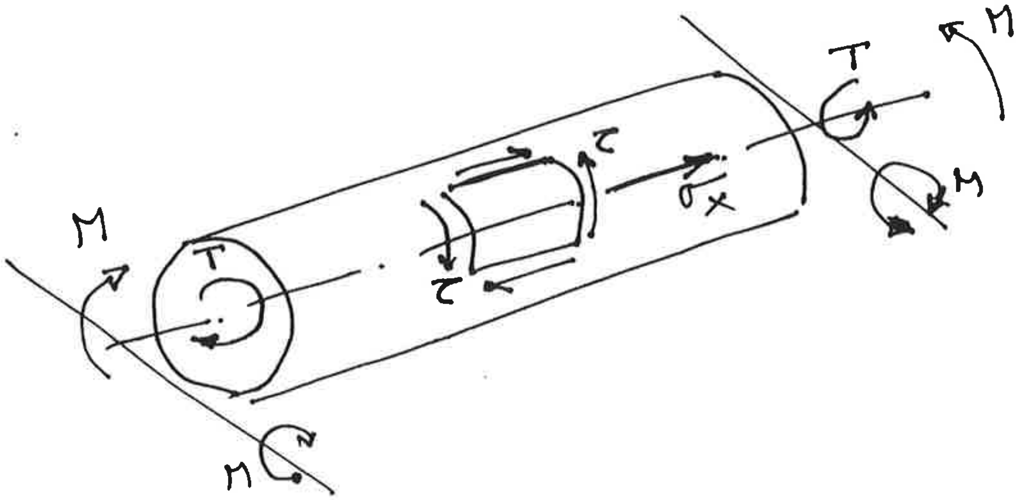
* For design it is necessary to find the principal stresses and Max. shear stress

$$\sigma_b = \frac{My}{I} \quad \text{———— (1)}$$

$$\sigma_b = \frac{M}{\frac{I}{y}} \quad \text{where } \frac{I}{y} = \text{modulus of section}$$

$$I = \frac{bh^3}{12} \text{ and } y = \frac{h}{2}$$

$$\text{Hence } \frac{I}{y} = \frac{2 \times bh^3}{12h} = \boxed{\frac{bh^2}{6}}$$



$$y = \frac{D}{2}$$

$$I = \frac{\pi}{64} D^4 \quad \text{———— *}$$

$$\sigma_{b_{max}} = \frac{M \cdot \frac{d}{2}}{\frac{\pi}{64} D^4} = \frac{32M}{\pi D^3} \quad \text{———— *}$$

The shear stress due to Torque T

$$\tau_{max} = \frac{T \cdot r}{J} = \frac{T \cdot \frac{D}{2}}{\frac{\pi}{32} D^4} = \frac{16T}{\pi D^3} \quad \text{———— *}$$

The principal stress σ_1 & σ_2

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$$\sigma_1 = \frac{\sigma_x - \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_y = 0 \Rightarrow \sigma_1 = \frac{\sigma_x}{2} \pm \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\sigma_1 = \frac{1}{2}\left(\frac{32M}{\pi D^3}\right) + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi D^3}\right)^2 + 4 \times \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{1}{2}\left[\frac{32M}{\pi D^3}\right] + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi D^3}\right)^2 + \left(\frac{32T}{\pi D^3}\right)^2}$$

$$= \frac{16M}{\pi D^3} + \frac{1}{2}\sqrt{\left(\frac{16}{\pi D^3}\right)^2\left[4M^2 + 4T^2\right]}$$

$$= \frac{16}{\pi D^3}\left[M + \left[\sqrt{M^2 + T^2}\right]\right]$$

$$\sigma_1 = \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$

max. stress

min stress $\sigma_2 = \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

Equivalent Bending Moment:

$$\sigma_f \quad \frac{32 M_e}{\pi D^3}$$

$$\sigma_1 = \frac{16}{\pi D^3} \left(M + \sqrt{M^2 + T^2} \right) \text{ and}$$

$$\boxed{M_{equivalent} = \frac{1}{2} \left(M + \sqrt{M^2 + T^2} \right)}$$

since

$$\frac{32 M_e}{\pi D^3} = \frac{16}{\pi D^3} \left(M + \sqrt{M^2 + T^2} \right)$$

$$M_{eq} = \frac{1}{2} \left(M + \sqrt{M^2 + T^2} \right) \longrightarrow \text{imp.}$$

Equivalent Torque T_e

$$\sigma_1 = \frac{16}{\pi D^3} \left(M + \sqrt{M^2 + T^2} \right)$$

$$\sigma_2 = + \frac{16}{\pi D^3} \left(M - \sqrt{M^2 + T^2} \right)$$

$$\frac{\sigma_1 - \sigma_2}{2} = \tau_{max}$$

$$\therefore \tau_{max} = \frac{1}{2} \left(\frac{16}{\pi D^3} \right) \left[M + \sqrt{M^2 + T^2} - \left(M - \sqrt{M^2 + T^2} \right) \right]$$

$$\tau_{max} = \frac{16}{\pi D^3} \left[\sqrt{M^2 + T^2} \right] = \frac{16}{\pi D^3} T_{eq.}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$\sigma_y = 0$

$$\sigma_1 = \frac{\sigma_x}{2} + \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\sigma_1 = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau_{max}^2}$$

$$= \frac{16M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{2\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left[M + \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}\right.$$

$$\sigma_1 = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\sigma_1 = \frac{32 M_{eq}}{\pi d^3} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$\boxed{M_{eq} = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]}$$

Equivalent Moment M_{eq}.

For Equivalent Torque:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \quad ---- (1)$$

$$\tau_{max} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right) - \left(M + \sqrt{M^2 + T^2}\right)\right]$$

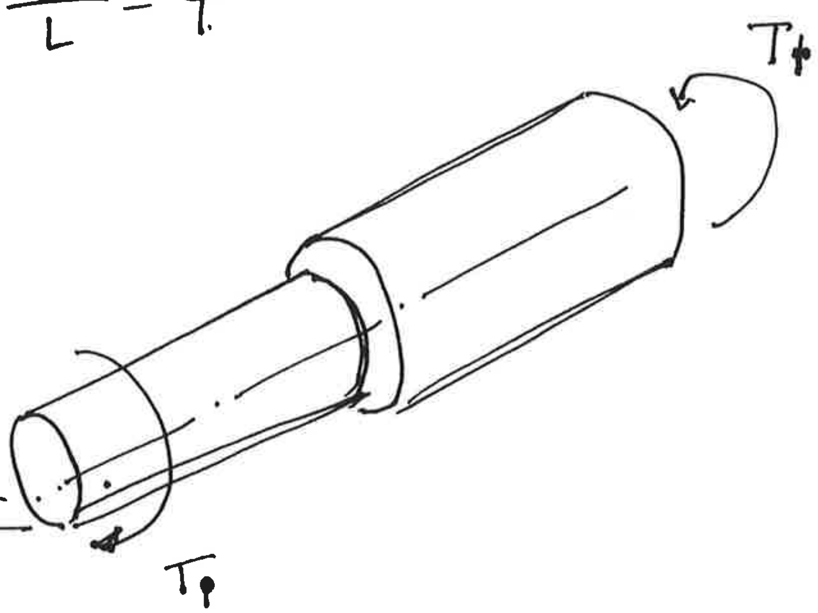
$$\tau_{max} = \frac{16}{\pi d^3}\left(2\sqrt{M^2 + T^2}\right. = \frac{16\, T_{eq}}{\pi d^3}$$

$$\boxed{T_{equivalent} = \sqrt{M^2 + T^2}}$$

Design on either M_{eq} or T_{equiv}.

$$T = \frac{G \cdot J\,\theta}{L} \qquad \frac{\theta}{L} = \phi$$

$$T = GJ\phi$$



$$T_1 = \frac{G_1 J_1 \theta_1}{L_1}$$

$$T_2 = \frac{G_2 J_2 \theta_2}{L_2}$$

$$T_1 = T_2$$

$$\frac{G_1 J_1 \theta_1}{L_1} = \frac{G_2 J_2 \theta_2}{L_2}$$

This equation is for composite shafts of Different material & cross-section

$\theta_1 = \theta_2 \quad G_1 = G_2$ for similar material

$$\therefore \frac{J_1}{L_1} = \frac{J_2}{L_2} \Rightarrow \frac{L_1}{L_2} = \frac{J_1}{J_2}$$

$$\frac{L_1}{L_2} = \frac{D_1^4}{D_2^4}$$

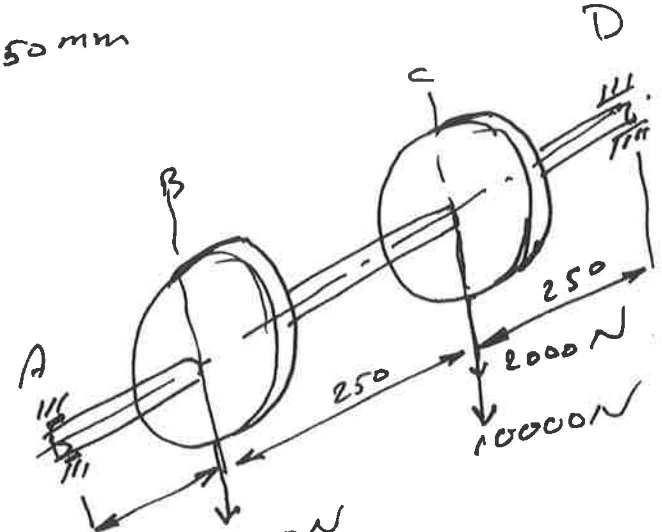
Design of shafts

Ex: A shaft carrying two pulleys each of 2000N. The shaft is 750 mm length. The two pulleys at 250 mm and 500 mm from the left end. Pull on the right pulley is 10000N vertically downward. The shaft transmits a torque of 3000N.m between the pulleys. assume $K_b = K_t = 1.5$ an $[\tau] = 70 N/mm^2$

Find the standard shaft diameter?

Data:

Distance length $l = 750mm$



$W_1 = 2000N$

$W_2 = 2000N$

$T = 3000 N.m$

$\Sigma M_A = 0$

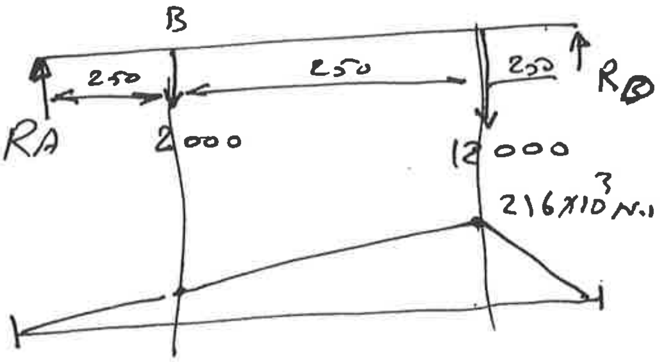
$R_B \times \overset{3}{\cancel{750}} = 12000 \times \overset{2}{\cancel{500}} + 2000 \times \cancel{250}$ 2000N

$R_B = \dfrac{24000 + 2000}{3}$

$R_B = \dfrac{26000}{3} = 8666 N$

$R_A = 14000 - 8666$

$R_A = 5334 N$



$M_b = 216000 N.mm$

$T = 3000 000 N.mm$

(2)

$$T_{eq} = \sqrt{(K_b M_b)^2 + (K_t T)^2}$$

$$T_{eq} = \sqrt{(1.5 \times 216 \times 10^3)^2 + (1.5 \times 3 \times 10^6)^2}$$

$$T_{eq} = 5.55 \times 10^6 \text{ N.mm}$$

$$\tau = \frac{16T}{\pi D^3}$$

$$D = \sqrt[3]{\frac{16 \times 5.55 \times 10^6}{70 \times \pi}}$$

$$D = 73.9 \approx 75 \text{ mm}$$

$\therefore$ Diameter of shaft is 75 mm.

Shaft Design (1)

Equivalent stress on a shaft:

$$\sigma_{eq} = \sqrt{\sigma_b^2 + (\alpha \tau)^2}$$

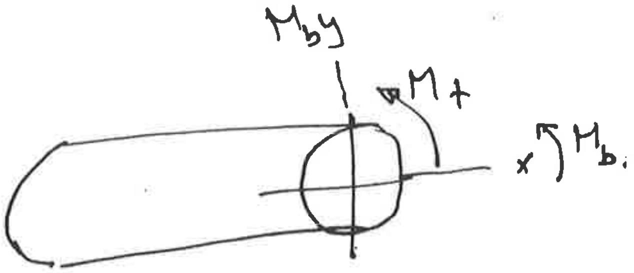
$\alpha_1 = \frac{\sigma_{all}}{\tau_{all}}$ — σ_{all} = allowable or permissible normal stress

τ_{all} = permissible shear st.

if $\alpha = 2$

$$\sigma_{equi} = \sqrt{\sigma^2 + 4\tau^2}$$

$$M_{br} = \sqrt{M_{bx}^2 + M_{by}^2}$$



$$M_{eq} = \sqrt{M_{br}^2 + \left(\frac{\alpha}{2} M_t\right)^2}$$

$\alpha_1 = \sigma_{all} / \tau_{all}$

$\sigma_b = \frac{M_{bo}}{W}$ where $W = \frac{I}{y}$ .... = modulus of section

These equations are for stationary shaft

Buckling of shaft:

Buckling stresses in bars:

P = Force = $\sigma \cdot A$ ⟶

if S_g is factor of safety against buckling.

$$P_g = \sigma_g \cdot F$$

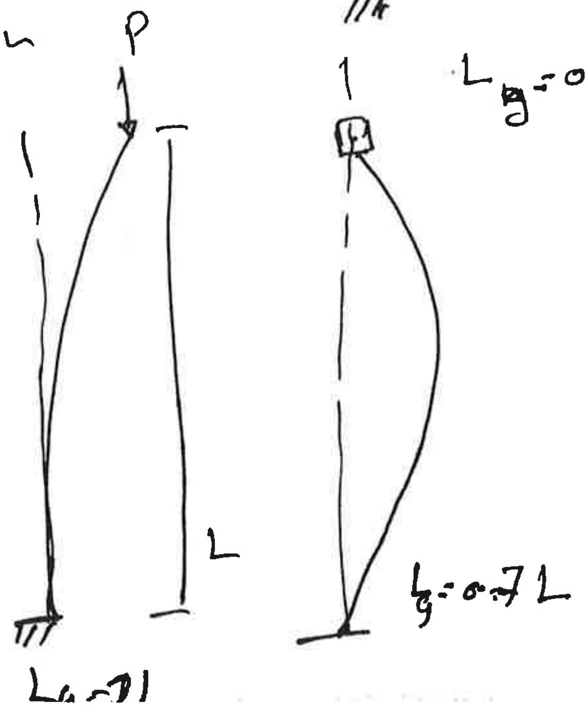
$$P = \frac{P_g}{S_g} = \frac{\sigma_g \cdot F}{S_g} = \sigma \cdot F$$

$$S_g = \frac{P_g}{P} = \frac{\sigma_g}{\sigma}$$

$$\lambda = \frac{L_g}{L}$$

L_g = free buckling length

L_g =



radius of gyration i

$$K = \sqrt{\frac{J}{A}}$$

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for
Pure ~~p~~ elastic buckling

$$\sigma_g = \frac{\pi^2 E}{\lambda^2} \quad \text{——— (1)}$$

$$P = \frac{P_g}{S_g} = \frac{\pi^2 \cdot E \cdot J}{L_g^2 \cdot S_g} \quad \text{——— (2)}$$

$\sigma_g \propto E$

For steels:

St. 37 σ	$E = 210000\ N/m^2$	~~σ_g~~ $L_g = 25d$
steel 60	E 210000 N/m^2	$L_g = 23d$
spring steel	210 000	$L_g = 15d$

Stress concentration factor of loaded shafts: K_t

$$\sigma_{max} = K_{t_b} \cdot \frac{MY}{I} \quad \text{——— (1)}$$ for stress concentration

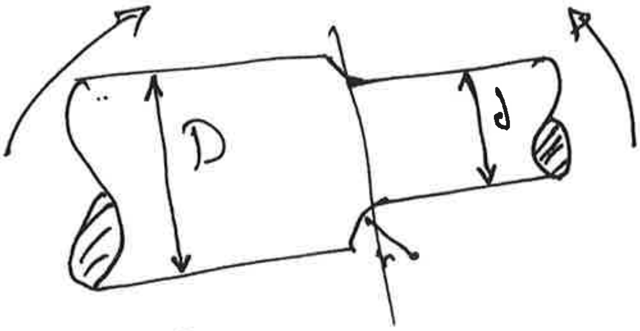
$$\tau_{max} = K_{t_T} \frac{\not{M} r}{J} \quad \text{——— (2)}$$

K_t = stress concentration factor
is find depending on type of loading.

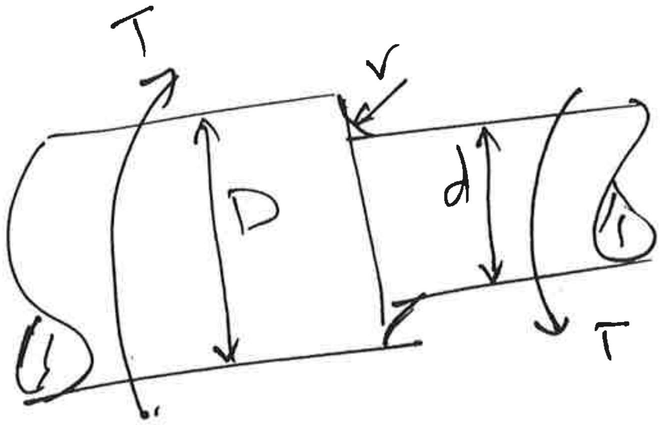
(4)

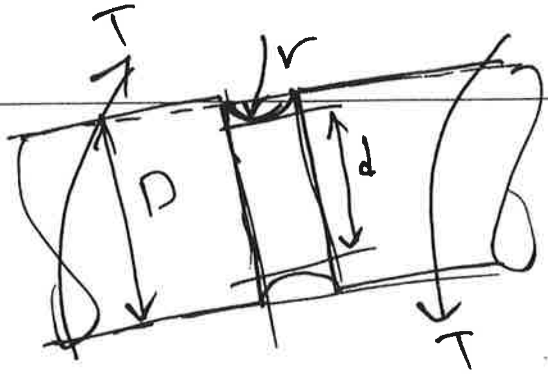
$\frac{D}{d} \longrightarrow$ (1)

$\frac{r}{d} \longrightarrow$ (2)



from charts for loading condition of shaft.





For bending of a Stationary shaft

$$\sigma_b = \frac{32 M}{\pi D^3} \quad \text{---} (1)$$

and

$$\tau = \frac{16 T}{\pi D^3} \quad \text{---} (2)$$

for hollow shafts

$$\sigma_b = \frac{32 M}{\pi D_o^3 (1 - K^4)} \quad \text{where } K = \frac{D_i}{D_o}$$

$K = 0$ for solid shaft.

$$\tau = \frac{16 T}{\pi D_o^3 (1 - K^4)}$$

& $\alpha = 1$ fo tensile = Column action factor

Max. shear stress Theory

$$(\tau) = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16}{\pi D_o^3 (1 - K^4)} \sqrt{\left[M + \frac{\alpha F \cdot D_o (1 + K^2)}{8}\right]^2 + T^2}$$

where $\sigma_x = \dfrac{32 M}{\pi D_o^3 (1 - K^4)} \pm \dfrac{4 \alpha F}{\pi D_o^2 (1 - K^2)}$

α is calculated from the quation

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K_g}\right)} \quad \text{for } \frac{L}{K_g} \le 115$$

due to buckling α should be introduced

$$\alpha = \frac{\sigma_{yc}}{\pi^2 \cdot n E}\left(\frac{L}{K_g}\right)^2 \quad \text{for } \frac{L}{K_g} \ge 115$$

$n = 1.$ for hinged

$n = 2.25$ for fixed ends

$n = 1.6$ for ends partly restrained.

K_g = radius of gyration

L = length of shaft (m)

σ_{yc} = yield stress in compression

$$K_g = \sqrt{\frac{J}{A}}$$ — radius of gyration

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* For a shaft subjected to Combined bending + torsion and tension -

$$\sigma_b = \frac{32\ M}{\pi D_o^3}$$

$$\tau = \frac{16\,T}{\pi D_o^3}$$

$$\sigma_t = \frac{4F}{\pi D_o^2}$$

$$\sigma_x = \frac{32M}{\pi D_o^3} \pm \frac{4F}{\pi D_o^2}$$

$$\tau_x = \frac{16T}{\pi D_o^3}$$

$$[\tau] = \frac{16}{\pi D_o^3}\sqrt{\left[M \pm \frac{FD_o}{8}\right]^2 + T^2}$$

* For Combined Torsion and Bending :

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2} = \frac{1}{2}\sqrt{\left(\frac{32M_b}{\pi D_o^3}\right)^2 + 4\times\left(\frac{16T}{\pi D_o^3}\right)^2}$$

$$\tau_{max} = \frac{16}{\pi D_o^3}\sqrt{M_b^2 + T^2}$$

For Flactuating loading

$$\tau_{max} = \frac{16}{\pi D_o^3}\sqrt{(K_b M_b)^2 + (K_t T)^2}$$

$$\tau_{max} = \frac{16}{\pi D_o^3} \sqrt{(K_b M_b)^2 + (K_t T)^2}$$

So shaft Diameter D_o

$$D_o = \left[\frac{16 \sqrt{(K_b M_b)^2 + (K_t T)^2}}{\pi \times \tau_{max}} \right]^{\frac{1}{3}}$$

K_b and K_t are factors and depend on whether the load is shock type or fatigue

Values of K_b and K_t are given in table below

Nature of loading	K_b	K_t
1. Stationary shaft		1.0
- Gradually applied load	→ 1.0	
- Suddenly applied load	→ 1.5 - 2.0	1.5 - 2.0
2- Revolving or turning shaft		1.0
- Steadily applied load	→ 1.5	
- Minor shock load	→ 1.5 - 2.0	1 - 1.5
- Heavy shock load	→ 2.0 3.0	1.5 - 3.0

Shaft Design

1- Material Selection (usually steel, unless you have good reasons)
2- Geometric Layout (fit power transmission equipment, gears, pulleys)

3- Failure strength
– Static strength
– Fatigue strength

4- Shaft deflection
– Bending deflection
– Torsional deflection
– Slope at bearings and shaft-supported elements
– Shear deflection due to transverse loading of short shafts

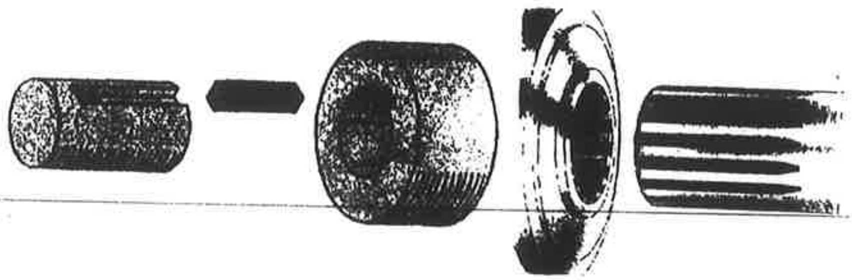
5- Critical speeds at natural frequencies

Shaft Materials

- Deflection primarily controlled by geometry, not material
- Strain controlled by geometry but material has a role in stress
- Strength, Yield or UTS is a material property. Cold drawn steel typical for d< 3 in.
- HR steel common for larger sizes. Should be machined all over.
- Low production quantities: Machining
- High production quantities: Forming

Torque Transmission

- Common means of transferring torque to shaft
  1. Keys
  2. Splines
  3. Setscrews
  4. Pins
  5. Press or shrink fits
  6. Tapered fits
- Keys are one of the most effective。
  - Slip fit of component onto shaft for easy assembly
  - Positive angular orientation of component
  - Can design the key to be weakest link to fail in case of overload



Shaft Design for Stresses

- Stresses are only evaluated at critical location
- Critical locations are usually
  1. On the outer surface
  2. Where the bending moment is large
  3. Where the torque is present

  4- Where stress concentrations exist

Instructional Objectives:

At the end of this lesson, the students should be able to understand:

- Definition of shaft
- Standard shaft sizes
- Standard shaft materials
- Design of shaft based on strength

8.1.1 Shaft

Shaft is a common and important machine element. It is a rotating member, in general,
has a circular cross-section and is used to transmit power. The shaft may be hollow or solid. The shaft is supported on bearings and it rotates a set of gears or pulleys for the purpose of power transmission. The shaft is generally acted upon by bending moment, torsion and axial force. Design of shaft primarily involves in determining stresses at critical point in the shaft that is arising due to aforementioned loading. Other two similar forms of a shaft are axle and spindle.

Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque. Spindle is simply defined as a short shaft. However, design method remains the same for axle and spindle as that for a shaft.

8.1.2 Standard sizes of Shafts

Typical sizes of solid shaft that are available in the market are,

Up to 25 mm	0.5 mm increments
25 to 50 mm	1.0 mm increments
50 to 100 mm	2.0 mm increments
100 to 200 mm	5.0 mm increments

8.1.3 Material for Shafts

The ferrous, non-ferrous materials and non metals are used as shaft material depending on the application. Some of the common ferrous materials used for shaft are discussed below.

Hot-rolled plain carbon steel

These materials are least expensive. Since it is hot rolled, scaling is always present on the surface and machining is required to make the surface smooth.

Cold-drawn plain carbon/alloy composition

Since it is cold drawn it has got its inherent characteristics of smooth bright finish. Amount of machining therefore is minimal. Better yield strength is also obtained. This is widely used for general purpose transmission shaft.

Alloy steels

Alloy steel as one can understand is a mixture of various elements with the parent steel to improve certain physical properties. To retain the total advantage of alloying materials one requires heat treatment of the machine components after it has been manufactured. Nickel, chromium and vanadium are some of the common alloying materials. However, alloy steel is expensive.

These materials are used for relatively severe service conditions. When the situation demands great strength then alloy steels are used. They have fewer tendencies to crack, warp or distort in heat treatment. Residual stresses are also less compared to CS(Carbon Steel).

In certain cases the shaft needs to be wear resistant, and then more attention has to be paid to make the surface of the shaft to be wear resistant. The common types of surface hardening methods are,

Hardening of surface

Case hardening and carburizing
Cyaniding and nitriding

8.1.4 Design considerations for shaft

For the design of shaft following two methods are adopted,

Design based on Strength

In this method, design is carried out so that stress at any location of the shaft should not exceed the material yield stress. However, no consideration for shaft deflection and shaft twist is included.

Design based on Stiffness

Basic idea of design in such case depends on the allowable deflection and twist of the shaft.

8.1.5 Design based on Strength

The stress at any point on the shaft depends on the nature of load acting on it. The stresses which may be present are as follows.

Basic stress equations :

Bending stress

$$\sigma_b = \frac{32M}{\pi d_0^3(1-k^4)}$$

(8.1.1)

Where,

M : Bending moment at the point of interest

d_o : Outer diameter of the shaft

k : Ratio of inner to outer diameters of the shaft (k = 0 for a solid shaft because inner diameter is zero)

Axial Stress

$$\sigma_a = \frac{4\alpha F}{\pi d_0^2(1-k^2)}$$

(8.1.2)

Where,

F: Axial force (tensile or compressive)

α : Column-action factor(= 1.0 for tensile load)

The term α has been introduced in the equation. This is known as column action factor. What is a column action factor? This arises due the phenomenon of buckling of long slender members which are acted upon by axial compressive loads.

Here, α is defined as,

$$\alpha = \frac{1}{1-0.0044(L/K)} \quad \text{for L/K} < 115$$

$$\alpha = \frac{\sigma_{yc}}{\pi^2 nE}\left(\frac{L}{K}\right)^2 \qquad \text{for } L/K > 115 \qquad (8.1.3)$$

Where,

n = 1.0 for hinged end
n = 2.25 for fixed end
n = 1.6 for ends partly restrained, as in bearing
K = least radius of gyration, L = shaft length
σ_{yc} = yield stress in compression

Stress due to torsion

$$\tau_{xy} = \frac{16T}{\pi d_0^3(1-k^4)}$$

(8.1.4)

Where,

T : Torque on the shaft

τ_{xy} : Shear stress due to torsion

Combined Bending and Axial stress

Both bending and axial stresses are normal stresses, hence the net normal stress is given by,

$$\sigma_x = [\frac{32M}{\pi d_0^3(1-k^4)} \pm \frac{4\alpha F}{\pi d_0^2(1-k^2)}]$$

(8.1.5)

The net normal stress can be either positive or negative. Normally, shear stress due to torsion is only considered in a shaft and shear stress due to load on the shaft is neglected.

Maximum shear stress theory

Design of the shaft mostly uses maximum shear stress theory. It states that a machine member fails when the maximum shear stress at a point exceeds the maximum allowable shear stress for the shaft material. Therefore,

$$\tau_{max} = \tau_{allowable} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

(8.1.6)

Substituting the values of σ_x and τ_{xy} in the above equation, the final form is,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + T^2}$$

(8.1.7)

Therefore, the shaft diameter can be calculated in terms of external loads and material properties. However, the above equation is further standarised for steel shafting in terms of allowable design stress and load factors in ASME design code for shaft.

8.1.6 ASME design Code

The shafts are normally acted upon by gradual and sudden loads. Hence, the equation (8.1.7) is modified in ASME code by suitable load factors,

$$\tau_{allowable} = \frac{16}{\pi d_0^3(1-k^4)}\sqrt{\left\{C_{bm}M + \frac{\alpha F d_0(1+k^2)}{8}\right\}^2 + (C_t T)^2}$$

(8.1.8)

where, C_{bm} and C_t are the bending and torsion factors. The values of these factors are given below,

	C_{bm}	C_t
For stationary shaft:		
Load gradually applied	1.0	1.0
Load suddenly applied	1.5 - 2.0	1.5 - 2.0
For rotating shaft:		
Load gradually applied	1.5	1.0

Load suddenly applied (minor shock)	1.5 - 2.0	1.0 - 1.5
Load suddenly applied (heavy shock)	2.0 - 3.0	1.5 - 3.0

ASME code also suggests about the allowable design stress, $\tau_{allowable}$ to be considered for steel shafting,

ASME Code for commercial steel shafting

= 55 MPa for shaft without keyway

= 40 MPa for shaft with keyway

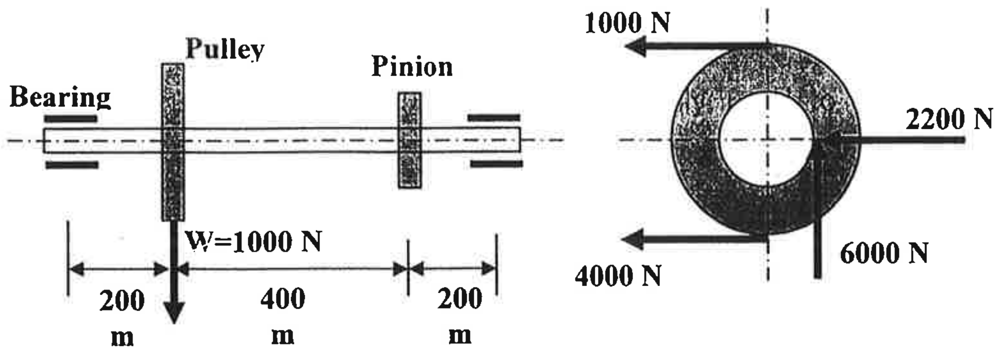
ASME Code for steel purchased under definite specifications

= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways.

The equations, (8.1.7) and (8.1.8) are commonly used to determine shaft diameter.

Sample problem

The problem is shown in the given figure. A pulley drive is transmitting power to a pinion, which in turn is transmitting power to some other machine element. Pulley and pinion diameters are 400mm and 200mm respectively. Shaft has to be designed for minor to heavy shock.

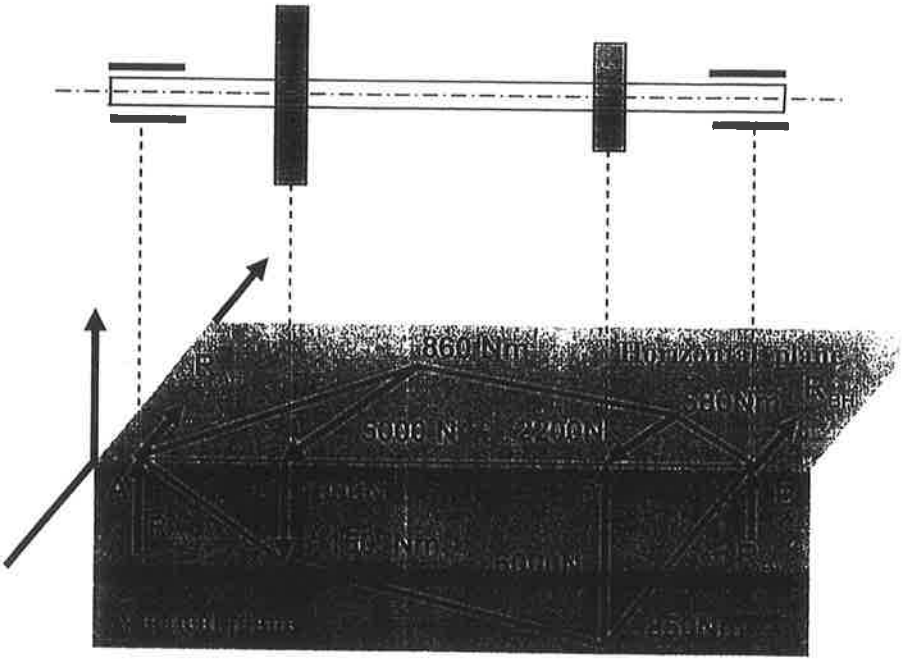


Solution

From the given figure, the magnitude of torque,

$$T = (4000 - 1000) \times 200 \text{ N mm} = 600 \times 10^3 \text{ Nmm}$$

It is observed that the load on the shaft is acting both in horizontal and vertical planes. The loading diagram, corresponding bearing reactions and bending moment diagram is given below.



Loading and Bending Moment Diagram

The bending moment at C:

For vertical plane, M_V: -150 Nm
For horizontal plane, M_H: 860 Nm
Resultant moment: 873 Nm

The bending moment at D:

For vertical plane, M_V: -850 Nm
For horizontal plane, M_H: 580 Nm
Resultant moment: 1029Nm

Therefore, section-D is critical and where bending moment and torsion is 1029 Nm and 600 Nm respectively.

ASME code for shaft design is suitable in this case as no other specifications are provided. In absence of any data for material property, the allowable shear for commercial steel shaft may be taken as 40 MPa, where keyway is present in the shaft.

For the given codition of shock, let us consider C_{bm} = 2.0 and C_t = 1.5.

From the ASME design code, we have,

$$d_o^3 = \frac{16\times10^3}{\tau_d\times\pi}\left(\sqrt{(C_{bm}\times1029)^2+(C_t\times600)^2}\right)$$

$$= \frac{16\times10^3}{40\times\pi}\left(\sqrt{(2.0\times1029)^2+(1.5\times600)^2}\right)$$

$$\therefore d_o = 65.88\text{ mm} \approx 66\text{mm}$$

From standard size available, the value of shaft diameter is also 66mm.

Questions and answers

Q1. What do you understand by shaft, axle and spindle?

A1. Shaft is a rotating member, in general, has a circular cross-section and is used to transmit power. Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque. Spindle is simply defined as a short shaft.

Q2. What are the common ferrous materials for a shaft?

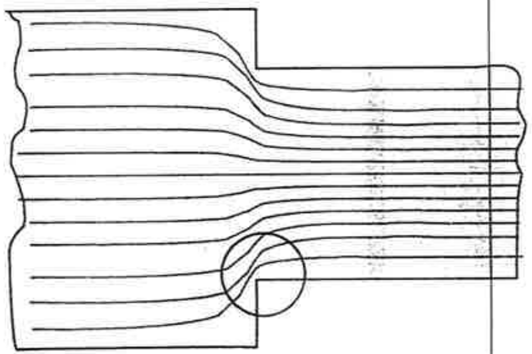
A2. Common materials for shaft are, hot-rolled plain carbon steel, cold-drawn plain carbon/alloy composition and alloy steels.

Q3. How do the strength of a steel material for shafting is estimated in ASME design code for shaft?

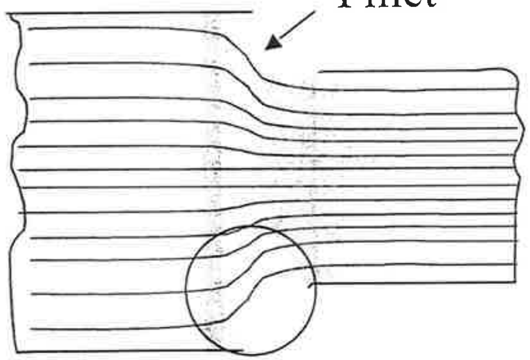
A3. Material property for steel shaft for ASME code is as follows,
For commercial steel shafting
= 55 MPa for shaft without keyway
= 40 MPa for shaft with keyway
For steel purchased under definite specifications
= 30% of the yield strength but not over 18% of the ultimate strength in tension for shafts without keyways. These values are to be reduced by 25% for the presence of keyways in the shaft.

Stress concentration

- Transition of cross sections ⟹ High stresses
- More abrupt transition ⟹ Higher stresses

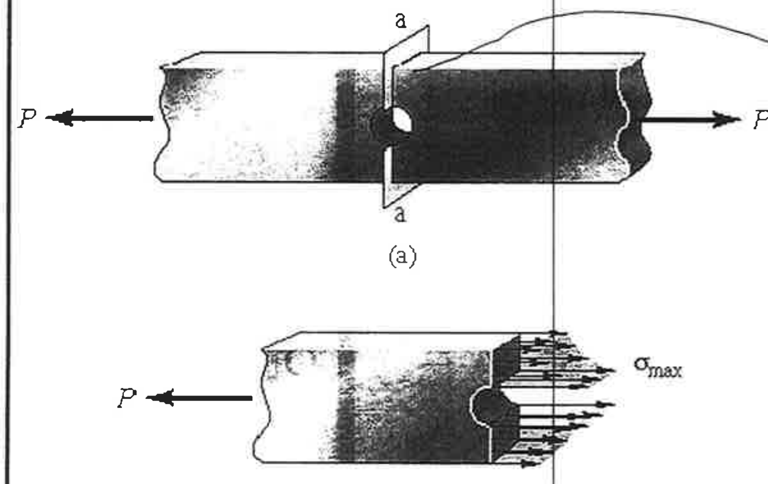


Abrupt change
Stress "flow lines" crowd
High stress concentration



Smoother change
"Flow lines" less crowded
Lower stress concentration

- Elementary stress equations don't apply in stress concentrations.



$$\sigma_{avg} = \frac{P}{A} = \frac{P}{(b-d)h}$$

Figure 6.1 Rectangular plate with hole subjected to axial load.

(a) Plate with cross-sectional plane.

(b) Half of plate with stress distribution.

$$\sigma_{max} = K_c \sigma_{avg}$$

Stress concentration factor ***from charts***

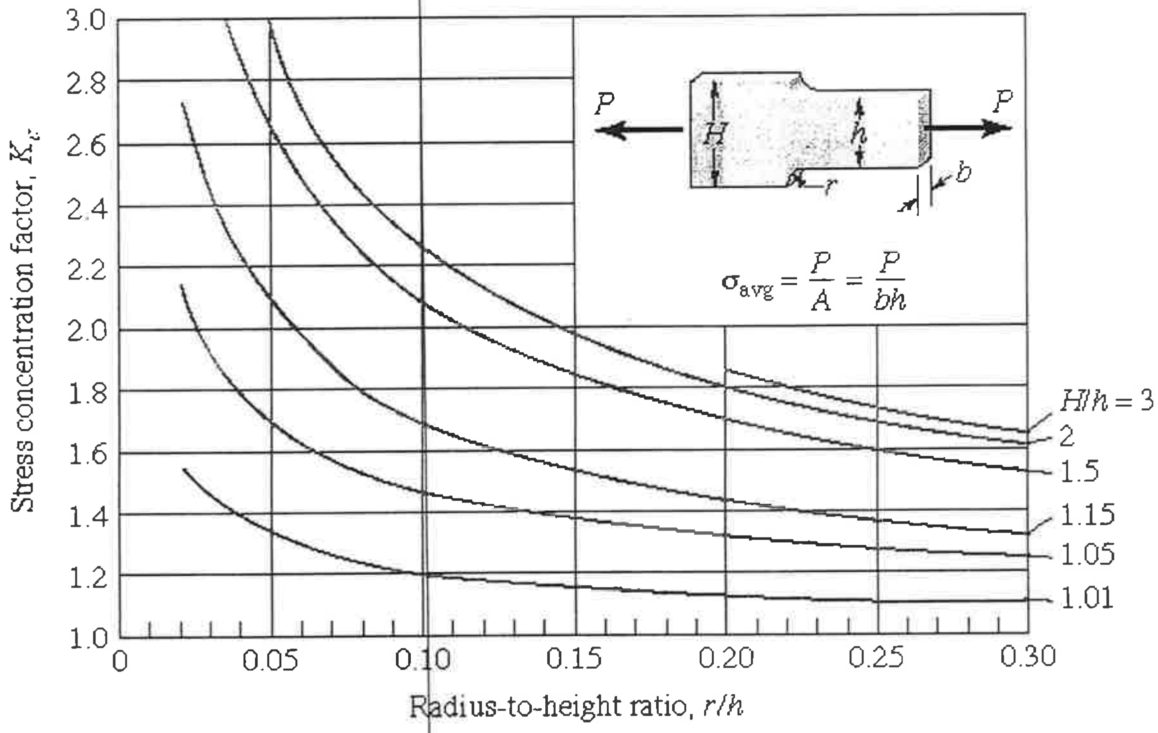
Text Reference: Figure 6.1, page 221

Stress concentration factor

K_c

- Obtained experimentally, analytically, etc
- Published in charts
- *Geometric* property
- Very important in ***brittle*** materials
- In ductile materials:
  - Important in ***fatigue*** calculation.
  - Important if safety is critical.
  - Localized yielding hardens material (strain hardening).
  - Redistributes stress concentration.

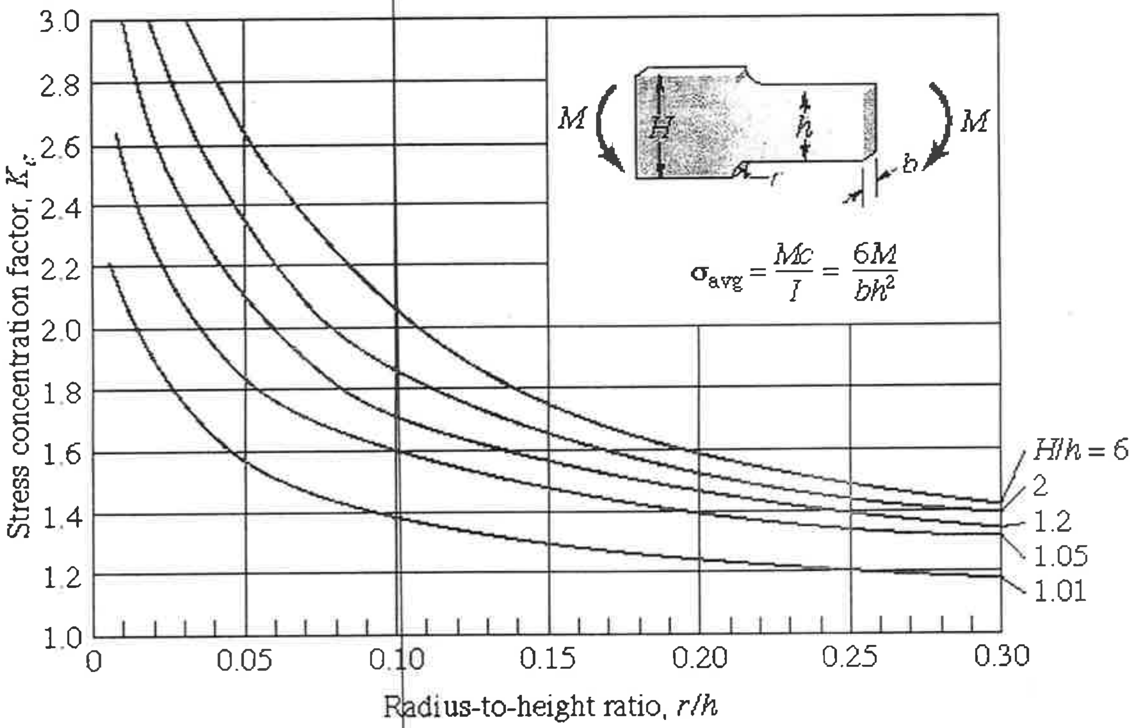
Stress Concentrations for Plate with Fillet



(a)

Figure 6.3 Stress concentration factor for rectangular plate with fillet. (a) Axial Load. [Adapted from Collins (1981).]

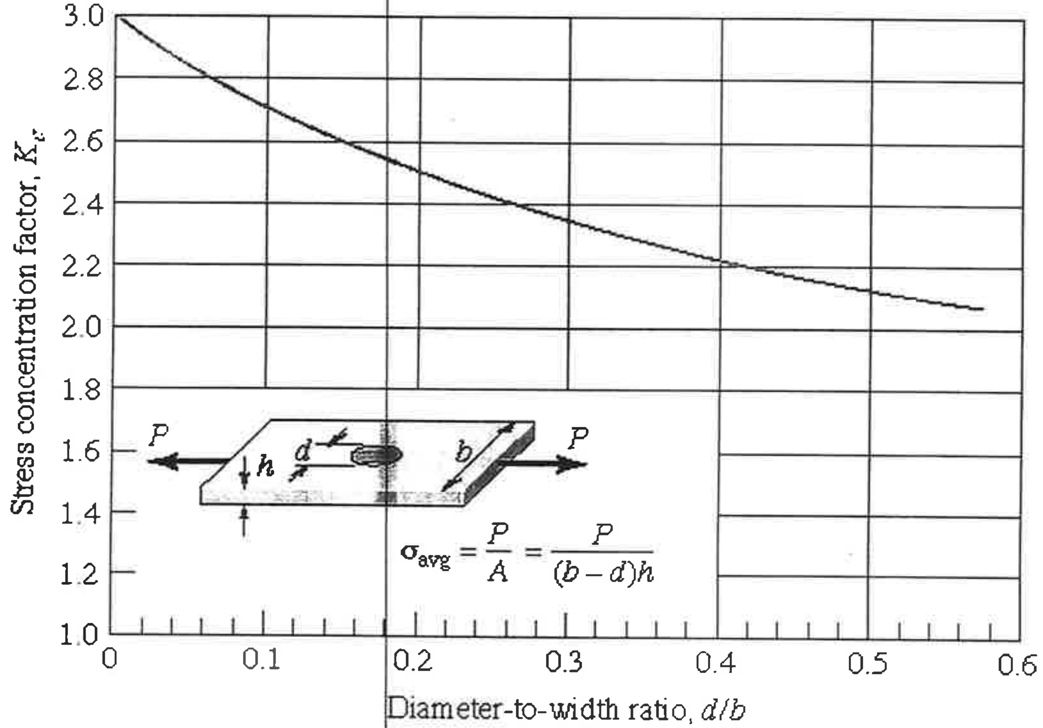
Stress Concentrations for Plate with Fillet (cont.)



