(where $T_{03'}$ = isentropic stagnation temperature at the diffuser outlet) or

$$\eta_c = \frac{T_{01}\left(T_{03'}/T_{01} - 1\right)}{T_{03} - T_{01}}$$

Let P_{01} be stagnation pressure at the compressor inlet and; P_{03} is stagnation pressure at the diffuser exit. Then, using the isentropic P–T relationship, we get:

$$\frac{P_{03}}{P_{01}} = \left(\frac{T_{03'}}{T_{01}}\right)^{\gamma/(\gamma-1)} = \left[1 + \frac{\eta_c(T_{03} - T_{01})}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{\eta_c \psi \sigma U_2{}^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)} \tag{4.5}$$

Equation (4.5) indicates that the pressure ratio also depends on the inlet temperature T_{01} and impeller tip speed U_2. Any lowering of the inlet temperature T_{01} will clearly increase the pressure ratio of the compressor for a given work input, but it is not under the control of the designer. The centrifugal stresses in a rotating disc are proportional to the square of the rim. For single sided impellers of light alloy, U_2 is limited to about 460 m/s by the maximum allowable centrifugal stresses in the impeller. Such speeds produce pressure ratios of about 4:1. To avoid disc loading, lower speeds must be used for double-sided impellers.

4.7 DIFFUSER

The designing of an efficient combustion system is easier if the velocity of the air entering the combustion chamber is as low as possible. Typical diffuser outlet velocities are in the region of 90 m/s. The natural tendency of the air in a diffusion process is to break away from the walls of the diverging passage, reverse its direction and flow back in the direction of the pressure gradient, as shown in Fig. 4.7. Eddy formation during air deceleration causes loss by reducing the maximum pressure rise. Therefore, the maximum permissible included angle of the vane diffuser passage is about 11°. Any increase in this angle leads to a loss of efficiency due to

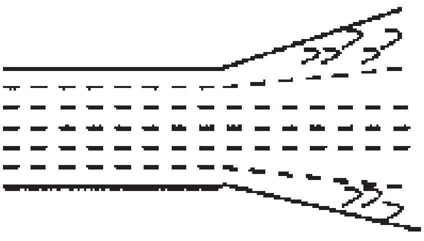


Figure 4.7 Diffusing flow.

boundary layer separation on the passage walls. It should also be noted that any change from the design mass flow and pressure ratio would also result in a loss of efficiency. The use of variable-angle diffuser vanes can control the efficiency loss. The flow theory of diffusion, covered in Chapter 2, is applicable here.

4.8 COMPRESSIBILITY EFFECTS

If the relative velocity of a compressible fluid reaches the speed of sound in the fluid, separation of flow causes excessive pressure losses. As mentioned earlier, diffusion is a very difficult process and there is always a tendency for the flow to break away from the surface, leading to eddy formation and reduced pressure rise. It is necessary to control the Mach number at certain points in the flow to mitigate this problem. The value of the Mach number cannot exceed the value at which shock waves occur. The relative Mach number at the impeller inlet must be less than unity.

As shown in Fig. 4.8a, the air breakaway from the convex face of the curved part of the impeller, and hence the Mach number at this point, will be very important and a shock wave might occur. Now, consider the inlet velocity triangle again (Fig. 4.5b). The relative Mach number at the inlet will be given by:

$$\mathrm{M}_1 = \frac{V_1}{\sqrt{\gamma \mathrm{R} T_1}} \tag{4.6}$$

where T_1 is the static temperature at the inlet.

It is possible to reduce the Mach number by introducing the prewhirl. The prewhirl is given by a set of fixed intake guide vanes preceding the impeller.

As shown in Fig. 4.8b, relative velocity is reduced as indicated by the dotted triangle. One obvious disadvantage of prewhirl is that the work capacity of