(b)

Figure 6.3 Stress concentration factor for rectangular plate with fillet. (b) Bending Load. [Adapted from Collins (1981).]

Stress Concentrations for Plate with Hole

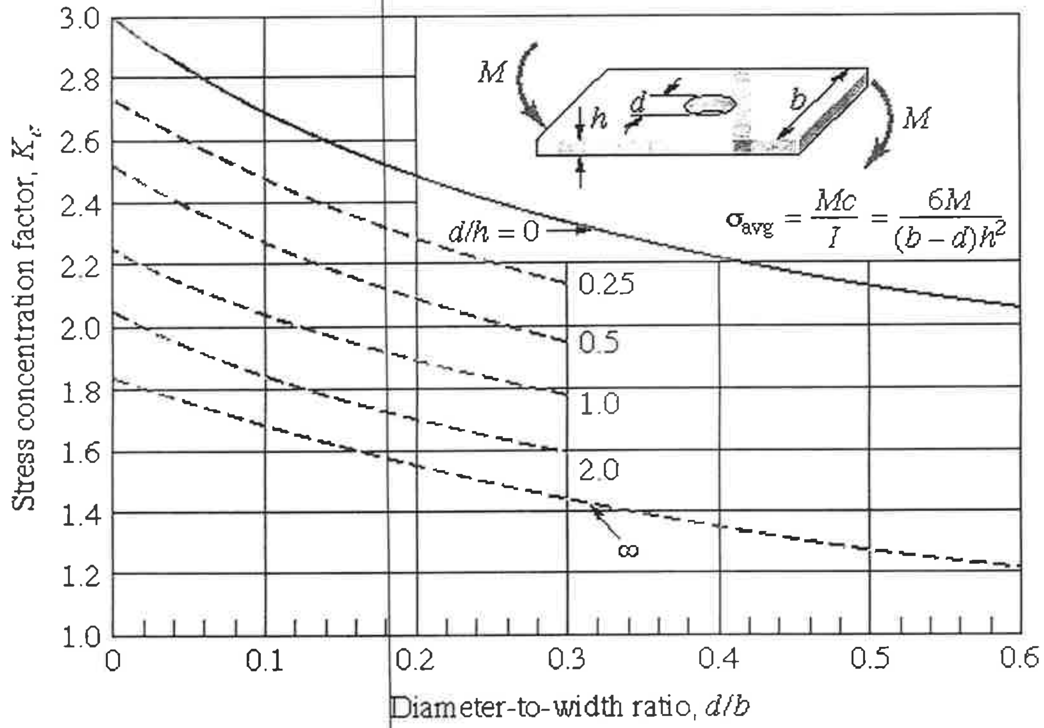


(a)

Figure 6.2 Stress concentration factor for rectangular plate with central hole. (a) Axial Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222

Stress Concentrations for Plate with Hole (cont.)



(b)

Figure 6.2 Stress concentration factor for rectangular plate with central hole. (b) Bending. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222

Stress Concentrations for Bar with Fillet (cont.)

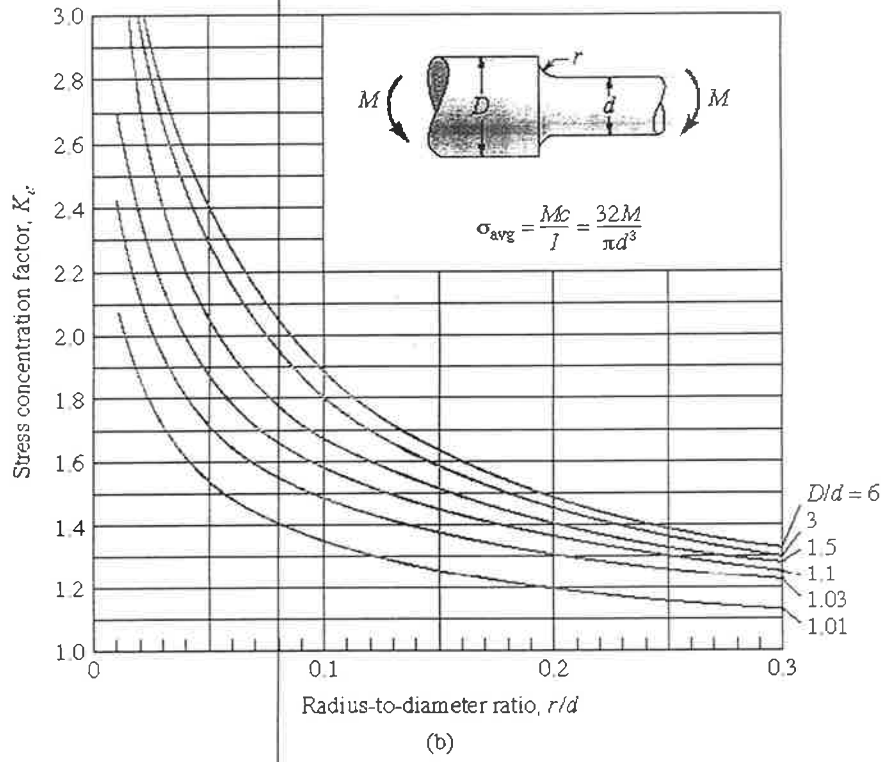


Figure 6.5 Stress concentration factor for round bar with fillet. (b) Bending. [Adapted from Collins (1981).]

Text Reference: Figure 6.5, page 225

Stress Concentrations for Bar with Fillet (cont.)

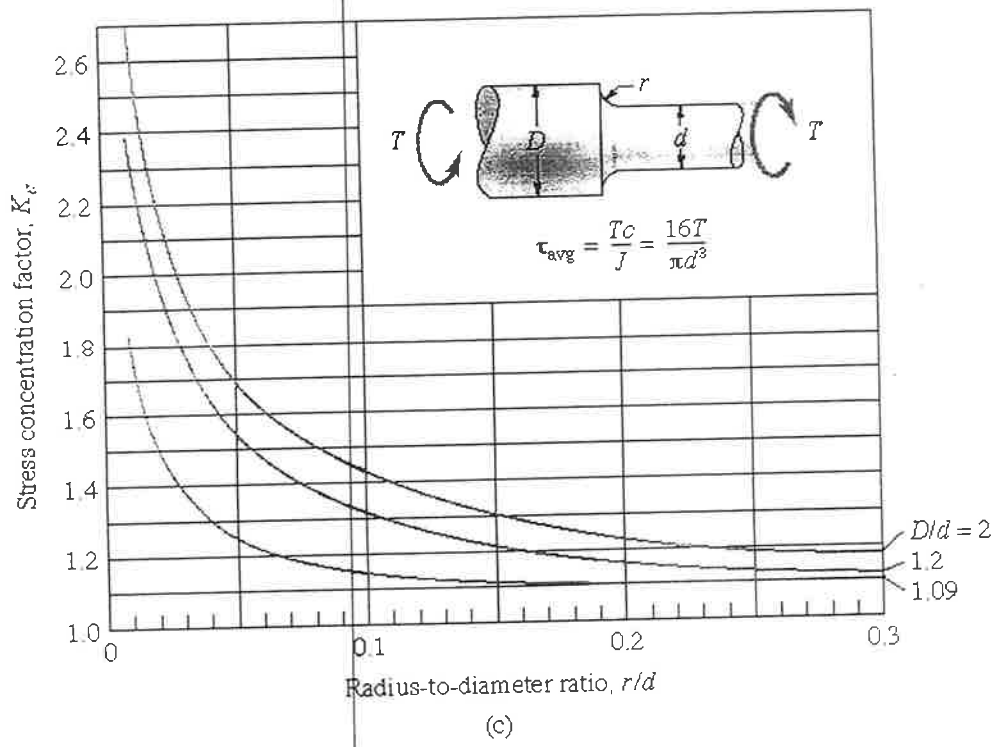


Figure 6.5 Stress concentration factor for round bar with fillet. (c) Torsion. [Adapted from Collins (1981).]

Stress Concentrations for Bar with Groove (cont.)

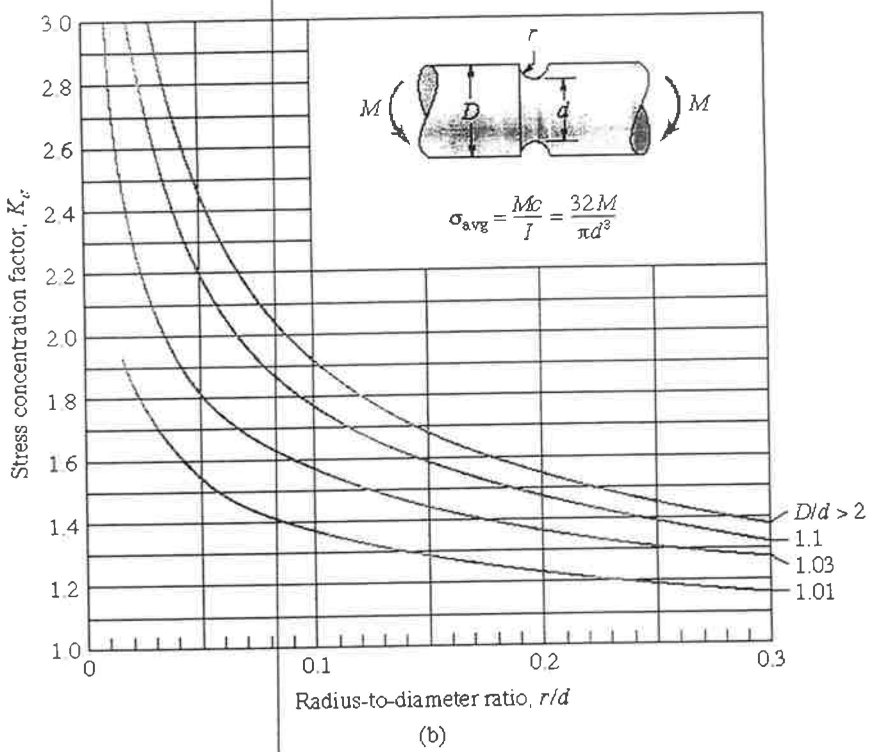
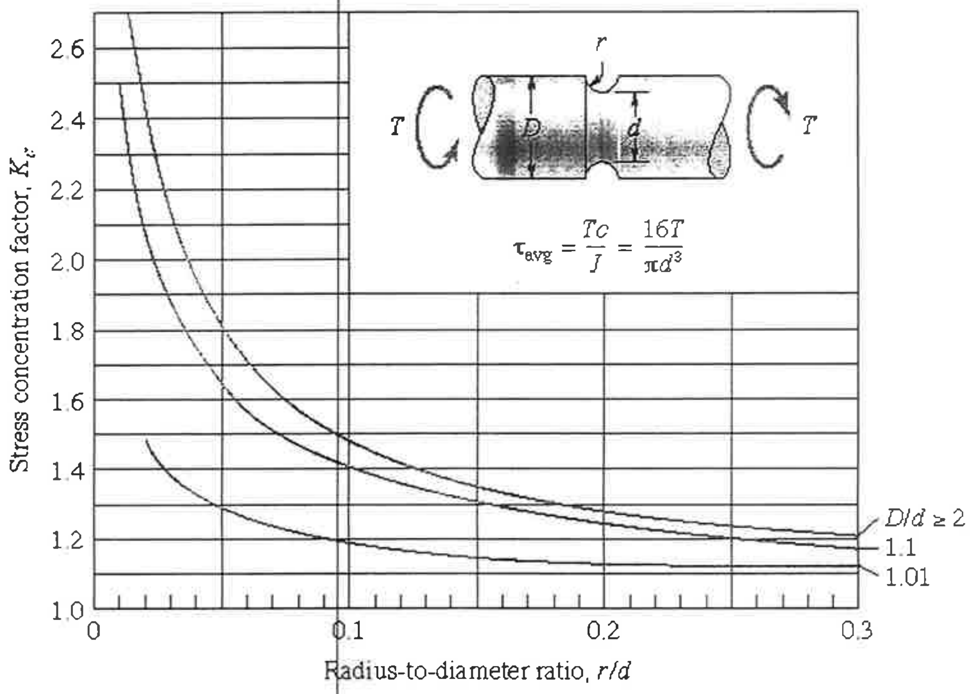


Figure 6.6 Stress concentration factor for round bar with groove. (b) Bending. [Adapted from Collins (1981).]

Stress Concentrations for Bar with Groove (cont.)



(c)

Figure 6.6 Stress concentration factor for round bar with groove. (c) Torsion. [Adapted from Collins (1981).]

Text Reference: Figure 6.6, page 226

Fracture mechanics

- All materials contain cracks.
- If crack is bigger than critical dimension, it propagates → Catastrophic failure.

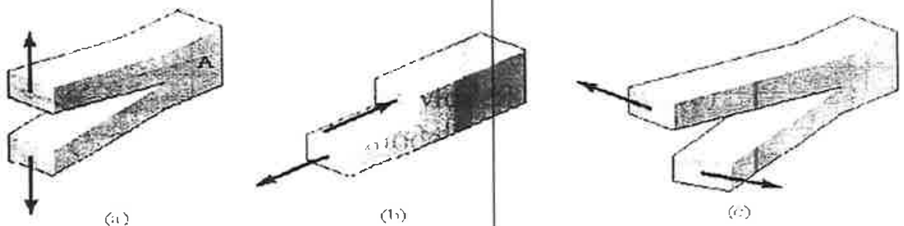


Figure 6.8 Three modes of crack displacement. (a) Mode I, opening; (b) mode II, sliding; (c) mode III, tearing.

- Fracture mechanics predict mode I crack propagation ***if crack size*** > 2 ***a***, where

$$a = \frac{1}{\pi}\left(\frac{K_{ci}}{Y\sigma_{nom}}\right)^2$$

K_{ci}: Fracture toughness, from tables

Y: Geometric correction factor

Yield Stress and Fracture Toughness Data (@room Temperature)

Material	Yield Stress, S_y ksi	Yield Stress, S_y Mpa	Fracture Toughness, K_{ci} ksi $in^{1/2}$	Fracture Toughness, K_{ci} Mpa $m^{1/2}$
Metals				
Aluminum alloy 2024-T351	47	325	33	36
Aluminum alloy 7075-T651	73	505	26	29
Alloy steel 4340 tempered at 260℃	238	1640	45.8	50.0
Alloy steel 4340 tempered at 425℃	206	1420	80.0	87.4
Titanium alloy Ti-6Al-4V	130	910	40-60	44-66
Ceramics				
Aluminum oxide			2.7-4.8	3.0-5.3
Soda-lime glass			0.64-0.73	0.7-0.8
Concrete			0.18-1.27	0.2-1.4
Polymers				
Polymethyl methacrylate			0.9	1.0
Polystyrene			0.73-1.0	0.8-1.1

Text Reference: Table 6.1, page 232

Stress intensity factor

- Recall critical crack length = $2a$.

$$a = \frac{1}{\pi}\left(\frac{K_{ci}}{Y\sigma_{nom}}\right)^2$$

K_{ci} = Fracture toughness, from tables

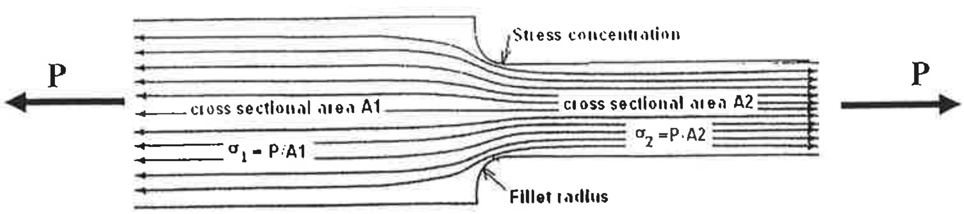
Y = Geometric correction factor

- So fracture toughness is $K_{ci} = Y\sigma_{nom}\sqrt{\pi a}$
- If the actual crack length is $2x$, the ***stress intensity factor*** is defined as $\boxed{K_i = Y\sigma_{nom}\sqrt{\pi x}}$
- Crack will propagate if $K_i > K_{ci}$, or $2x > 2a$.
- Safety factor against crack propagation is K_{ci}/K_i.

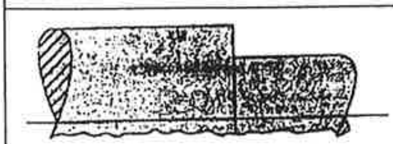
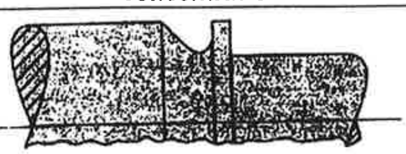
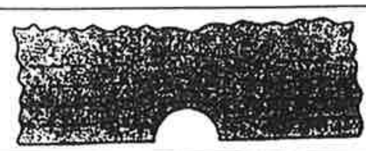
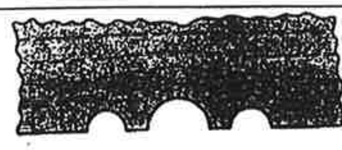
2-4: Stress Concentration Caused by Sudden Change in Form

A change in the geometric shape of a part gives rise to additional stress over and above the calculated stress, which is known as stress concentration.

In the part shown below, the tensile stress changes from $\sigma_1 = P/A_1$ to $\sigma_2 = P/A_2$ and, as $A_1 > A_2$, $\sigma_2 > \sigma_1$. Within the part, the internal stress gets redistributed from low to high value at the region where the cross-sectional area changes. In this example, this redistribution occurs at the region of the fillet radius joining the two geometric forms. If a large fillet radius is provided between the two sections, that is the cross-sectional area is changed more gradually, then the internal stresses gets ample space to get redistributed evenly. However, if the fillet radius is small, that is the change in shape is more abrupt, then the internal stresses do not get enough space to get redistributed evenly. As a result of this, at the base of the fillet in the smaller side, the actual stress becomes more than the theoretical stress ($\sigma_2 = P/A_2$). **This increase in stress due to sudden change of geometric shape is called stress concentration.**



Similar to the fillet radius, **holes, notches, or grooves** also bring in sudden change in the geometric form. This means all these features will also be associated with stress concentration effect. **Generally, more abrupt the change in geometric form, higher is the stress concentration effect.** Because stress concentration increases mechanical stress, **a better design approach is to strive to reduce stress concentration effect at the critical stressed areas.** Following design alternatives are some creative design approach to reduce stress concentration. In each of the cases, the change is section geometry has been introduced more gradually.

High stress concentration	Better design and less stress concentration	Remark
		Gradual reduction in diameter of the stepped shaft
		More gradual change of cross-section

In many design situations, it is not possible to altogether avoid the effect of stress concentration. For example, collars and groves are needed in shafts for bearing mounting; keyways in shafts are needed for couplings. Often holes are needed for fasteners. All these discontinuities in the part geometry cause stress concentration. **In mechanical design, stress concentration effects can be minimized by placing these geometric features in a noncritical stress area.** However, in some cases, the effect of stress concentration must be accounted for in design calculation. It is also worthwhile to note that tool marks, porosity in castings etc. may also give rise to stress concentration effect.

Geometric stress concentration factor

To account for stress concentration effect, the actual maximum stresses have been determined either experimentally or by using more sophisticated stress analysis methods, such as finite element analysis, for **common types of geometric features**. Based on such calculations the **geometric stress concentration factors (K)** are determined for these types of features. The stress concentration factor is defined as,

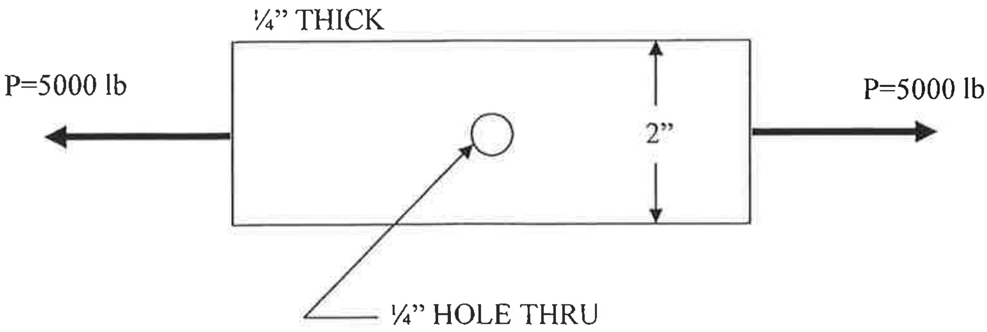
$$K = \frac{\textit{highest value of actual stress on fillet, notch, hole, etc.}}{\textit{calculated stress for the minimum cross section}}$$

The value of the factor K varies from 1 to about 3 in most cases. K= 1 means no stress concentration, that is, calculated value of stress = actual value of stress. **When K = 3, the actual stress is three times the calculated value.**

The values of stress concentration are provided in chart forms in the textbook (**Figures 2-8 to 2-21**). There are **14 different charts**; each chart is for a specific combination of (i) **type of section, (ii) type of geometric feature and (iii) type of loading.**

Example

Determine the maximum theoretical stress and actual stress considering stress concentration effect in the following loading condition:



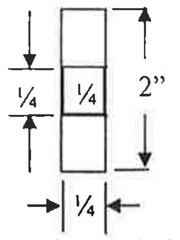
THE MAX STRESS WILL OCCUR AT THE CROSS SECTION WITH THE HOLE.

THE AREA OF THE CROSS SECTION

$A = (2-1/4)*1/4 = 0.4375 \text{ in}^2$

Thus the MAX THEORETICAL STRESS ON THE PART IS $\sigma_{Theoretical} = P/A = 5000/.4375 = 11{,}428$ **psi**

FOR ACTUAL STRESS, FIND STRESS CONCENTRATION FACTOR K_t **FROM FIGURE 2-20 PAGE 145:**



Cross-section with the hole

$d/W = .25/2 = 0.125$

$K_t = 2.63$

Thus the actual stress

$\sigma_{Actual} = \sigma_{Theoretical} * K_t = 11{,}428*2.63 = 30{,}057$ **psi**

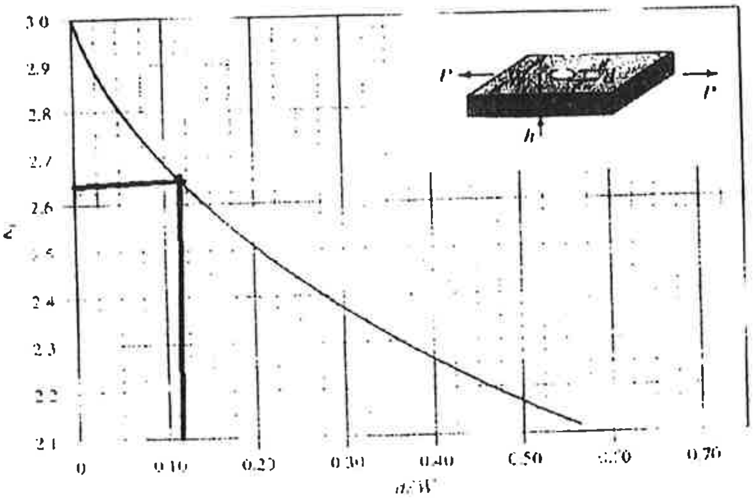


Figure 2-20 Stress concentration factor for a flat bar with a transverse hole in axial tension. (Spreadsheet Module 2-17)

24. Find the value of the maximum stress on the fillet in Fig. 2-50 if ~~the stress concentration factor is equal to 1.75.~~ $D/d = 1.5$. What is the F_s if the part is made of cast iron; $\sigma_{ult} =$ 200 MPa?
Ans. $F_s = 2.74$.

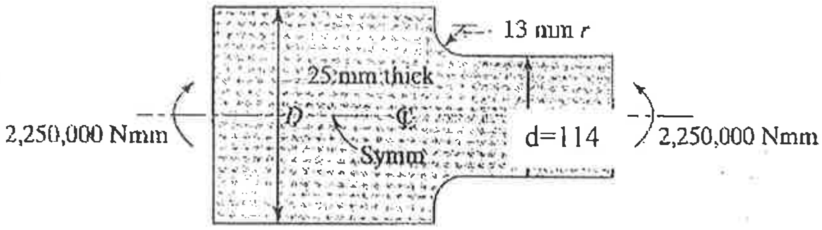


Figure 2-50 Problem 24.

The stress will be maximum at the top and bottom points, at the root of the fillets in the thinner portion of the part. Equal tensile and compressive stress will be developed.

M = 2,250,000 N-mm
$I = bd^3/12 = 25*114^3/12 = 3086550$ mm^4
$\sigma = Mc/I = 2{,}250{,}000 * 57/3086550 = 41.55$ MPa

From Figure 2-17: D/d = 1.5; r/d = 13/114 = 0.11 -> $K_t = 1.8$

$\sigma_{Actual} = \sigma * K_t = 41.55*1.8 =$ 74 MPa

Cast Iron is a brittle material. Thus the maximum normal stress theory is applicable.
Since, cast iron is weaker in tension,
$N_{fs} = S_{ut}/\sigma_{Actual} = 200/74 =$ 2.7

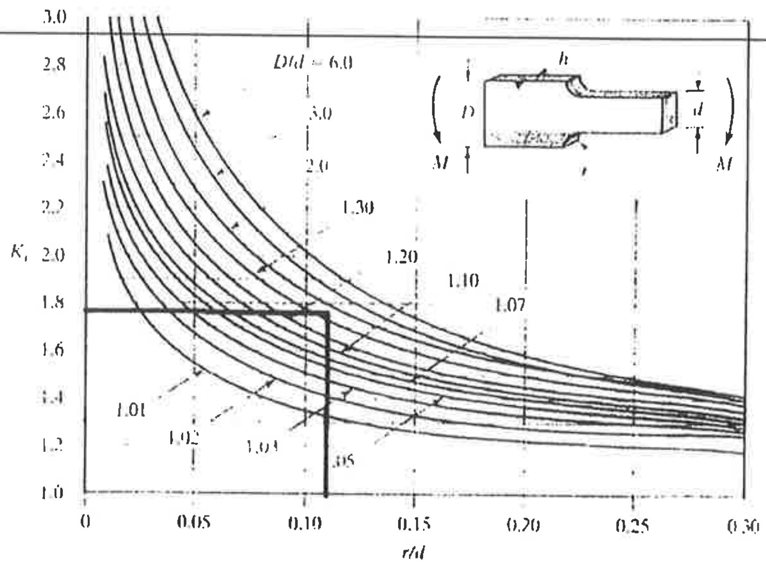


Figure 2-17 Stress concentration factor for a flat bar with a fillet in bending. (Spreadsheet Module 2-14.)

Ductile and brittle material, loading type, operating condition and applicability of geometric stress concentration factor

DUCTILE MATERIAL

Steady stress: In ductile materials, when stress exceeds yield strength (S_{yp}) due to stress concentration at a point, plastic deformation is initiated at that point. It has been observed that in such a situation the plastic deformation proceeds in such a way that it reduces and ultimately eliminates the effect of stress concentration. Plastic deformation due to stress concentration is limited in a small area does not generally constitute failure of the part in most design situations. As a result of this, **for ductile materials, when the stress is applied gradually and the stress is steady (not changing too much), stress concentration effects are neglected (K=1) in mechanical design calculations.**

Impact loading: Instead of gradual loading, if the load is applied suddenly, such as an impact load, the stress inside the material may reach to ultimate tensile strength (S_u) due to stress concentration. The critically stress point may not get enough time to plastically deform to mitigate the effect of stress concentration. When stress reaches S_u, a brittle failure occurs, that is a crack will form at the critically stressed point. This crack will form a geometric discontinuity and will cause more stress concentration, and as a result the crack will propagate and a **rapid fracture of the part can occur**. Thus, if there is an **impact load on ductile material, stress concentration effect must be considered.**

Cold environment: A similar rapid fracture of a part may occur if the part operates at a low temperature condition. At a low temperature ductile materials can fail as a brittle material; that is no yielding but directly fracture; failure from formation of crack rather than a plastic deformation. Thus, **if a part is expected to operate in low temperature environment, geometric stress concentration factor should be used to determine the actual stress, even if the part is made up of ductile material.**

Cyclic stress: Another form of stress is **cyclic stress** where the direction of stress is continuously changing from positive to negative stress throughout the life cycle of the part. Consider the axial stress in the piston rod in a reciprocating compressor. As long as the compressor is running, in each forward and backward stroke will induce compressive and tensile axial stress in the piston rod. Similar kind of alternating stresses are also common for rotating shafts with bending load in one direction. **The bending stresses in the outer layer of the shaft alternates between tensile and compressive stress, as the part rotates.** For all gear, belt or chain drives, similar alternating bending stress may arise. **Failure of a ductile part due to purely cyclic stress occurs in a special way.** The failure is called **fatigue failure** and occurs at a stress level known as **endurance strength (S_e)** of the material. We will learn more about this fatigue loading in the next section. What is important at this point is that, **the appearance of the failure surface due to fatigue loading resembles brittle failure, even if the part is made of ductile material.** No plastic deformation is noticed at the failure initiation point, rather than the failure surface appears to be initiated from a crack (separation of molecular plane). Due this type of failure characteristics, we may intuitively conclude that stress concentration has an important role in failure for cyclic loading.

Bbased on extensive testing of ductile materials in cyclic loading, it has been found out that the effect of stress concentration in cyclic loading is not as pronounced as in static loading. In other words the **fatigue stress concentration factor (K_f) has a lower value than the geometric stress concentration factor (K).** These two stress concentration factors are related to each other by a material property called **sensitivity index or notch sensitivity (q)** in the following way:

$$q = \frac{K_f - 1}{K - 1}$$

The value of the q can vary between zero and 1.

When q=0, K_f=1, that is, no fatigue stress concentration effect

When q=1, K_f=K, that is, fatigue stress concentration has equal value of geometric stress concentration factor.

The value of the sensitivity index of a material depends on two factors, the type of the material and its micro structure. For some commonly used ductile materials q values are provided in **table 2-6 in the textbook, which varies between 0.07 & 0.57.** For other materials q values are available from material handbooks. **If q value is not readily available for a material, a conservative approach is to use q=1**, that is geometric and fatigue stress concentration factors to have the same value.

BRITTLE MATERIAL: Finally the application of stress concentration factors for brittle materials. As the brittle materials always fail by propagation of crack (separation in atomic plane) and not by yielding (plastic deformation), stress concentration factor will never be mitigated by plastic deformation. As a result of this, **designing with brittle materials, it requires the use of geometric stress concentration factors to determine the actual stress at a point, for all kinds of loading situations.**

TABLE 2-5 WHEN TO CONSIDER STRESS CONCENTRATIONS.

Type of Material	Loading Conditions	Consider or Neglect	Stress Concentration Factor	Type of Failure
Brittle	Any	Consider	K	Rapid Fracture
Ductile	Low Temp.	Consider	K	Rapid Fracture
Ductile	Rapid Application	Consider	K	Rapid Fracture
Ductile	Cyclic	Consider	K_f	Progressive Fatigue Failure
Ductile	Static Loads at Room Temperature	Neglect	1	No Failure Redistribution of Stress.

Appendix A

Mechanical Properties

A.1 Non-Metals

A few non-metals are listed in Table A.1. Collectively the majority of these have limited load bearing capacity where they show (i) low strength, (ii) brittle failure and (ii) a tendency to creep. Compared to metallic materials (see Table A.2) the properties of non-metals may appear undefined on account of the absence of a linear-elastic region for many polymers and the directional behaviour of woods and composites. The latter have by far the greatest application to weight/strength applications [1–4] where they can be used to maximum effect by aligning the grain/fibre direction with the direction of the major principal tensile stress arising from the loading applied.

Table A.1 Weight, stiffness and strength properties of polymers and composites

Non-Metal[1–4]	Density ρ, kg/m^3	Modulus[a] E, GPa	Strength[c] MPa	% El[d]	Comments
ABS	1045	1.4–3.1	17–58	10–140	
Acetate	1220–1340	1.0–2.0	25–65	5–55	Cellulose
Acrylic	1185	2.7–3.5	50–70	5–8	
Cartilage		0.024			
Carbon	3300	1200		–	Diamond
Carbon	1900–2300	700	20	–	Graphite fibre
Ceramics	3600	276	2480/220	–	90% alumina, C/T
Carbides	1500	400	5170–5930	–	94%WC, 6% Co
CFR epoxy	1700	165	1500	2	Along fibre
Concrete	2400	14	4	2	Compression, 70% water
Douglas fir	500–520	12.5	50/138	5	Along grain (C/T)
Epoxy	1150	3.5–8	50–100	–	Cast, thermoset
GFR polyester	1800	20	350	4	50/50 along fibre

(continued overleaf)

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Table A.1 *(continued)*

Non-Metal[1-4]	Density ρ, kg/m^3	Modulus[a] E, GPa	Strength[c] MPa	% El[d]	Comments
Glasses	2600–4100	70	50	–	Al, B, soda silicates
HDPE	935–970	0.4–1.3	22–38	50–300	
LDPE	910–970	0.1–0.3	8–16	300–800	
Nylon 66	1150	2.8–3.3	60–80	60–300	Polyamide
Oak	650	12	100	5	Along grain
Perspex	1190	3	50	2–7	
Polyester	1100–1350	2.4	40–55	650	Thermoset
PTFE	2200	0.34	14–35	200–600	
PC	1200	2.1–2.4	55–65	60–100	
PP	900	1.1–1.6	30–40	50–600	
PS	1060	2.5–4.1	35–60	2–40	
PU	1100–1250	0.01	40	650	
PVC	1400–1700	2.4–4.1	48–58	2–40	Rigid unplasticised
Rubber	910	0.007	17	500	Polyisoprene
Tendon		0.6			

Note: C/T = compression/tension.
For footnotes, see Table A.2.

A.2 Metals and Alloys

The properties listed in Table A.2 apply to both metals and alloys. Particular applications of these materials in engineering structures are normally confined to those alloys that

Table A.2 Weight, stiffness and strength properties of metals and alloys

Metal/Alloy[5-16]	Density ρ, kg/m^3	Elastic moduli[a] E, GPa	G, GPa	0.1% PS[b] σ_y, MPa	UTS[c] σ_{ult}, MPa	% El[d]	Comments
Aluminium[10,11]	2710	70–71	27	50	80	30	99.5% pure
Al-Cu alloy[11,12]	2800	75	28.5	290	425	20	4.4% Cu (2014 -T4)
Al-Mg alloy[11,12]	2725	71	26.5	270	330	8	3.5% Mg (5154- H38)
Alloy steel[5,6]	7900	210	83	750	1000	15	2.5% Ni, Cr, Mo-En25
Antimony	6680	78	29.3				
Bismuth	9800	32	12				
Brass[13]	8500	104	39	140	440	8	Free cutting
Bronze[13]	8800	117	45	100	190	10	+ phosphorous
Cast iron	7150	97	41	310	500(C)	7	Nodular type (BS 2789)
Cobalt	8900	206	79		500		
Constantan	8880	170	63.9				

Table A.2 *(continued)*

Metal/Alloy[5-16]	Density ρ, kg/m^3	Elastic moduli[a] E, GPa	G, GPa	0.1% PS[b] σ_y, MPa	UTS[c] σ_{ult}, MPa	% El[d]	Comments
Copper	8950	96–117	38	65	175	45	
Gold	19300	79	27	80	120	40	
Invar[15]	8000	145	56	280	480	40	64% Fe, 36% Ni
Iron	7850	200–206	82	165	300	45	Pure
Lead	11370	17–18	6	12	15	50	
Magnesium	1740	44	17.1	95	190	5	
Manganese	8500	120	45.1				
Mild-steel[6,7]	7860	207	81	300	510	35	0.4% C
Monel	8800	206	79	240	420–520	40	70% Ni, 30% Cu
Nickel	8900	198	80	60	300	30	
Ni-Alloy[16]	9000	200	79	800	1000	10	76% Ni, 20% Cr
P-Bronze[13]	8800	120	43.5	420	560	30	C510 wrought
Platinum	21040	164	51	250	350		
Silver	10530	78	29	150	180	45	
Stainless steel[7,8]	7930	200	77	210	510	60	18% Cr, 8% Ni (austenitic)
Tin	7300	40	14.7		30		
Titanium	4540	118	45	480	620	20	
Ti-Al alloy[9]	4430	110	42	800	860	15	5% Al (α-alloy)
Ti-Cu alloy[9]	4700	115	44	700	750	15	2.5% Cu
Ti-Sn alloy[9]	4600	105	40	1000	1300	12	11% Sn
Tungsten	19300	410	157				
Zinc	6860	86	38	100	150	50	
Zn-Alloy[14]	6800	80	31	250	330	50	4% Al (β-alloy)

[1-16]These refer to the list of works at the end of this appendix.

[a]E is the modulus of elasticity (Young's modulus), as found from the gradient to the linear-elastic, stress–strain response in a tensile test. Where non-linear behaviour is observed, in polymers and rubbers, E expresses the gradient to the stress–strain curve at its origin. G is the modulus of rigidity (shear modulus) most usually found from the gradient to the linear-elastic, shear stress versus shear strain response in a torsion test. Fewer values of G are available for non-metals than metals though given E for a non-metal, a value for G may be approximated by $G = E/3$.

[b]0.1% proof stress (PS) refers to the stress corresponding to an offset strain of 0.1%. This is a common measure of the yield strength for a metallic material that does not show a sharply defined yield point. The offset is the amount of permanent set (plastic strain) that would arise following an unloading from the quoted proof stress level. Offsetting by lesser or greater amounts of strain (typically 0.05% or 0.2%) is permitted depending upon the amount of permanent set that is tolerable. This yield strength measure does not appear as a listed property for non-metals that respond to stress either in a non-linear or in a brittle manner.

[c]Ultimate tensile strength (UTS) is the greatest stress a material sustains at failure. It would normally be calculated by dividing the maximum load observed in a tensile test by the original cross-sectional area, i.e. without correction for the area reduction under maximum load.

[d]Percentage elongation (% El) refers to the percentage amount by which the initial parallel length of a tensile testpiece has been stretched to failure. It is also the limiting percentage engineering strain for a material since this strain is also defined identically as (extension ÷ original length) × 100.

Table B-1
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-27 60-30	J03000	60	415	30	205	24	35	--
SA-27 65-35	J03001	65	450	35	240	24	35	--
SA-27 70-36	J03501	70	485	36	250	22	30	--
A-27 70-40	J02501	70	485	40	275	22	30	--
A-27 N1	J02500	Mechanical testing not required						
A-27 N2	J03500	Mechanical testing not required						
SA-53 Gr. A	K02504 (Note 4)	48	330	30	205	(Based on size)	(Based on size)	--
SA-53 Gr. B	K03005 (Note 4)	60	415	35	240	(Based on size)	(Based on size)	--
SA-105	K03504 (Note 4)	70	485	36	250	22	30	187 Brinell hardness (HB)
SA-106 Gr. B	K03006 (Note 4)	60	415	35	240	16.5	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-106 Gr. C	K03501 (Note 4)	70	485	40	275	16.5	--	--
SA-134 Gr. 283A	K01400	45–60	310–415	24	165	30	--	--
SA-134 Gr. 283B	K01702	50–65	345–450	27	185	28	--	--
SA-134 Gr. 283C	K02401	55–75	380–515	30	205	25	--	--
SA-134 Gr. 283D	K02702	60–80	415–550	33	230	23	--	--
SA-134 Gr. 285A	K01700	45–65	310–450	24	165	30	--	--
SA-134 Gr. 285B	K02200	50–70	345–485	27	185	28	--	--
SA-134 Gr. 285C	K02801	55–75	380–515	30	205	27	--	--
SA-135 Gr. A	K02509	48	331	30	207	35	--	--
SA-135 Gr. B	K03018	60	414	35	241	30	--	--
A-139 Gr. A	--	48	330	30	205	35	--	--
A-139 Gr. B	K03003 (Note 4)	60	415	35	240	30	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
A-139 Gr. C	K03004 (Note 4)	60	415	42	290	25	--	--
A-139 Gr. D	K03010	60	415	46	315	23	--	--
A-139 Gr. E	K03012	66	455	52	360	22	--	--
SA-155 KC55 (Note 1)	K02001 (Note 5)	55–75	380–515	30	205	27	--	From A-515 Gr. 55
SA-155 KC60 (Note 1)	K02401	60–80	415–550	32	220	25	--	From A-515 Gr. 60
SA-155 KC65 (Note 1)	K02800	65–85	450–585	35	240	23	--	From A-515 Gr. 65
SA-155 KC70 (Note 1)	K03101	70–90	485–620	38	260	21	--	From A-515 Gr. 70
SA-155 KCF55 (Note 1)	K01800	55–75	380–515	30	205	27	--	From A-516 Gr. 55
SA-155 KCF60 (Note 1)	K02100	60–80	415–550	32	220	25	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-155 KCF65 (Note 1)	K02403	65–85	450–585	35	240	23	--	--
SA-155 KCF70 (Note 1)	K02700	70–90	485–620	38	260	21	--	From A-516 Gr. 70
SA-178 Gr. A	K01200	47	325	26	180	35	--	--
SA-178 Gr. C	K03503	60	415	37	255	30	--	--
SA-178 Gr. D	K02709	70	485	40	275	30	--	--
SA-179	K01200 (Note 4)	47	325	26	180	35	--	--
SA-181 Cl. 60	K03502	60	415	30	205	22	35	--
SA-181 Cl. 70	K03502	70	485	36	250	18	24	--
SA-192	K01201	47	325	26	180	35	--	--
SA-210 Gr. A1	K02707	60	415	37	255	30	--	--
SA-210 Gr. C	K03501	70	485	40	275	30	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-212 Gr. B (Note 1)	--	70–85	485–585	38	260	21 for flange; 22 for firebox	--	--
SA-214 (Note 2)	K01807	47	325	26	180	--	--	72 Rockwell hardness (B scale) (HRB)
SA-216 WCB	J03002	70–95	485–655	36	250	22	35	--
SA-216 WCC	J02503	70–95	485–655	40	275	22	35	--
SA-226 (Note 1)	K01201 (Note 5)	47	325	26	180	35	--	72 HRB
SA-234 WPB	K03006	60–85	415–585	35	240	30	--	--
SA-234 WPC	K03501	70–95	485–655	40	275	30	--	--
SA-266 Gr. 1	K03506 (Note 4)	60–85	415–585	30	205	23	38	--
SA-266 Gr. 2	K03506 (Note 4)	70–95	485–655	36	250	20	33	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-266 Gr. 3	K05001 (Note 4)	75–100	515	690	37.5	260	19	30
SA-266 Gr. 4	K03017	70–95	485–655	36	250	20	33	--
SA-283 Gr. A	K01400	45–60	310–415	24	165	30	--	--
SA-283 Gr. B	K01702	50–65	345–450	27	185	28	--	--
SA-283 Gr. C	K02401	55–75	380–515	30	205	25	--	--
SA-283 Gr. D	K02702	60–80	415–550	33	230	23	--	--
SA-285 Gr. A	K01700	45–65	310–450	24	165	30	--	--
SA-285 Gr. B	K02200	50–70	345–485	27	185	28	--	--
SA-285 Gr. C	K02801	55–75	380–515	30	205	27	--	--
SA-299	K02803	75–95	515–655	42 ≤ 1 in. 40 > 1 in.	290 ≤ 1 in. 275 > 1 in.	19	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-333 Gr. 1	K03008	55	380	30	205	35	--	13 feet/pound (ft-lb) @ -50°F
SA-333 Gr. 6	K03006	60	415	35	240	30	--	13 ft-lb @ -50°F
SA-334 Gr. 1	K03008	55	380	30	205	35	--	HRB 85 13 ft-lb @ -50°F
SA-334 Gr. 6	K03006	60	415	35	240	30	--	HRB 90 13 ft-lb @ -50°F
SA-350 LF1	K03009	60–85	415–585	30	205	28	38	13 ft-lb @ -20°F

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-350 LF2	K03011	70–95	485–655	36	250	30	30	15 ft-lb @ -50°F for Cl. 1 20 ft-lb @ 0°F for Cl. 2
SA-352 LCA	J02504	60.0–80.0	415–585	30.0	205	24	35	13 ft-lb @ -25°F
SA-352 LCB	J03003	65.0–90.0	450–620	35.0	240	24	35	13 ft-lb @ -50°F
SA-352 LCC	J02505	70.0–95.0	485–655	40.0	275	22	35	15 ft-lb @ -50°F
SA-372 Gr. A	K03002	60–85	415–585	35	240	20	--	121 HB
SA-372 Gr. B	K04001	75–100	515–690	45	310	18	--	156 HB
A-381Y35	K02601	60	415	35	240	26	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
A-381 Y42	K02601	60	415	42	290	25	--	--
A-381 Y46	K02601	63	435	46	316	23	--	--
A-381 Y48	K02601	62	430	48	330	21	--	--
A-381 Y50	K02601	64	440	50	345	21	--	--
A-381 Y52	K02601	66	455	52	360	20	--	--
A-381 Y56	K02601	71	490	56	385	20	--	--
A-381 Y60	K02601	75	515	60	415	20	--	--
SA-433 Gr. L-45 (Note 1)		45–55	--	24	--	30	--	--
SA-433 Gr. L-50 (Note 1)		50–60	--	27	--	28	--	--
SA-433 Gr.L-55 (Note 1)		55–65	--	30	--	27	--	--
SA-433 Gr. LK-55 (Note 1)		55–65	--	30	--	27	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-433 Gr. LK-60 (Note 1)		60–72	--	32	--	24	--	--
SA-433 Gr. LK-65 (Note 1)		65–77	--	35	--	22	--	--
SA-433 Gr. LK-70 (Note 1)		70–85	--	38	--	20	--	--
SA-442 Gr. 55 (Note 1)	K02202 (Note 5)	55–75	380–515	30	205	26	--	--
SA-442 Gr.60 (Note 1)	K02402 (Note 5)	60–80	415–550	32	220	23	--	--
SA-455	K03300	75–95	515–655	38	260	22	--	≤ 0.375-in. thick
	--	73–93	505–640	37	255			>0.375 in. ≤0.580 in.
A-465 Gr. L-I (Note 1)	--	60	414	30	207	22	35	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
A-465 Gr. L-II (Note 1)	--	60	414	30	207	25	38	--
A-465 Gr. L-III (Note 1)	--	70	483	36	248	18	24	--
A-465 Gr. L-IV (Note 1)	--	70	483	36	248	22	30	--
SA-508 Gr. 1	K13502	70–95	485–655	36	250	20	38	--
SA-515 Gr. 55 (Note 1)	K02001 (Note 5)	55–75	380–515	30	205	27	--	--
SA-515 Gr. 60	K02401	60–80	415–550	32	220	25	--	--
SA-515 Gr. 65	K02800	65–85	450–585	35	240	23	--	--
SA-515 Gr. 70	K03101	70–90	485–620	38	260	21	--	--
SA-516 Gr. 55	K01800	55–75	380–515	30	205	27	--	--
SA-516 Gr. 60	K02100	60–80	415–550	32	220	25	--	--
SA-516 Gr. 65	K02403	65–85	450–585	35	240	23	--	--
SA-516 Gr. 70	K02700	70–90	485–620	38	260	21	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-524 Gr. I	K02104	60–85	414–586	35	240	30	--	--
SA-524 Gr. II	K02104	55–80	380–550	30	205	35	--	--
SA-537 Cl. 1	K12437	70–90 for ≤ 2.5 in. 65–85 for > 2.5 in.	485–620 for ≤ 2.5 in. 450–585 for > 2.5 in.	50 for ≤ 2.5 in. 45 for > 2.5 in.	345 for ≤ 2.5 in. 310 for > 2.5 in.	22	--	--
SA-541 Gr. 1	K03506	70–95	485–660	36	250	20	38	15 ft-lb @ 40°F
SA-541 Gr.1A	K03020	70–95	485–660	36	250	20	38	15 ft-lb @ 40°F
A-573 Gr. 58	K02301	58–71	400–490	32	220	24	--	--
A-573 Gr. 65	K02404	65–77	450–530	35	240	23	--	--
A-573 Gr. 70	K02701	70–90	485–620	42	290	21	--	--

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-587	K11500	48	331	30	207	40	--	--
SA-671 Gr. CA55	K02801	55–75	380–515	30	205	27	--	Fabricated from SA-285 Gr. C
SA-671 Gr. CB60	K02401	60–80	415–550	32	220	25	--	Fabricated from SA-515 Gr. 60
SA-671 Gr. CB65	K02800	65–85	450–585	35	240	23	--	Fabricated from SA-515 Gr. 65
SA-671 Gr. CB70	K03101	70–90	485–620	38	260	21	--	Fabricated from SA-515 Gr. 70
SA-671 Gr. CC60	K02100	60–80	415–550	32	220	25	--	Fabricated from SA-516 Gr. 60

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-671 Gr. CC65	K02403	65–85	450–585	35	240	23	--	Fabricated from SA-516 Gr. 65
SA-671 Gr. CC70	K02700	70–90	485–620	38	260	21	--	Fabricated from SA-516 Gr. 70
SA-671 Gr. CD70	K12437	70–90 for ≤ 2.5 in. 65–85 for > 2.5 in.	485–620 for ≤ 2.5 in. 450–585 for > 2.5 in.	50 for ≤ 2.5 in. 45 for > 2.5 in.	345 for ≤ 2.5 in. 310 for > 2.5 in.	22	--	Fabricated from SA-537 Cl.1
SA-671 Gr. CE55 (Note 1)	K02202 (Note 5)	55–75	380–515	30	205	26	--	Fabricated from SA-442 Gr. 55

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-671 Gr. CE60 (Note 1)	K02402 (Note 5)	60–80	415–550	32	220	23	--	Fabricated from SA-442 Gr. 60
SA-671 Gr. CK75	K02803	75–95	515–655	42 ≤ 1 in. 40 > 1 in.	290 ≤ 1 in. 275 > 1 in.	19	--	Fabricated from SA-299
SA-672 Gr. A45	K01700	45–65	310–450	24	165	30	--	Fabricated from SA-285 Gr. A
SA-672 Gr. A50	K02200	50–70	345–485	27	185	28	--	Fabricated from SA-285 Gr. B
SA-672 Gr. A55	K02801	55–75	380–515	30	205	27	--	Fabricated from SA-285 Gr. C

Table B-1 (continued)
Mechanical Properties of Selected Carbon Steels

ASME/ASTM Material Specification	UNS Number (Note 3)	Mechanical Properties						
		Tensile Strength		Yield Strength		Minimum Elongation (2 in./ 50mm), %	Minimum Reduction in Area, %	Other
		ksi	MPa	ksi	MPa			
SA-672 Gr. B55 (Note 1)	K02001	55–75	380–515	30	205	27	--	Fabricated from SA-515 Gr. 55
SA-672 Gr. B60	K02401	60–80	415–550	32	220	25	--	Fabricated from SA-515 Gr. 60
SA-672 Gr. B65	K02800	65–85	450–585	35	240	23	--	Fabricated from SA-515 Gr. 65
SA-672 Gr. B70	K03101	70–90	485–620	38	260	21	--	Fabricated from SA-515 Gr. 70
SA-672 Gr. C55	K01800	55–75	380–515	30	205	27	--	Fabricated from SA-516 Gr. 55
SA-672 Gr. C60	K02100	60–80	415–550	32	220	25	--	Fabricated from SA-516 Gr. 60

CITY

Design web

Factor of Safety Method

- **Factor of safety method,** the classical method of design, employs reduced values of strengths that are used in the design to determine the geometrical dimensions of the parts.
- A design factor of safety N_d, some times called simply design factor N, is defined by the relation

$$N = \frac{Loss\ of\ function\ Load}{Allowable\ Load} = \frac{Strength}{Stress} = \frac{S}{\sigma}$$

- The failure stress (strength) can be anything the designer chooses it to be. Often such strengths as minimum, mean, yield, ultimate, shear, fatigue as well as others are used; of course it must correspond in type and units to the induced stress.

Material	Load	Factor of safety value N
Exceptionally reliable	Certainly known	1.25 to 1.50
Well known	Known	1.50 to 2.00
Known	Well known	1.50 to 2.00
Less tried	Known orAverage	2.00 to 2.50

PRODUCT DESCRIPTION

This report, one in an ongoing series of metallurgical reports, is devoted to iron-based alloys that contain only residual amounts of elements other than the primary alloying element, carbon—the definition of *carbon steel*. Because of its attractive cost, wide availability, and ease of fabrication and weldability, carbon steel is one of the most commonly used materials in the electric power generation industry. Carbon steels in which carbon represents 0.15–0.35%—those used most often as boiler and piping materials—are the focus of this *Carbon Steel Handbook*.

Although carbon steel is available in virtually all product forms, it is the pressure-containing applications that are of primary interest in this report: pipes, tubes, plates, castings, forgings, and wrought fittings.

Results and Findings

The report presents technical background information on carbon steels and the various international standards that apply to them; applicable American Society of Mechanical Engineers (ASME) and ASTM International (ASTM) codes; the metallurgy of carbon steels; the physical, mechanical, creep, graphitization, fatigue, and grain growth properties of carbon steels; oxidation resistance; and fabrication and welding issues. Two appendices—one containing a table of material chemical compositions and the other containing a table of mechanical properties of selected carbon steels—are included.

Challenges and Objectives

Maintaining an accurate knowledge of the full range of boiler materials has become increasingly challenging: even for well-established alloys, the information base continues to expand, and new alloys with complex metallurgies are regularly introduced. The intent of this report and the others in the series is to provide a comprehensive materials reference that organizes relevant information in a concise manner for each material.

$$\sigma_b = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

using Factor of safety of 2.0

$n = 2.0$

$$\sigma_d = \frac{380}{2} = 190 \text{ N/mm}^2$$

$$190 = \frac{16}{\pi D^3}\left[2406685 + \sqrt{2406685^2 + 1710000^2}\right]$$

$$D^3 = \frac{16\left[2406685 + \sqrt{2406685^2 + 1710000^2}\right]}{190 \times \pi}$$

$$D = \sqrt[3]{}$$

$D = 40$ mm

Design Based on Max. Torsion :

$$\tau = \frac{380}{2} = 190 \text{ N/mm}^2$$

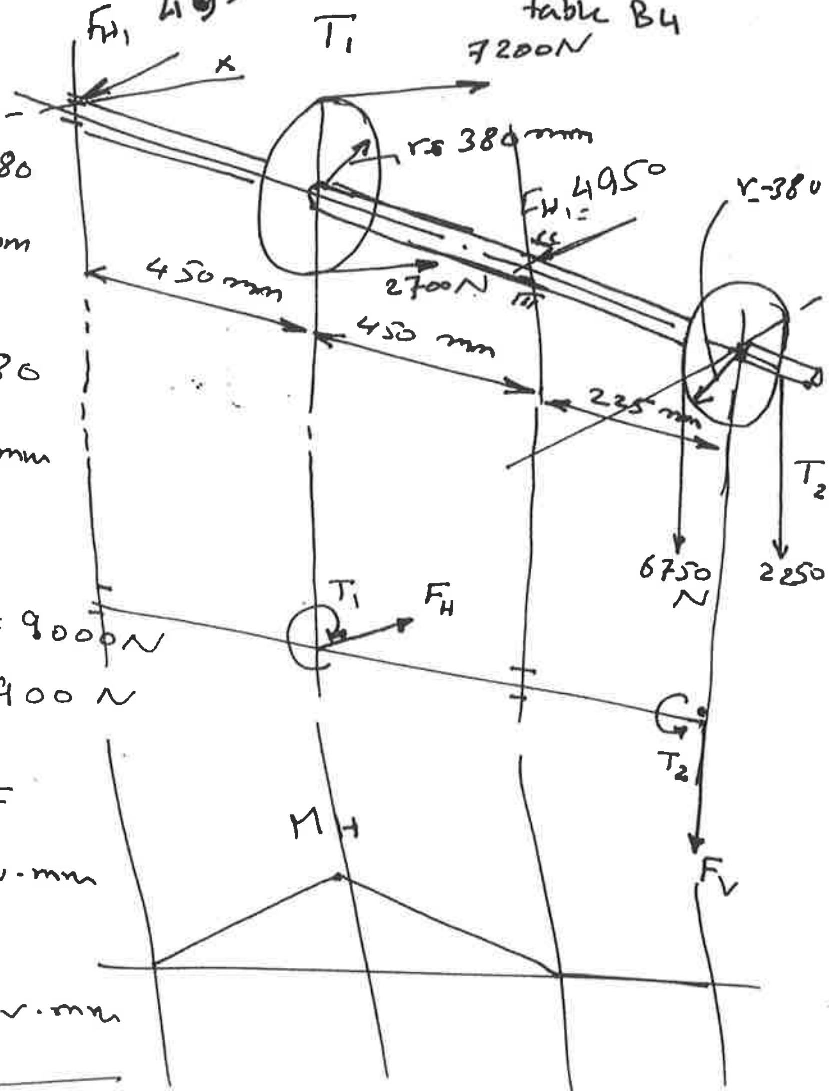
$$[\tau] = \frac{190}{2} = 95 \text{ N/mm}^2$$

$$\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2 + T^2}\right]$$

$D = 54$ mm

$\therefore$ $D = 54$ mm is the best for the loading system

Example (1): Design a shaft holding Two pulleys if the material is steel A-139 Gr.E table B4



$T_1 = (7200 - 2700) \times 380$

$T_1 = 1\,710\,000$ N.mm

$T_2 = (6750 - 2250)\,380$

$T_2 = 1\,710\,000$ N.mm

Vertical force: $F_V = 9000$ N

Horizontal force = 9900 N

$M_H = 4950 \times 450 =$

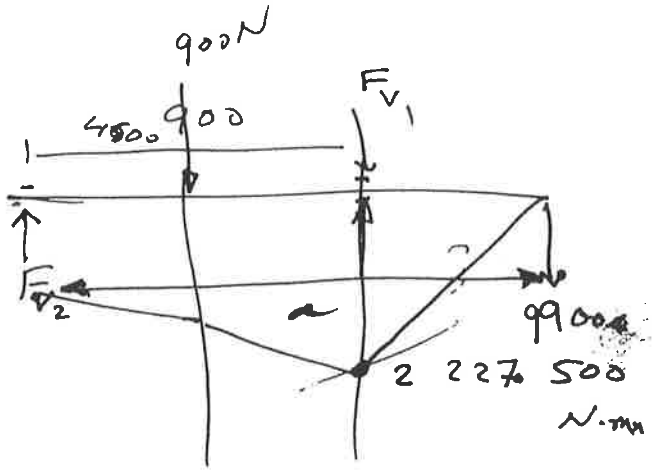
$M_H = 2\,227\,500$ N.mm

$M_V = 2\,227\,500$ N.mm

$M_R = \sqrt{M_V^2 + M_H^2}$

$M_R = 2\,406\,685$ N.mm

$T_R = 1\,710\,000$ N.m



Designing based on Max. Bending:

$\sigma_{max} = 158.255\ N/mm^2$

$\sigma_{min} = -28.25\ N/mm^2$

$$\tau_y = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{158.255 + 28.25}{2} = 93.25\ N/mm^2$$

$\tau = 93.25\ N/mm^2$

$$f.s = \frac{180}{93.25} = \boxed{1.93}$$

For max stress $\sigma_{max} = 158.255\ N/mm^2$

$$f.s = \frac{360}{158.255} = \boxed{2.274}$$

[illegible] max [illegible] 132.6 [illegible]

For σ due to Combined M + T

$$\sigma = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right] \quad \text{——— (1)}$$

$$\sigma = \frac{16}{\pi\, 60^3}\left[2717300 + \sqrt{2717300^2 + 808000^2}\right]$$

$$\sigma = \frac{16}{\pi\, 60^3}\left[2717300 + 2834886\right]$$

$$= \frac{16}{\pi\, 60 \times 60 \times 60}\left[5552186.8\right]$$

$$\sigma_{max} = \frac{16 \times 5552186.8}{\pi \times 60 \times 3600} = \boxed{130\ N/mm^2}$$

$$\tau_{max} = \frac{16\left(\sqrt{M^2 + T^2}\right)}{\pi D^3} = \frac{16 \times 2834886}{\pi \times 60 \times 3600}$$

$$\tau_{max} = \boxed{66.87\ N/mm^2}$$

$$\sigma_{1max} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{130}{2} + \sqrt{65^2 + 66.87^2}$$

$$\sigma_1 = \quad = 65 + \sqrt{4225 + 4472.43}$$

$$\sigma_1 = 65 + 93.25 = \quad N/mm^2$$

$$\sigma_{max} = 158.255\ N/mm^2$$

$$\sigma_{min} = 65 - 9325 = \boxed{-28.25\ N/mm^2}$$

$$\tau_q = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{2020}{\frac{\pi}{4}(60)^2}$$

=

$$\tau_q = \frac{\frac{4}{3} \times 2020 \times 4}{3.14 \times 60 \times 60} = 1.925 \ N/mm^2$$

τ_R = ~~19.064~~ 1.9~~3~~5 =

$$\tau_R = 19.065 + 1.935 = 21.0 \ N/mm^2$$

For Principal stresses:

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{129.29}{2} + \sqrt{\left(\frac{129.29}{2}\right)^2 + 21^2}$$

$$= 64.645 + \sqrt{}$$

$$\sigma_{1\,max} = 64.645 + 67.970 = 132.6 \ N/mm^2$$

$$\sigma_{2\,min} = 64.645 - 67.97 = -3.325 \ N/mm^2$$

For yield theory

$$\tau = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{132.6 - (-3.325)}{2} = \boxed{67.96 \ N/mm^2}$$

$$\tau = \frac{1}{2}\sigma_y = \frac{360}{2} = 180 \ N/mm^2$$

$$\therefore F.S = \frac{180}{67.96} = \boxed{2.65} \longrightarrow *$$

1

$P_1 = 2.020\ kN = 2020\ N$

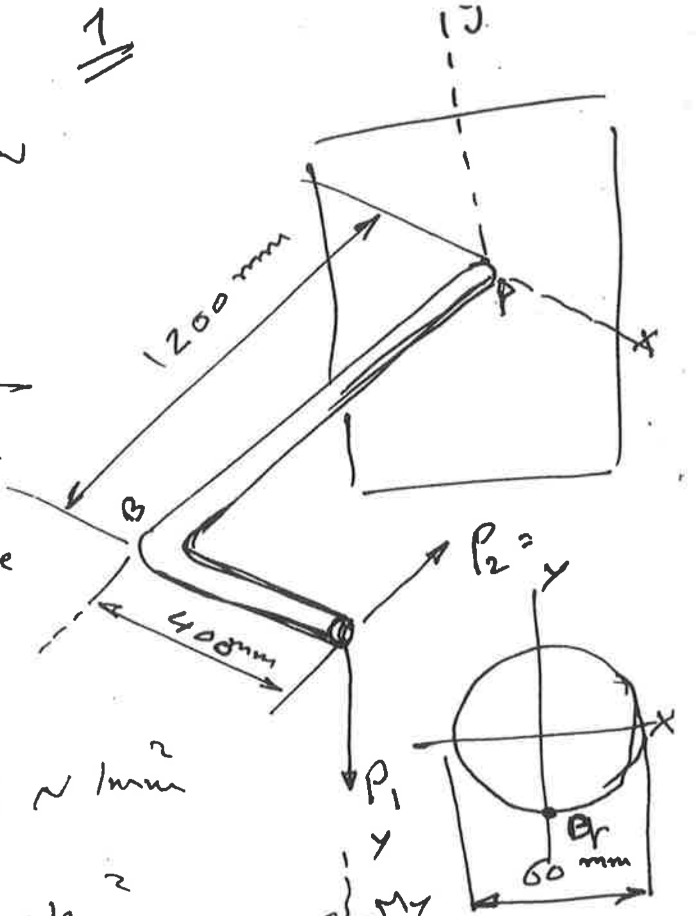
$P_2 = 3.070\ kN = 3070\ N$

$\sigma_y = 360\ N/mm^2$

Find ① principal stresses

② Max. shear stress.

③ - what factor of safety should be taken.



Stresses on the beam, Z

$\sigma_c = \dfrac{3070}{\frac{\pi}{4}(60)^2} = 1.086\ N/mm^2$

$T = 2020 \times 400 = 808\,000\ N/mm^2$

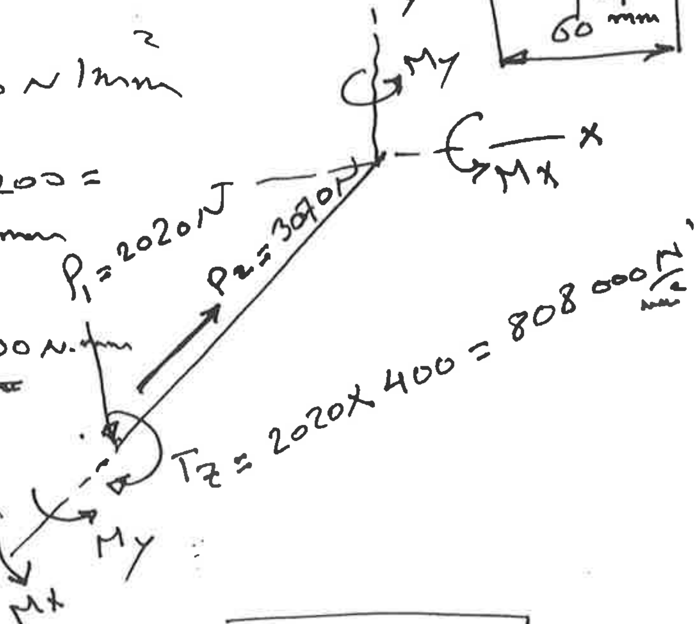
$M_x = P_1 \times 1200 = 2020 \times 1200 =$

$M_x = 2424.000\ N.mm$

$M_y = 3070 \times 400 = 1228000\ N.mm$

$M_R = \sqrt{M_x^2 + M_y^2}$

$M = 2717300\ N.mm$



$\sigma_b = \dfrac{32\ M}{\pi D^3} = \dfrac{32 \times 2717300}{\pi \times 60^3} = \boxed{128\ N/mm^2}$

$\sigma_R = \sigma_b + \sigma_c = 128 + 1.086 = \boxed{129.29\ N/mm^2}$

$\tau = \dfrac{16\ T}{\pi D^3} = \dfrac{16 \times 808000}{\pi \times 60 \times 60 \times 60} = 19.06\ N/mm^2$

(2)

Factor of safety for A:

$$n = \frac{\sigma_y}{\sigma_{ym}} = \frac{280}{99.1} = \boxed{2.8}$$

For point B

$$\sigma_x = \frac{P}{A} = \frac{4 \times 8000}{\pi \times 20^2} = 25.47 \ N/mm^2$$

$$\tau = \frac{16T}{\pi D^3} + \frac{4}{3}\frac{V}{A} = \frac{16 \times 30000}{\pi (20)^3} + \frac{4 \times 550}{3 \ \frac{\pi}{4}(20)^2}$$

$$\tau = 21.43 \ N/mm^2$$

$\sigma_{max} =$

$$\sigma_{max} = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma_{max} = \frac{1}{2}\left(25.47 + \frac{1}{2}\sqrt{25.47^2 + 4 \times 21.43^2}\right)$$

$+ 1836.97$

$$\sigma_{max} = 37.66 \ N/mm^2$$

$$f.s = \frac{280}{37.66} = \boxed{7.43}$$

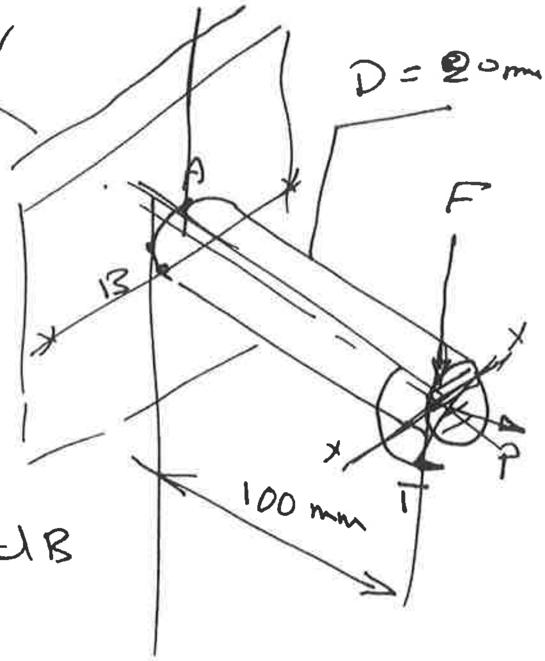
(1)

Ex: 2

A bar is loaded by bending load F of 0.55 kN, tensile load P of 8 kN and a torque of 30 N.m

If the material is steel AISI 1006 cold drawn steel where $\sigma_y = 280\ N/mm^2$

Determine the factor of safety for points A and B



Solution:

for Point A:

$$\sigma_x = \frac{P}{A} + \frac{(F \cdot L)\frac{d}{2}}{\frac{\pi}{64} D^4} \quad (1)$$

$$= \frac{8000}{\frac{\pi}{4}(20)^2} + \frac{550 \times 100 \times 10}{\frac{\pi}{64} \times 20^4} = 95.49\ N/mm^2$$

$$\tau = \frac{T \cdot r}{J} = \frac{30000 \times 16}{\pi\ (20)^3} = 19.10\ N/mm^2$$

$$\sigma_{ymax} = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\left(\frac{\sigma}{2}\right)^2 + 4\tau^2} =$$

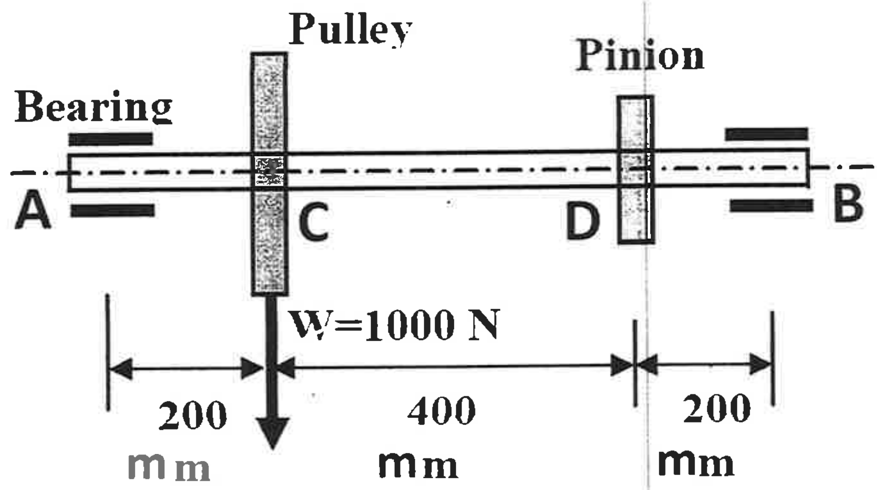
$$\sigma_{max} = \frac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 4\tau^2}\right)$$

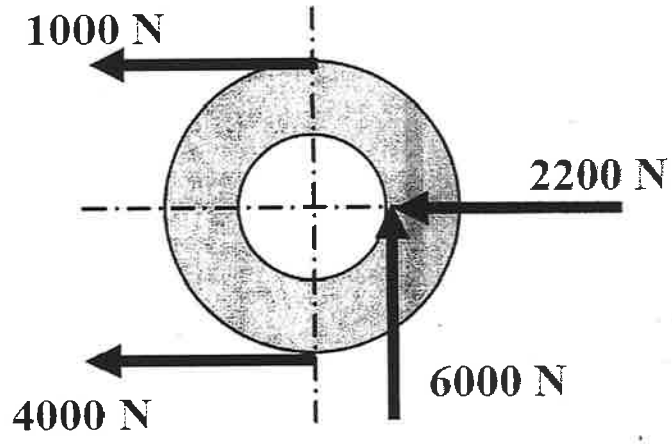
$$= \frac{1}{2}\left(95.49 + \sqrt{95.49^2 + 4 \times 19.1^2}\right.$$

$$(95.49 + 102) = 99.1\ N/mm^2$$

Example: problem

A pulley drive is transmitting power to a pinion, which in turn is transmitting power to some other machine element. Pulley and pinion diameters are 400 mm and 200 mm respectively. Shaft has to be designed for minor to heavy shock.





Example: solution

Torsion:

$T_D = 6000 \times (D_{pinion}/2)$

$= 6000 \times (200/2)$

$= 6x10e5$ N.mm

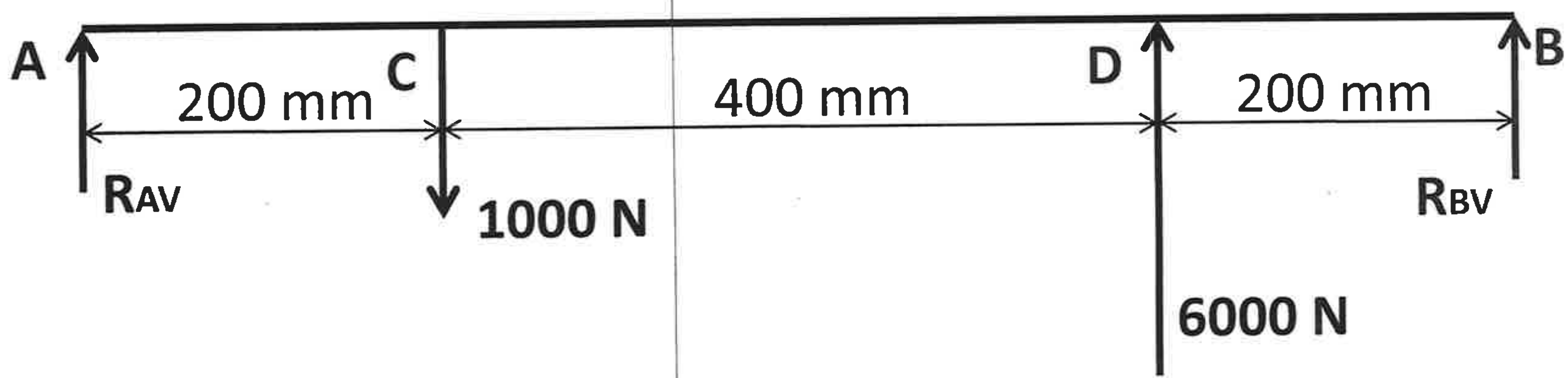
OR

$T_C = (4000 - 1000) \times (D_{pulley}/2)$

$= 3000 \times (400/2) = 6x10e5$ N.mm

Example: solution

Bending (vertical plane):



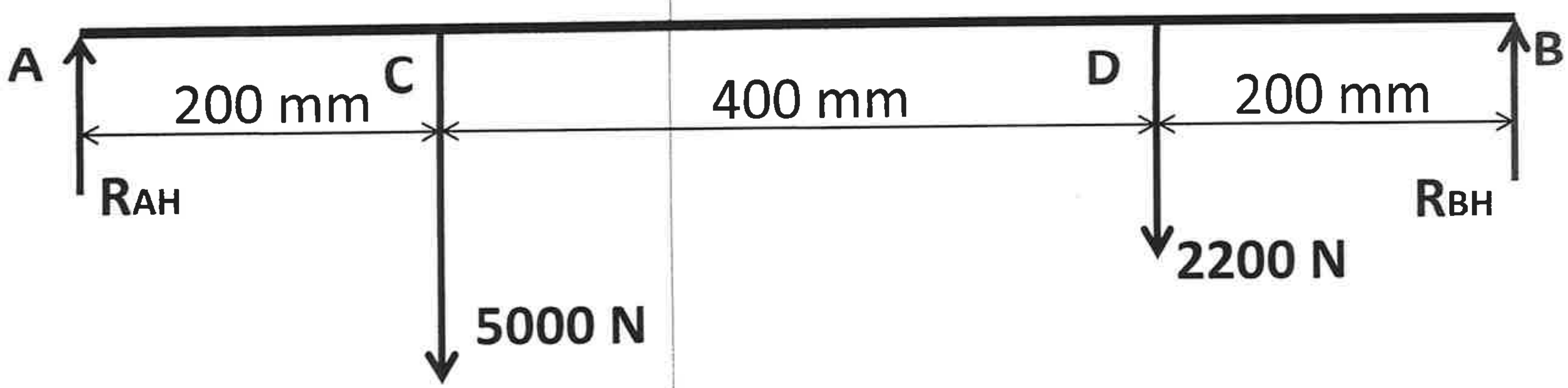
R_{BV}= (1000x200- 6000x(400+200))/(200+400+200) = -4250 N

M_{DV}= -4250x200 = -8.5e5 N.mm

M_{CV}= 6000x400 - 4250x600 = -1.5e5 N.mm

Example: solution

Bending (horizontal plane):



$R_{BH} = (5000 \times 200 + 2200 \times (400+200))/(200+400+200)$

$= 2900$ N

$M_{DH} = 2900 \times 200 = 5.8e5$ N.mm

$M_{CH} = 2900 \times 600 - 2200 \times 400 = 8.6e5$ N.mm

Example: solution

Bending (resultant):

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2}$$

$$= 10.29 \times 10^5 \, N.mm$$

Similarly,

$$M_C = \sqrt{\left(1.5 \times 10^5\right)^2 + \left(8.6 \times 10^5\right)^2}$$

$$= 8.73 \times 10^5 \, N.mm$$

Since $T_C = T_D$ and $M_D > M_C$, section-D is critical.

Example: solution

ASME code:

Under minor to heavy shock, let us consider k_m= 2 and k_t = 1.5. Also let us assume the shaft will be fabricated from commercial steel, i.e. $\tau_{allowable}$ = 40 Mpa.

$$d_o^3 = \frac{16}{40\times\pi}\sqrt{\left(2\times10.29\times10^5\right)^2 + \left(1.5\times6\times10^5\right)^2}$$

$$d_o = 65.88\ mm$$

The value of standard shaft diameter is 66 mm.

Shaft design based on strength

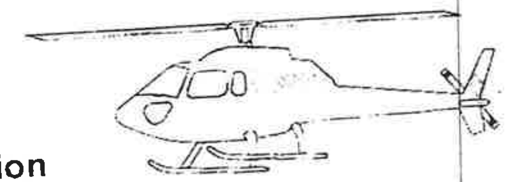
ASME design code:

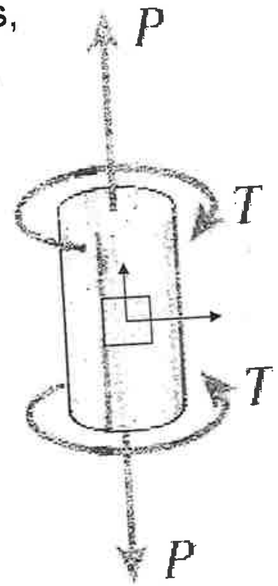
Combined shock and fatigue factors

Type of load	Stationary shaft		Rotating shaft	
	k_m	k_t	k_m	k_t
Gradualy applied load	1	1	1.5	1
Suddenly applied load, minor shock	1.5-2	1.5-2	1.5-2	1-1.5
Suddenly applied load, heavy shock	---	---	2-3	1.5-3

Example The rotor shaft of an helicopter drives the rotor blades that provide the lifting force to support the helicopter in the air. As a consequence, the shaft is subjected to a combination of torsion and axial loading.

For a ***50mm*** diameter shaft transmitting a torque T = ***2.4kN.m*** and a tensile force ***P = 125kN***, determine the maximum tensile stress, maximum compressive stress, and maximum shear stress in the shaft.



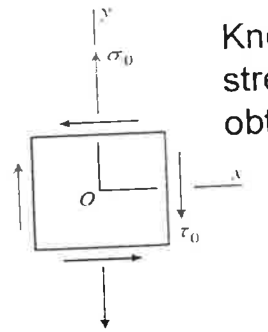


Solution

The stresses in the rotor shaft are produced by the combined action of the axial force ***P*** and the torque T. Therefore the stresses at any point on the surface of the shaft consist of a tensile stress σ_o and a shear stress τ_o.

The tensile stress $$\sigma = \frac{P}{A} = \frac{125kN}{\pi/4\,(0.05m)^2} = 63.66MPa$$

The shear stress to is obtained from the torsion formula $$\tau_{Torsion} = \frac{Tr}{I_P} = \frac{(2.4kN.m)\left(\frac{0.05}{2}\right)}{\frac{\pi(0.05)^4}{32}} = 97.78MPa$$



Knowing the stresses σo and τo, we can now obtain the principal stresses and maximum shear stresses . The principal stresses are obtained from

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$$\sigma_{1,2} = \left(\frac{0 + 63.66}{2}\right) \pm \sqrt{\left(\frac{0 - 63.66}{2}\right)^2 + (-97.78)^2}$$

$\sigma_1 = 135MPa$

$\sigma_2 = -71MPa$

The maximum in-plane shear stresses are obtained using the formula

$$\tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{135-(-71)}{2} = 103\ N/mm^2$$

Because the principal stresses σ_1 and σ_2 have opposite signs, the maximum in-plane shear stresses are larger than the maximum out-of-plane shear stresses. Therefore, the maximum shear stress in the shaft is 103MPa.

Will it fail if σ_{yield}=480MPa? & $\tau = \frac{1}{2}\sigma = 240\ N/mm^2$, so

$$MSST \Rightarrow SF = \frac{480MPa/2}{103MPa} = 2.33$$

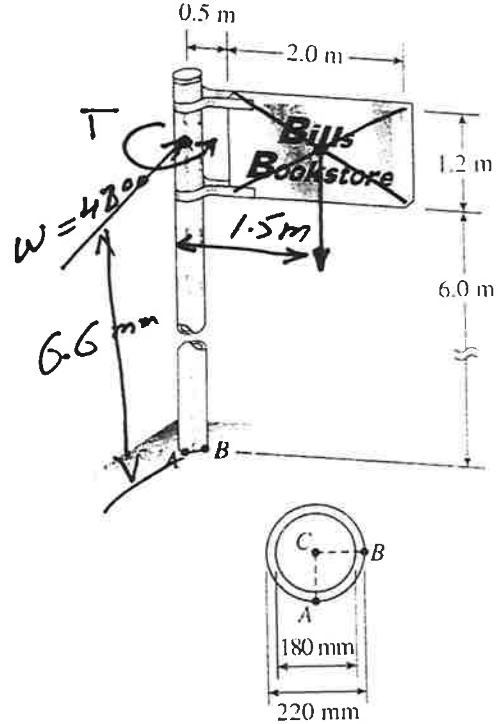
in Torsion

$$\sigma_{VM} = \sqrt{(135)^2 - (135)(-71) + (-71)^2} = 181.2MPa$$

$$DET \Rightarrow SF = \frac{480MPa}{181.2MPa} = 2.65$$

Factor of Safety is 2.65 in tension

Example



A sign of dimensions ***2.0mx1.2m*** is supported by a hollow circular pole having outer diameter ***220mm*** and inner diameter ***180mm*** (see figure). The sign offset ***0.5m*** from the centerline of the pole and its lower edge is ***6.0m*** above the ground.

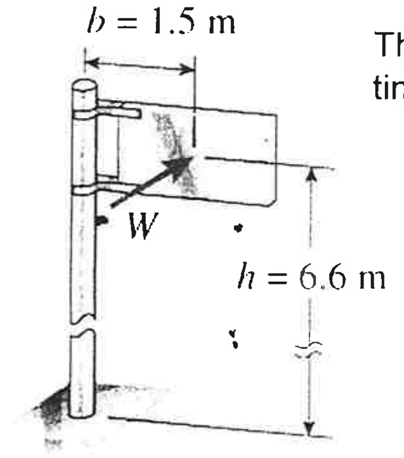
Determine the principal stresses and maximum shear stresses at points ***A*** and ***B*** at the base of the pole due to wind pressure of ***2.0kPa*** against the sign.

Solution

Stress Resultant: The wind pressure against the sign produces a resultant force ***W*** that acts at the midpoint of the sign and it is equal to the pressure ***p*** times the area ***A*** over which it acts:

$$W = pA = (2.0kPa)(2.0m \times 1.2m) = 4.8kN \quad = 4800N$$

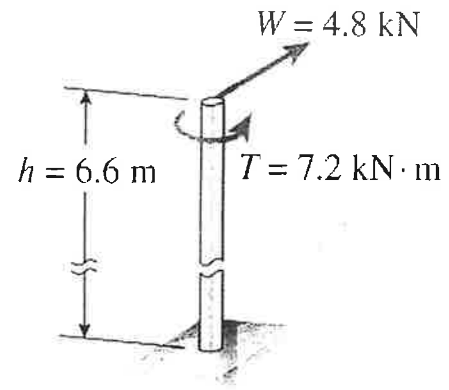
The line of action of this force is at height ***h* = 6.6m** above the ground and at distance ***b* = 1.5m** from the centerline of the pole.
The wind force acting on the sign is statically equivalent to a lateral force ***W*** and a torque T acting on the pole.



The torque is equal to the force ***W*** times the distance ***b***:

$$T = Wb = (4.8kN)(1.5m)$$

$$T = 7.2kN \cdot m$$



The stress resultant at the base of the pole consists of a bending moment M, a torque T and a shear force V. Their magnitudes are:

M = Wh = (4.8kN)(6.6m) = 31.68kN.m

T = 7.2kN.m

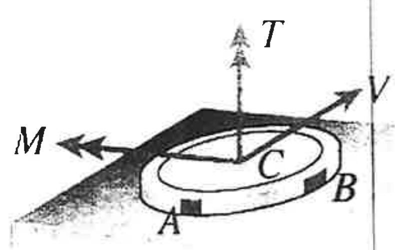
V = W = 4.8kN

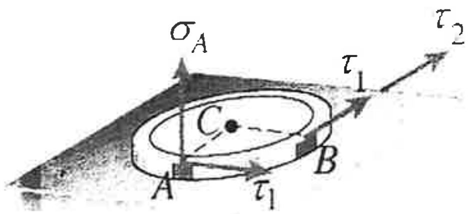
Examination of these stress resultants shows that maximum bending stresses occur at point ***A*** and maximum shear stresses at point ***B***.

Therefore, ***A*** and ***B*** are critical points where the stresses should be determined.

Stresses at points A and B

The bending moment ***M*** produces a tensile stress σ_a at point ***A***, but no stress at point ***B*** (which is located on the neutral axis)





$$\sigma_a = \frac{M\left(\frac{d_2}{2}\right)}{\left[\frac{\pi\left(d_2^4 - d_1^4\right)}{64}\right]} = \frac{(31.68kN)(0.11m)}{\left[\frac{\pi\left(0.22^4 - 0.18^4\right)}{64}\right]} = 54.91MPa$$

N/mm²

The torque *T* produces shear stresses τ_1 at points ***A*** and ***B***.

$$\tau_{Torsion} = \frac{T\left(\frac{d_2}{2}\right)}{\left[\frac{\pi\left(d_2^4 - d_1^4\right)}{32}\right]} = \frac{(7.2kN.m)(0.11m)}{\left[\frac{\pi\left(0.22^4 - 0.18^4\right)}{32}\right]} = 6.24MPa$$

N/mm²

Finally, we need to calculate the direct shear stresses at points ***A*** and ***B*** due to the shear force ***V***.

The shear stress at point ***A*** is zero, and the shear stress at point ***B*** (τ_2) is obtained from the shear formula for a circular tube

$$\tau_{2,Max} = \frac{2V}{A} = \frac{2(4800)}{0.01257m^2} = 0.7637MPa$$

~ N/mm²

$$\tau_2 = \frac{VQ}{Ib}$$

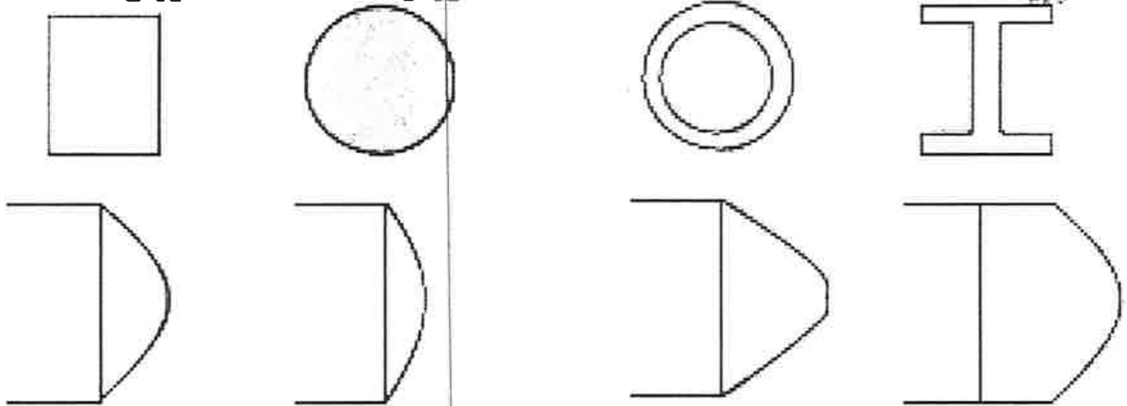
$$I = \left[\frac{\pi\left(d_2^4 - d_1^4\right)}{64}\right]$$

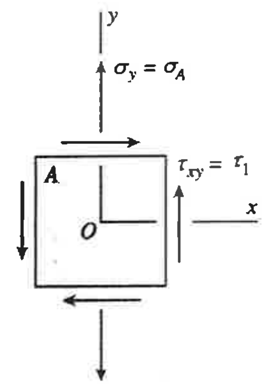
$$Q = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

$$b = 2(r_2 - r_1)$$

The stresses acting on the cross section at points ***A*** and ***B*** have now been calculated.

$\tau_{max} = \frac{3}{2}\frac{V}{A}$ $\tau_{max} = \frac{4}{3}\frac{V}{A}$ $\tau_{max} = 2\frac{V}{A}$ $\tau_{max} = \frac{V}{A_{web}}$





Stress Elements

For both elements the y-axis is parallel to the longitudinal axis of the pole and the x-axis is horizontal.

Point A :

$\sigma_x = 0$

$\sigma_y = \sigma_a = 54.91\text{MPa}$ ✓

$\tau_{xy} = \tau_1 = 6.24\text{MPa}$ ✓

Principal stresses at Point ***A***

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

Substituting $\sigma_{1,2} = 27.5\text{MPa} \pm 28.2\text{MPa}$

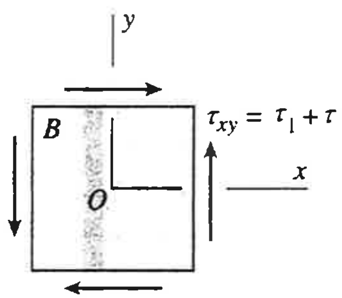
$\sigma_1 = 55.7MPa$ ✓ *and* $\sigma_2 = -0.7MPa$

The maximum in-plane shear stresses can be obtained from the equation

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} = 28.2\,MPa$$

Because the principal stresses have opposite signs, the ***maximum in-plane shear stresses are larger than the maximum out-of-plane shear stresses.***

Then, $\tau_{max} = 28.2MPa.$ ✓



Point B :

$\sigma_x = \sigma_y = 0$

$\tau_{xy} = \tau_1 + \tau_2$

$\tau_{xy} = 6.24MPa + 0.76MPa = 7.0MPa$

Principal stresses at point ***B*** are

σ_1 **= *7.0MPa*** σ_2 **= *- 7.0 MPa***

And the maximum in-plane shear stress is

τ_{max} **= *7.0MPa***

The maximum out-of-plane shear stresses are half of this value.

Note

If the largest stresses anywhere in the pole are needed, then we must also determine the stresses at the critical point diametrically opposite point ***A***, because at that point the compressive stress due to bending has its largest value.

The principal stresses at that point are

σ_1 ***= 0.7MPa*** ***and*** σ_2 ***= - 55.7MPa***

The maximum shear stress is ***28.2MPa.***

(In this analysis only the effects of wind pressure are considered. Other loads, such as weight of the structure, also produce stresses at the base of the pole).

Design of Spur Gears

F = Force on gear tooth N

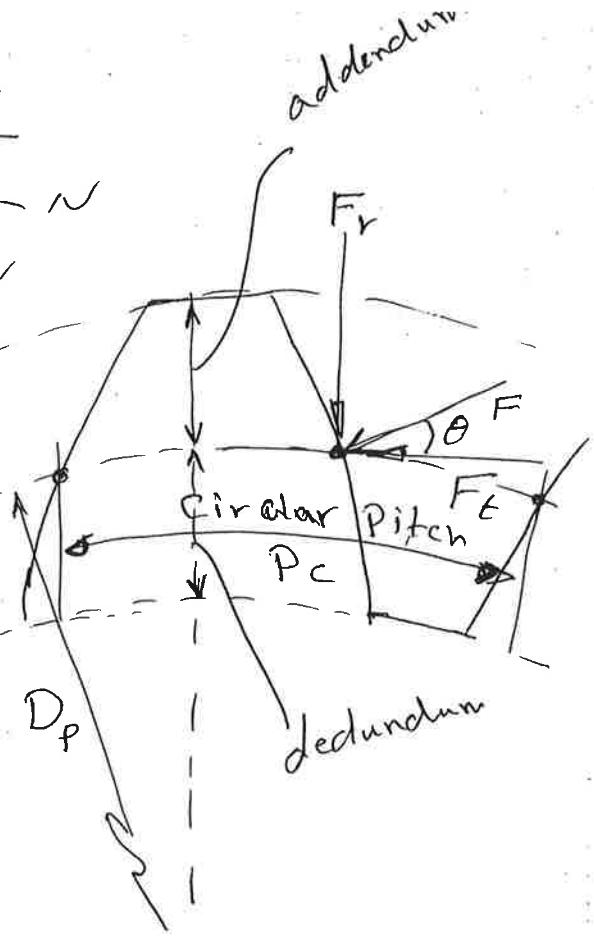
F_t = Tangential Force N

F_r = Radial Force on Gear Tooth

θ = pressure angle (20°)

D_p = Pitch Circle diameter (mm)

P_c = Circular pitch



$$P = \frac{N}{D_p} \quad \text{Pitch} \quad \text{—— (1)}$$

N = Number of teeth

$$\text{Modul } M = \frac{1}{P} = \frac{D_p}{N} \text{ (mm)} \quad \text{—— (2)}$$

$$P_c = \text{circular pitch} = \frac{\pi D_p}{N} \quad \text{—— (3)}$$

$$P_c = \pi\left(\frac{D_p}{N}\right) = m\pi \quad \text{—— (4)}$$

add = modul = m

dedd = (1.25) m

Tooth depth = add + dedd = m + 1.25 m = 2.25 m

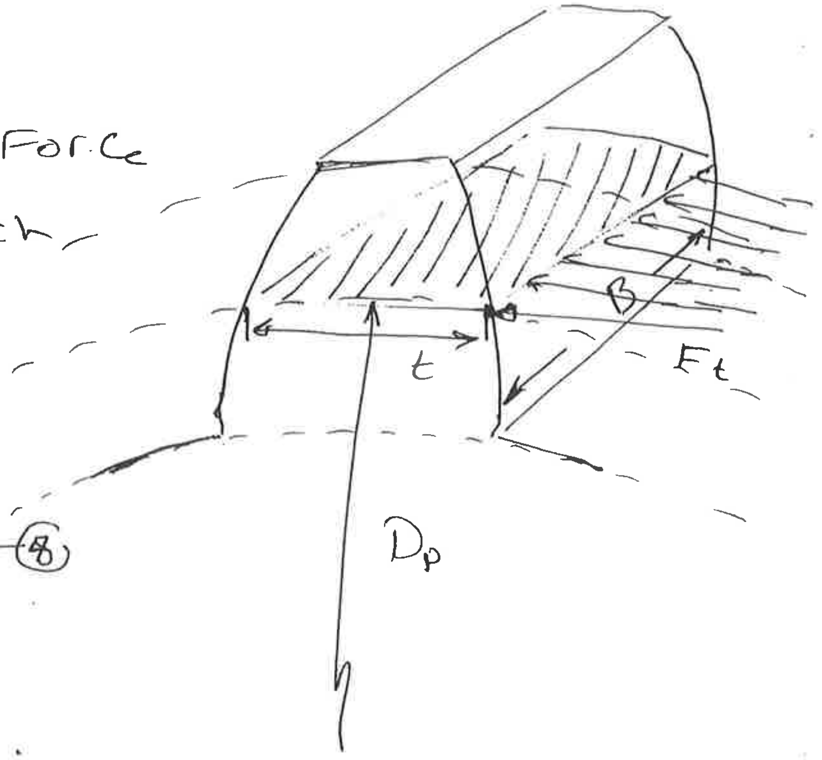
$$D_o = D_p + 2\text{ add} = D_p + 2m \quad \text{—— (5)}$$

$$D_p = Nm \quad \therefore D_o = m(N+2) \quad \text{—— (6)}$$

Tooth thickness $t = \frac{1}{2} P_c = \frac{1}{2} \pi m$ ——— (7)

Forces on Gear Tooth :

Max. Tangential Force effect is at pitch Circule diameter



$$Torque = F_t \cdot \frac{D_p}{2}$$

$$T = \frac{F_t \cdot D_p}{2} \quad \text{——— (8)}$$

$$F_t = \frac{2\,T}{D_p}$$

B = Tooth Face width (mm)

t = tooth thickness (mm)

To Find The torque transmitted :

$$Power = \frac{\pi n T}{60} \left(\frac{N \cdot m}{sec}\right) \text{ watt} \quad \text{——— (9)}$$

$$T = \frac{60\ Power}{\pi\ n}$$

n = angular speed RPM (Revolution per minute)

shear stress τ on the tooth at Circular pitch Diameter

$$\tau = \frac{F_t}{B \cdot t} \quad \text{——— (10)}$$

$$\tau = \frac{F_t}{B \cdot t} \qquad \text{if } t = \frac{1}{2} P_c = \frac{1}{2} m\pi$$

$$\tau = \frac{2 F_t}{B \cdot m \cdot \pi} \qquad (11)$$

or

$$\tau = \frac{2 \cdot \left(\frac{2T}{D_P}\right)}{B \cdot m \cdot \pi} = \frac{4T}{D_P \cdot B \cdot m \cdot \pi} \qquad (12)$$

or

$$\tau = \frac{4 \times 60 \text{ Power}}{\pi^2 \cdot n \cdot B \cdot m \cdot D_p} \leq [\tau] \qquad (13)$$

For Max Bending stress on the gear tooth the effective hight of tooth is

l = add + dedd = 2.25 m (mm)

Moment on the tooth = $M = F_t \cdot l$ — (14)

$$M = F_t \cdot (2.25 m) \qquad (15)$$

$$\sigma_b = \frac{M y}{I}$$

For the tooth : $y = \frac{t}{2}$

$$I = \frac{B t^3}{12}$$

So

$$\sigma_b = \frac{6 F_t \cdot (2.25 m)}{B \cdot t^2} \qquad (16)$$

Design Based on Max Shear:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} \leq [\tau]$$

$$\boxed{\tau_{max} = \sqrt{\left[\frac{13.5 \cdot m \cdot F_t}{2 \times B\, t^2}\right]^2 + \left[\frac{2 F_t}{B \cdot m \cdot \pi}\right]^2} \leq [\tau]}$$

$[\tau]$ = allowable Shear Stress $= \dfrac{\sigma_y/2}{f.s}$

or

$$\tau_{max} = \sqrt{\left[\frac{13.5 \cdot m \cdot F_t}{2 \cdot B \cdot t^2}\right]^2 + \left[\frac{F_t}{B \cdot t}\right]^2}$$

$$\tau_{max} = \frac{F_t}{B \cdot t}\sqrt{\left(\frac{13.5\, m}{2\, t}\right)^2 + 1} \leq [\tau]$$

Design Based on Max. Bending Stress

$$\sigma_{max} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\boxed{\sigma_{max} = \frac{6 F_t \cdot (2.25 m)}{2 \times B\, t^2} + \sqrt{\left[\frac{6 F_t (2.25 m)}{2\, B t^2}\right]^2 + \left[\frac{F_t}{B \cdot t}\right]^2} \leq [\sigma]}$$

$[\sigma]$ allowable stress $= \dfrac{\sigma_y}{f.s}$

Radial force $F_r = F_t \tan\alpha$ N/mm^2

where $\alpha = 20^\circ$ pressure angle.

Tooth Bending stress. By Barth

$$\sigma = \frac{K_v \, W_t}{F \cdot m \cdot Y}$$ Tooth strength N/mm²

$$K_v = \frac{3.05 + V}{3.05}$$ For Cast iron Cast. في حديد الزهر.

$$K_v = \frac{6.1 + V}{6.1}$$ Milled or Cut. بالتفريز

$$K_v = \frac{3.56 + \sqrt{V}}{3.36}$$ hobbed or shaped بالقطع

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}}$$

$$V = \frac{\pi D_p n}{60} \text{ m/sec}$$ D_p = pitch circle diagmeter.

n = RPM.

Y is Taken from table lewis Form Factor depending on number of teeth.

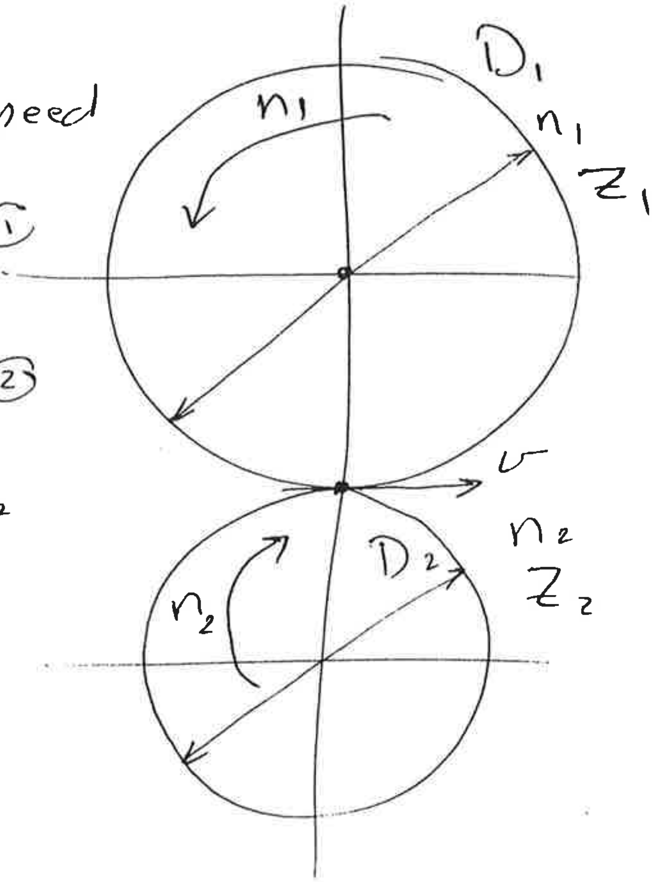
Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

$$V = \frac{\pi D n}{60} \quad mm/sec$$

V = tangential speed

$V = \pi D_1 n_1$ —— (1)
for penion

$V = \pi D_2 n_2$ —— (2)



$\cancel{\pi} D_1 n_1 = \cancel{\pi} D_2 n_2$

$$\frac{D_1}{D_2} = \frac{n_2}{n_1}$$

$D_p = m N = m Z$

$D_1 = m Z_1$

$D_2 = m Z_2$

$$\boxed{\frac{D_1}{D_2} = \frac{n_2}{n_1} = \frac{Z_1}{Z_2}}$$

n = angular speed R.P.M دورة/دقيقة

Z = No. of teeth.

D = pitch circle diameter (mm)

there are many occasions when, because of the unavailability of the correct hob, the teeth are pre-cut with standard cutters and then ground. This means the grinding wheel will have to grind into the root and may leave a "step". The fillet radius will probably also be a lot smaller than a properly cut tooth aid all of this must be taken into account at the "design for loading" stage.

Grinding into the root of a surface hardened gear may release the residue compressive stresses, thus making the bending strength no better than the core material, and also create a serious stress raiser.

TABLE 2.5. SPUR GEAR DESIGN FORMULAE

To Obtain	From Known	Use This Formula[1)]
Pitch Diameter	Module and Number of Teeth	$D = m \cdot z$
Number of Teeth	Module and Pitch Diameter	$z = \frac{D}{m}$
Outside Diameter	Module and Pitch Diameter or Number of Teeth	$D_0 = D + 2 \cdot m = m(z + 2)$
Root Diameter	Pitch Diameter and Module	$D_R = D - 2.5 \cdot m$
Base Circle Diameter	Pitch Diameter and Pressure Angle	$D_b = D \cdot cos\alpha$
Base Pitch	Module and Pressure Angle	$p_b = m \cdot \pi \cdot cos\alpha$
Tooth Thickness at Standard Pitch Diameter	Module	$t_{std} = \frac{\pi \cdot m}{2}$
Center Distance	Module and Number of Teeth	$a = \frac{m(z_1 + z_2)}{2}$
Contact Ratio	Outside Diameters, Base Circle Diameters, Center Distance, Pressure Angle	$n_p = \frac{\sqrt{\frac{D_{01} - D_{b1}}{2}} + \sqrt{\frac{D_{02} - D_{b2}}{2}} - a \cdot sin\alpha}{m \cdot \pi \cdot cos\alpha}$
Backlash (linear)	Change in Center Distance	$bl = 2 \cdot D \cdot a \cdot tan\alpha$
Backlash (linear)	Change in Tooth Thickness	$bl = D \cdot t_{std}$
Minimal Number of Teeth for No Undercutting	Pressure Angle	$z_c = \frac{2}{sin\alpha}$

8.. All linear dimensions in millimeters

EX: The figure below shows a sketch of a Spur gear reducer. The shaft is mounted on two ball bearings A and B.

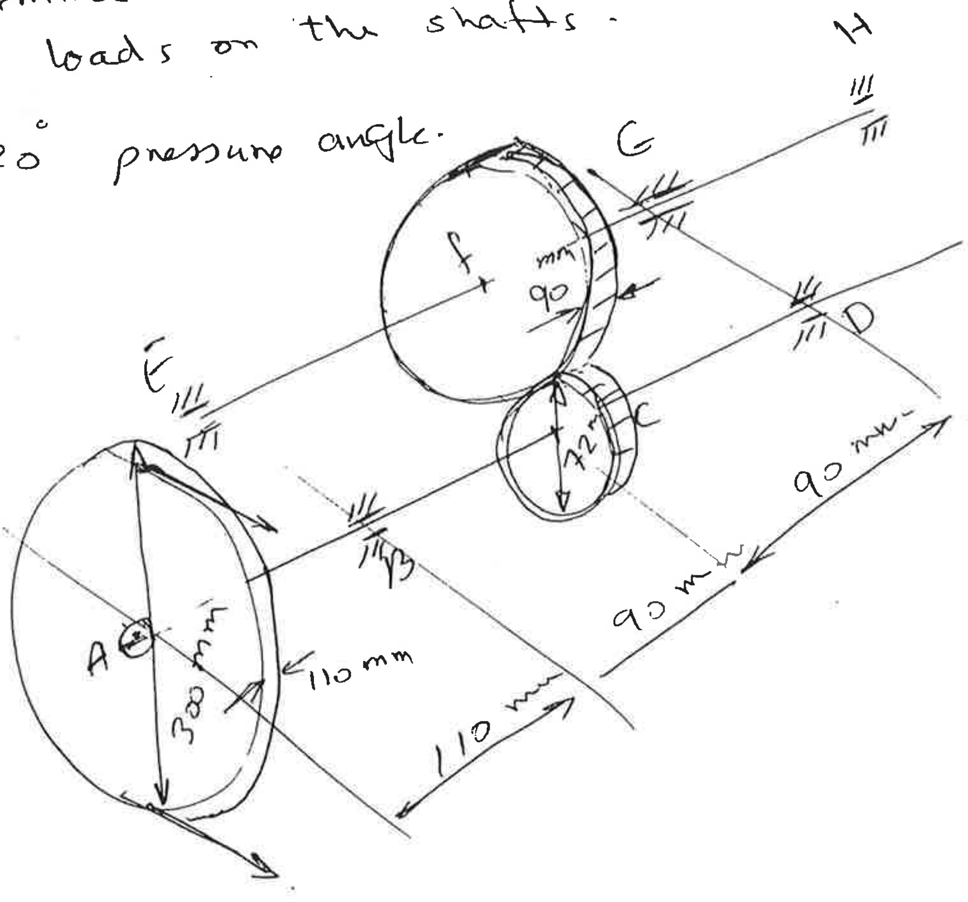
The diameters of the penion and pulley are 72 mm and 300 mm and their widths are 90 and 110 mm respectively.

* ∴ 22KWatt is transmitted at 610 RPM from the pulley to the penion.

* Power is transmitted at 3.95 : 1 from penion to Gear (F).

Determine the shaft minimal diameters and the loads on the shafts.

$\alpha = 20^\circ$ pressure angle.



Solution :

Given Data:

Power = 22 KWatt

n = 610 RPM

D_{penion} = 72 mm B_p = width = 90 mm

D_{pulley} = 300 mm D_{pull} = 110 mm $\alpha = 30^\circ$

Torque transmitted :-

$$Power = T \cdot n \cdot \pi$$

$$\Rightarrow T = \frac{power}{n \cdot \pi} = \frac{22000}{610 \times 3.14} = 344 \ N \cdot m$$

For the pulley :

$$T = (F_{max} - F_{min}) \times \frac{D_{pull}}{2}$$

$$F_{max} - F_{min} = \frac{344 \times 2 \times 1000}{300} = 2290 \ N$$

Hint ملاحظة مهمة $F_{max} = 5 F_{min}$.   for belts بالنسبة للأحزمة على شكل -V

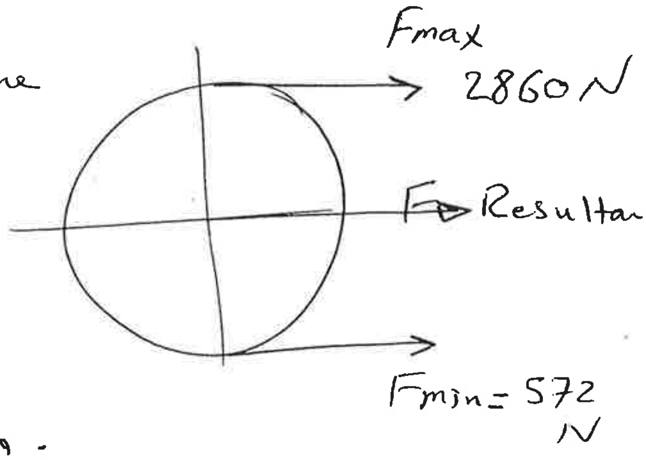
$$5 F_{min} - F_{min} = 2290 \ N$$

$$F_{min} = \frac{2290}{4} = 572 \ N$$

The total force on the pulley:

Resultant force on the pulley shaft =

$F_R = 3432\ N$



For the spur pinion:

Torque transmitted = 344 N.m

$344 \times 10^3 = F_t \times \frac{72}{2}$

$F_t = \frac{2 \times 344 \times 10^3}{72}$

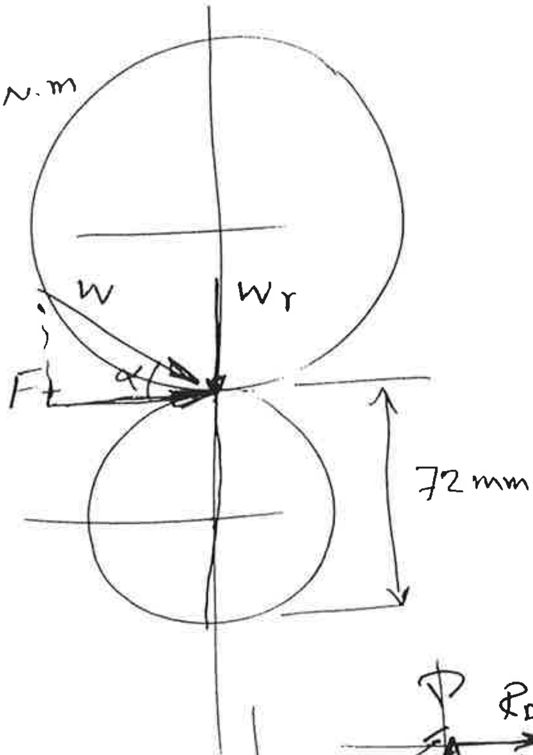
$F_t = 9556\ N$

W_r = Radial force

$W_r = W_t \tan \alpha$

$W_r = 9556 \times \tan 20^\circ$

$W_r = 3478\ N$



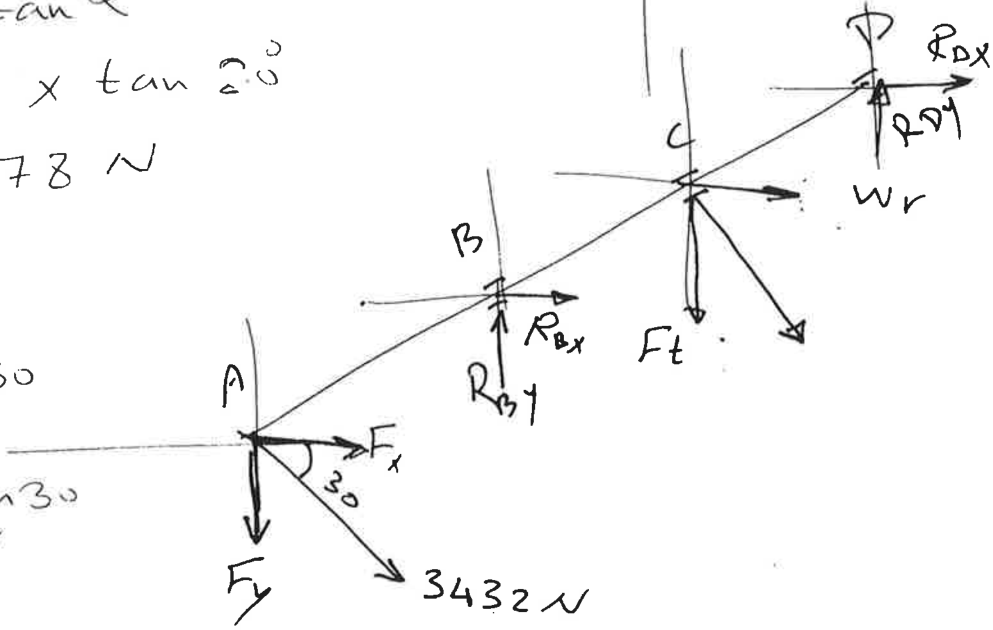
For pulley

$F_x = 3442 \cos 30$

$F_x = 2980\ N$

$F_y = 3432 \sin 30$

$F_y = 1716\ N$



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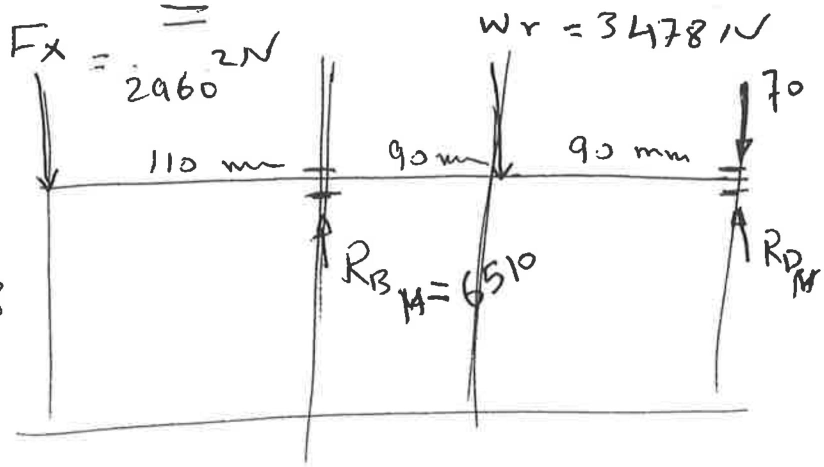


$R_{BH} + R_{DH} = 2960 + 3478$

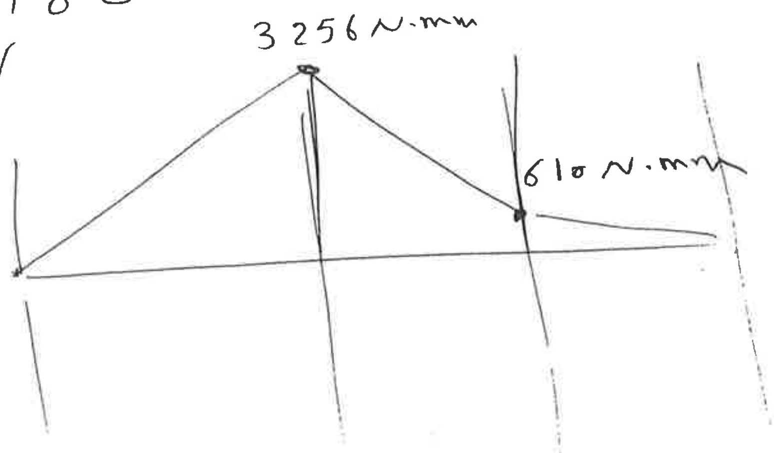
$R_{BH} + R_{DH} = 6580$ N

$\Sigma M_B = 0$

$2960 \times 110 + R_{DH} \times 180 = 3478 \times 90$

$R_{DH} = \dfrac{2960 \times 110 - 3478 \times 90}{180} = -70$ N

$R_{BH} = 6510$ N



Moments.

at 110 mm

$M = 2960 \times 110$

$= 3.256$ N-m

$M = 3256$ N.mm

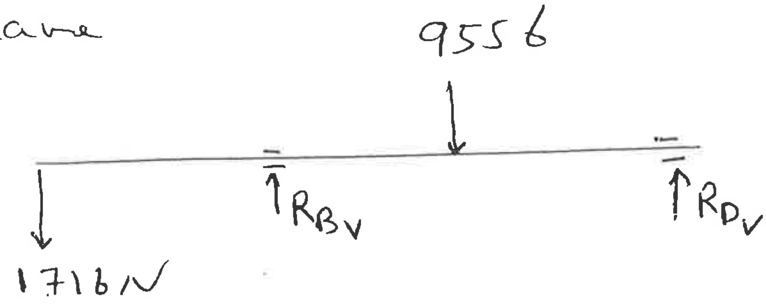
For vertical plane

$\Sigma M_B = 0$

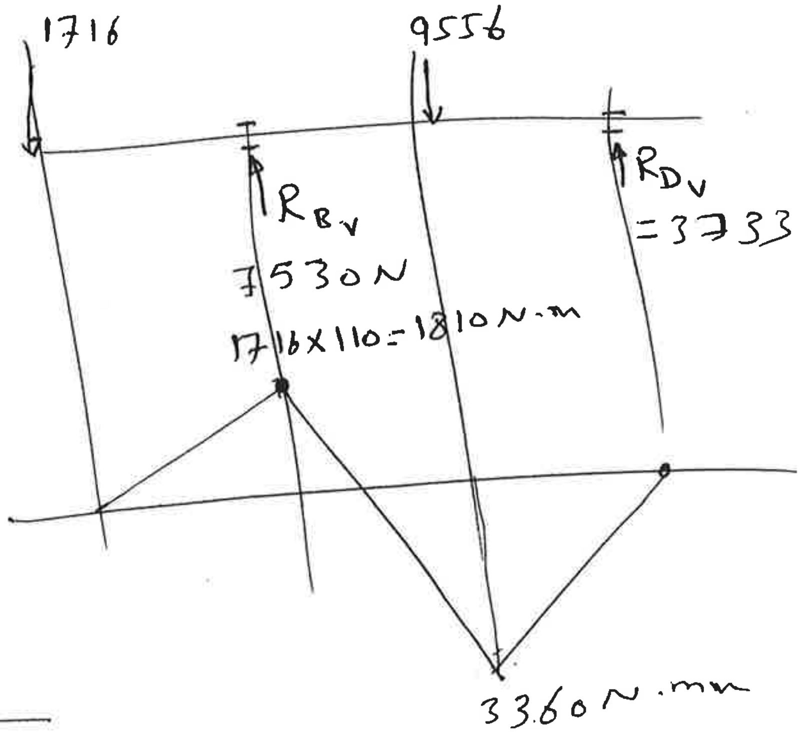
$1716 \times 110 + R_{D_V} \times 180$

$= 9556 \times 90$

$R_{D_V} = 3733$ N



$R_{BV} = 7530\,N$



$M_x = 1810$ N.mm

$M_y = 3256$

$M_B = \sqrt{M_x^2 + M_y^2}$

$M_B = 3730$ N.mm

$M_c = \sqrt{3360^2 + 610^2}$

$M_c = 3420$ N.mm

Forces on Bearings

$R_B = \sqrt{R_{Bx}^2 + R_{By}^2} = \sqrt{650^2 + 7533^2} = 9956\,N$

$R_D = \sqrt{R_{Dx}^2 + R_{Dy}^2} = \sqrt{70^2 + 3733^2} = 3730\,N$

Minimal shaft Diameter: for high speed

$$D_B = \left[\frac{32\ F_{s.f}}{\pi}\sqrt{\left(\frac{F_{cf}\ M_B}{S_n}\right)^2 + \frac{3}{4}\left(\frac{T}{\sigma_y}\right)^2}\right]^{\frac{1}{3}}$$

$F_{s.f} = 2.0$ and $F_{c.F} = 3$

stress concentration factor K_t

S_n = endurance strength

σ_y = yield strength

$$D_B = \left[\frac{16\ F_s}{\pi}\sqrt{\left(\frac{K_t \cdot M}{2}\right)^2 + T^2}\right]^{\frac{1}{3}}$$

$D_B = 43.6$ mm $\Rightarrow$ 44 mm

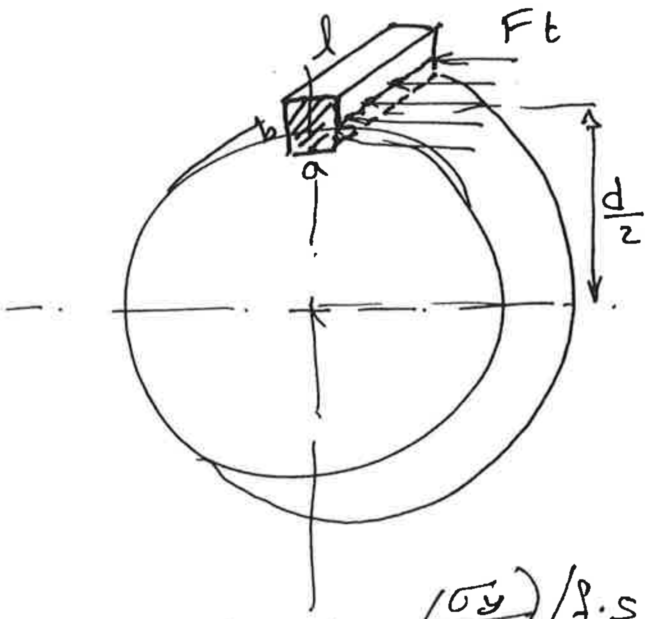
Design of Keys:

$T = F_t \cdot \frac{d}{2}$

$F_t = \frac{2T}{d}$

d = shaft diameter.

$\tau = \frac{F_t}{a \cdot l} \leq [\tau]$

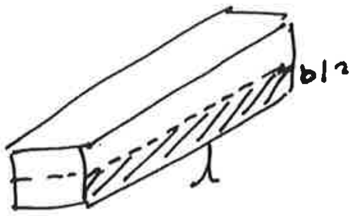


a & b cross-section area dimension of key

l = Length of key

$[\tau]$ shear allowable of key material $= \left(\frac{\sigma_y}{2}\right) / f.s$

For crushing stress σ_c



$\sigma_c = \frac{F_t}{\frac{b}{2} \cdot l}$

$\sigma_c = \frac{2 F_t}{b \cdot l} \leq \sigma_y$

For shear strength:

minimum key length $l = \frac{F_t}{[\tau] a} = \frac{2T}{d \cdot a [\tau]}$

min key length for crushing or compression $l = \frac{F_t}{[\sigma_c] \cdot \frac{b}{2}} = \frac{2 \times 2T}{b \{\sigma\} \cdot d}$

11–3 MATERIALS FOR KEYS

Keys are made from plain carbon steel, alloy steels, stainless steels, and some nonferrous metals. Even plastics are used for small devices under low loads. This book will focus primarily on steels for general industrial applications. Table 11–4 lists a variety of steels and one aluminum alloy to illustrate the types of materials available. (See Internet site 20.)

For problem solutions in this book, it is recommended that the low-carbon steel SAE 1018 be considered for most applications. It is a low-cost, readily available material and its strength is generally adequate.

If higher strength is necessary to produce a design with a reasonable length, medium-carbon SAE 1035 or 1045, or alloy steels SAE 4140 or 8630 are recommended. The high-carbon steel SAE 1095 may be used but it may have low ductility.

Where corrosion resistance is necessary, the stainless steels listed in Table 11–4 may be considered. Aluminum 6061 is less frequently used for keys but may be desirable for material compatibility reasons.

11–4 STRESS ANALYSIS TO DETERMINE KEY LENGTH

There are two basic modes of potential failure for keys transmitting power: (1) shear across the shaft/hub interface and (2) compression failure due to the bearing action between the sides of the key and the shaft or hub material. The analysis for either failure mode requires an understanding of the forces that act on the key. Figure 11–4 shows the idealized case in which the torque on the shaft creates a force on the left side of the key. The key in turn exerts a force on the right side of the hub keyseat. The reaction force of the hub back on the key then produces a set of opposing forces that place the key in direct shear over its cross section, $W \times L$. The magnitude of the shearing force can be found from

$$F = T/(D/2)$$

The shearing stress is then

$$\tau = \frac{F}{A_s} = \frac{T}{(D/2)(WL)} = \frac{2T}{DWL} \qquad (11\text{–}1)$$

In design, we can set the shearing stress equal to a design stress in shear for the maximum shear stress theory of failure:

$$\tau_d = 0.5s_y/N$$

Then the required length of the key is

→ Minimum Required Key Length for Shear

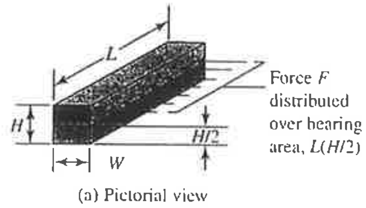
$$L_{min} = \frac{2T}{\tau_d D W} \qquad (11\text{–}2)$$

TABLE 11–4 Examples of Materials Used for Keys

Material designation	Tensile strength, s_u (ksi)	Tensile strength, s_u (MPa)	Yield strength, s_y (ksi)	Yield strength, s_y (MPa)
Carbon steels (SAE)				
1018	[illegible]	[illegible]	[illegible]	[illegible]
1035	[illegible]	496	[illegible]	[illegible]
1045	91	627	77	[illegible]
1095	140	965	83	[illegible]
Alloy steels (SAE)				
4140	102	702	90	[illegible]
8630	100	690	95	655
Stainless steels (SAE)				
[illegible]	90	[illegible]	35	241
304	85	586	35	241
316	85	586	35	241
416	75	517	40	276
Aluminum				
6061	[illegible]	[illegible]	[illegible]	[illegible]

Source: [illegible]

Note: Strength properties typical, not guaranteed.



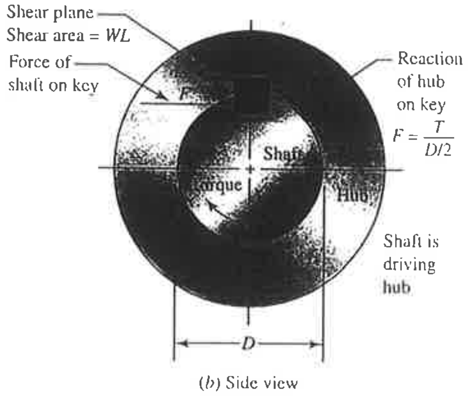


FIGURE 11–4 Forces on a key

The failure in bearing is related to the compressive stress on the side of the key, the side of the shaft keyseat, or the side of the hub keyseat. The area in compression is the same for either of these zones, $L \times (H/2)$. Thus, the failure occurs on the surface with the lowest compressive yield strength. Let's define a *design stress for compression* as

$$\sigma_d = s_y/N$$

Then the compressive stress is

$$\sigma = \frac{F}{A_c} = \frac{T}{(D/2)(L)(H/2)} = \frac{4T}{DLH} \qquad \textbf{(11–3)}$$

Letting this stress equal the design compressive stress allows the computation of the required length of the key for this mode of failure:

➲ Minimum Required Key Length for Compression

$$L_{min} = \frac{4T}{\sigma_d DH} \qquad (11\text{–}4)$$

For the design of a square key in which the strength of the key material is lower than that of the shaft or the hub, Equations (11–2) and (11–4) produce the same result. Substituting the design stress into either equation would give

➲ Minimum Required Key Length if Key Material Is Weakest

$$L_{min} = \frac{4TN}{DWs_y} \qquad (11\text{–}5)$$

But be sure to evaluate the length from Equation (11–4) if either the shaft or the hub has a lower yield strength than the key.

DESIGN PROCEDURE FOR PARALLEL KEYS

1. Complete the design of the shaft into which the key will be installed, and specify the actual diameter at the location of the keyseat.
2. Select the size of the key from Table 11–1.
3. Specify a suitable design factor, N. In typical industrial applications, $N = 3$ is adequate to accommodate accidental overloads and shock.
4. Specify the material for the key, usually SAE 1018 steel. A higher-strength material can be used.
5. Determine the yield strength of the materials for the key, the shaft, and the hub.
6. If a square key is used and the key material has the lowest strength, use Equation (11–5) to compute the minimum required length of the key. This length will be satisfactory for both shear and bearing stress.
7. If a rectangular key is used, or if either the shaft or the hub has a lower strength than the key, use Equation (11–4) to compute the minimum required length of the key based on bearing stress. Also, use Equation (11–2) or (11–5) to compute the minimum required length based on shear of the key. The larger of the two computed lengths governs the design. Check to be sure that the computed length is shorter than the hub length. If not, a higher-strength material must be selected and the design process repeated. Alternatively, two keys or a spline can be used instead of a single key.
8. Specify the actual length of the key to be equal to or longer than the computed minimum length. A convenient standard size should be specified using the preferred basic sizes shown in Appendix A2–1. ***The key should extend over all or a substantial part of the length of the hub. But the keyseat should not run into other stress raisers such as shoulders or grooves.***
9. Complete the design of the keyseat in the shaft and the keyway in the hub using the equations in Figure 11–2. ANSI Standard B17.1 should be consulted for standard tolerances on dimensions for the key and the keyseats.
10. See also Chapter 15 for additional details concerning tolerancing and chamfering.

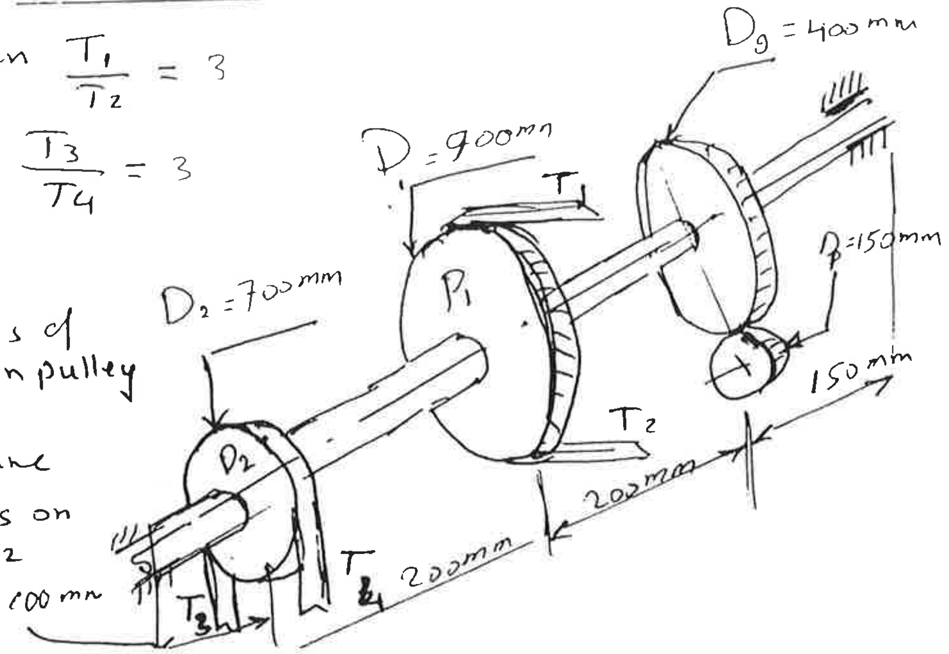
Ex: A line shaft recieves through a gear and a pinion. The pinion is connected to an electric motor delivering 20 KW at 1000 RPM. of which 14 Kwat is supplied to a milling Machine through a horizontal pulley drive at P_1 and the remaining of the power is supplied to a planer machine through pulley P_2 by a vertical belt. The diameters of the gear and pinion are 400 mm and 150 mm respectively. The diameters of the puley P_1 & P_2 are 700 mm and 900 mm respectively. The ratio of belt tensions is 3. Design the shaft. Then design the gear tooth dimensions if the module is 2.5 for angle of pressure $\alpha = 20^\circ$. Shaft material is selected to have yield strength of 360 N/mm² and gears of 415 N/mm². Select a suitable factor of Safety of 2.0

Given $\frac{T_1}{T_2} = 3$

and $\frac{T_3}{T_4} = 3$

where T_1, T_2 tensions of belt on pulley 1

T_3, T_4 are Tensions on pulley P_2



مثال:

خط محور يستلم خلال ترس كبير (A) من صغير قدرة مقدارها 20 كيلوواط، بسرعة دورانية 1000 دورة في الدقيقة 14 كيلوواط تنقل عن طريق بكرة دافعة P_1 والباقي ينقل عن طريق بكرة P_2 وبواسطة الأحزمة. فاذا كان قطر الترس الكبير 400 ملم والترس الصغير 150 ملم وقطر البكرة الكبيرة P_1 هو 900 ملم والصغيرة P_2 هو 700 ملم، ونسبة الشد على طرفي الحزام لكل بكرة هي (3). صمم المحور ثم صمم التروس اذا كان الموديول 2.5 وزاوية الضغط (20) درجة والمعدن المستخدم لصنع المحور له اجهاد خضوع σ_y = 360 نيوتن/ملم² والمعدن المستخدم في صناعة التروس له اجهاد خضوع 415 نيوتن/ملم². اختر معامل أمان (2).

Example:

Design the one-stage spur gear speed reducer, shown to the left, so that it has a power output of 15Kwatt, an input speed of 1,750 rpm, an output speed 700 rpm, uniform power source with moderate shock, maximum gearbox size 36 cm x 36 cm base – 56 cm height, gear life at least 12000 hours .

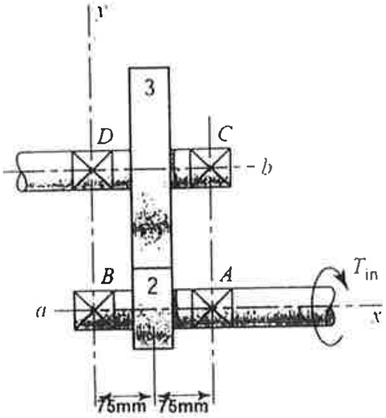
Known

module = 2.5 mm

pressure angle = 20°

and Number of teeth

gear pinion $Z_1 = 16$ tooth



Solution

For pinion

Power $P = 2\pi n T / 60$

$$15000 = \frac{2\pi \times 700 \times T_1}{60}$$

$T_1 = 204.7$ N.m ——— (1)

$$T_2 = \frac{1500 \times 60}{2\pi \times 1750} = 81.9 \text{ N.m}$$ ——— (2)

$$\frac{n_1}{n_2} = \frac{Z_1}{Z_2} = \frac{D_2}{D_1} = 2.5 \qquad \frac{16}{Z_2} = \frac{700}{1750} \Rightarrow Z_2 = 40$$ — (3)

For $\alpha = 20°$ if module = 2.5

$Dp_1 = m Z_1$

$Dp_1 = 2.5 \times 16 = 40$ mm ——— 4

$Dp_2 = 2.5 \times 40 = 100$ mm ——— 5

For Pinion and Gear and for $\alpha = 20^\circ$

$add = m = 2.5$ mm —— 6

$dedd = 1.25 m = 3.125$ mm —— 7

Gear tooth depth $l = 5.625$ mm

Tooth thickness $t = \frac{1}{2} P_c$ —— 8

$$P_c = \frac{\pi D_{p1}}{Z_1} = \frac{\pi \times 40}{16} = 7.85 \text{ mm} \quad \text{—— 9}$$

$$\therefore t = \frac{1}{2} \times 7.85 = 3.92 \text{ mm} \quad \text{—— 10}$$

$$T_1 = F_{t1} \times \frac{D_{p1}}{2}$$

$$81900 = F_{t1} \times \frac{40}{2} = 4850 \text{ N} \quad \text{—— 11}$$

$$\sigma_b = \frac{F_t \cdot l}{B t^2}$$

B is taken allway $2.5 t = 2.5 \times 3.92 \approx 10$ mm

$$\sigma_b = \frac{4095 \times 5.625}{10 \times (3.92)^2} = 899 \text{ N/mm}^2 > \text{ N/mm}^2$$

So taking $B = 40$

$\sigma_b = 448 < 240$ N/mm²

$\sigma_b = 224 < [240]$

③

For max. shear

taking Factor of Safety = 2

$$\tau = \frac{\sigma_y/2}{f.s} = \frac{480/2}{2} = 120\ N/mm^2$$

$$\tau = \frac{F_t}{B \cdot t} = \frac{40950}{40 \times 3.92} = 261 > 120$$

so taking B = 60 mm

$$\tau = \frac{40950}{60 \times 3.92} = 174\ N/mm^2 > 120\ N/mm^2$$

if B = 100 mm

$$\tau = \frac{40950}{100 \times 3.92} = 104\ N/mm^2 < [120]\ N/mm^2$$

OK

For max. Bending

if B = 100

$$\sigma_b = \frac{40950 \times 5.625}{100 \times (3.92)^2} = 150\ N/mm^2 < 240\ N/mm^2$$

taking factor of Safety = 1.5

$[\sigma_y] = 320\ N/mm^2$

$[\tau] = 160\ N/mm^2$

Fo shear theory

$$\tau = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{150}{2} + \sqrt{\left(\frac{150}{2}\right)^2 + (104)^2}$$

$$\sigma_{max} = 203 \ N/mm^2 < 320$$

$$\sigma_{min} = \frac{150}{2} - \sqrt{\left(\frac{150}{2}\right)^2 + (104)^2}$$

$$\sigma_{min} = -53 \ N/mm^2$$

$$\tau = \frac{\sigma_{mx} - \sigma_{min}}{2} = \frac{203+53}{2} = \frac{256}{2} = 128 \ N/mm^2 < 160$$

if.

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\tau_{max} = \sqrt{\left(\frac{150}{2}\right)^2 + (104)^2}$$

$$\tau_{max} = 128 \ N/mm^2 < 160 \ N/mm^2 \quad \therefore \ \underline{OK}$$

For the Radial force F_r :

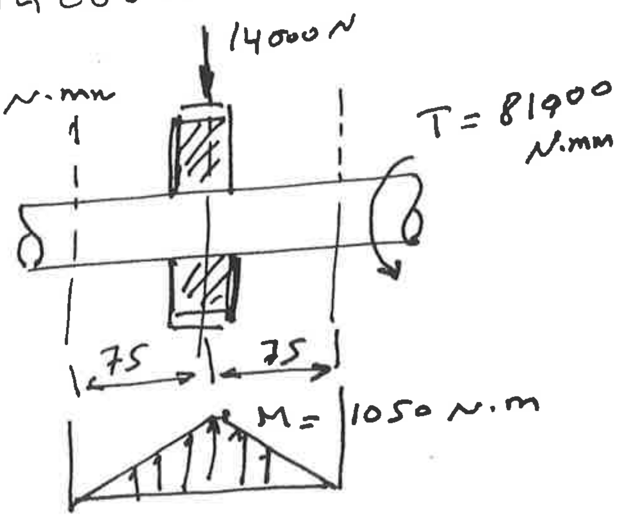
$F_r = F_t \tan\alpha$ where α = pressure angle $= 20°$

$F_r = 40950 \times \tan 20 = 14000 \ N$

$M = 14000 \times 75 = 1050000 \ N.mm$

$T = 81900 \ N.mm$

To design shaft diameter holding pinion :



Based on max. Shear

$$[\tau] = \frac{16}{\pi D^3}\left[\sqrt{M^2 + T^2}\right]$$

$$120 = \frac{16}{\pi D^3}\left[(1050\times10^3)^2 + 81.9\times10^3\right]^{\frac{1}{2}}$$

$$[1102500 + 6707.6$$

$$120 = \frac{16\times10^3\times1053}{\pi D^3}$$

$$\therefore D = \sqrt[3]{\frac{16\times10^3\times1053}{120\times3.14}} = 35.5\ mm$$

$\therefore$ Diameter of shaft is taken = 40 mm

For the shaft holding the gear where

$T = 204\,000$ N.mm

$M = 1050\,000$ N.mm

$$[\tau] = 120 = \frac{16}{\pi D^3}\sqrt{(1050\,000)^2 + (204\,000)^2}$$

$$120 = \frac{16\times1069\times10^3}{\pi D^3} = 35.68\ mm \Rightarrow 40\ mm$$