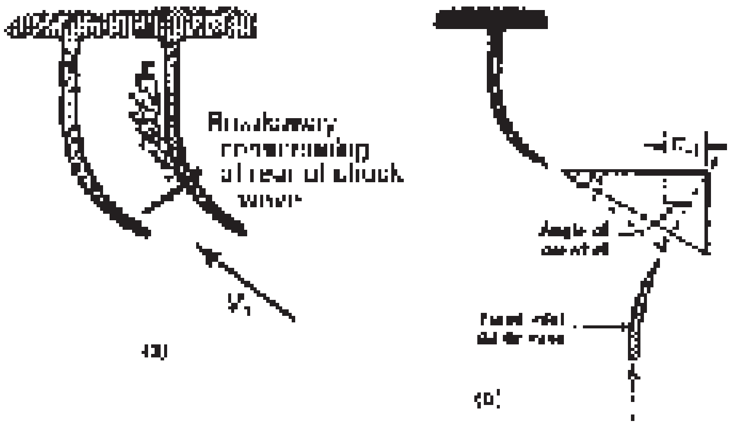


Figure 4.8 a) Breakaway commencing at the aft edge of the shock wave, and b) Compressibility effects.

the compressor is reduced by an amount U_1C_{w1}. It is not necessary to introduce prewhirl down to the hub because the fluid velocity is low in this region due to lower blade speed. The prewhirl is therefore gradually reduced to zero by twisting the inlet guide vanes.

4.9 MACH NUMBER IN THE DIFFUSER

The absolute velocity of the fluid becomes a maximum at the tip of the impeller and so the Mach number may well be in excess of unity. Assuming a perfect gas, the Mach number at the impeller exit M_2 can be written as:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} \tag{4.7}$$

However, it has been found that as long as the radial velocity component (C_{r2}) is subsonic, Mach number greater than unity can be used at the impeller tip without loss of efficiency. In addition, supersonic diffusion can occur without the formation of shock waves provided constant angular momentum is maintained with vortex motion in the vaneless space. High Mach numbers at the inlet to the diffuser vanes will also cause high pressure at the stagnation points on the diffuser vane tips, which leads to a variation of static pressure around the circumference of the diffuser. This pressure variation is transmitted upstream in a radial direction through the vaneless space and causes cyclic loading of the impeller. This may lead to early fatigue failure when the exciting frequency is of the same order as one of the natural frequencies of the impeller vanes. To overcome this concern, it is a common a practice to use prime numbers for the impeller vanes and an even number for the diffuser vanes.

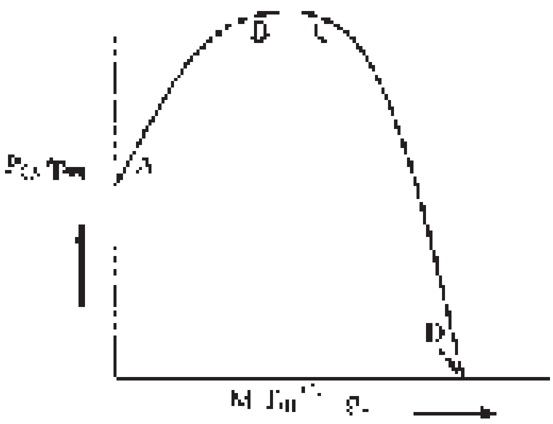


Figure 4.9 The theoretical centrifugal compressor characteristic.

4.10 CENTRIFUGAL COMPRESSOR CHARACTERISTICS

The performance of compressible flow machines is usually described in terms of the groups of variables derived in dimensional analysis (Chapter 1). These characteristics are dependent on other variables such as the conditions of pressure and temperature at the compressor inlet and physical properties of the working fluid. To study the performance of a compressor completely, it is necessary to plot P_{03}/P_{01} against the mass flow parameter $m\frac{\sqrt{T_{01}}}{P_{01}}$ for fixed speed intervals of $\frac{N}{\sqrt{T_{01}}}$. Figure 4.9 shows an idealized fixed speed characteristic. Consider a valve placed in the delivery line of a compressor running at constant speed. First, suppose that the valve is fully closed. Then the pressure ratio will have some value as indicated by Point A. This pressure ratio is available from vanes moving the air about in the impeller. Now, suppose that the valve is opened and airflow begins. The diffuser contributes to the pressure rise, the pressure ratio increases, and at Point B, the maximum pressure occurs. But the compressor efficiency at this pressure ratio will be below the maximum efficiency. Point C indicates the further increase in mass flow, but the pressure has dropped slightly from the maximum possible value. This is the design mass flow rate pressure ratio. Further increases in mass flow will increase the slope of the curve until point D. Point D indicates that the pressure rise is zero. However, the above-described curve is not possible to obtain.

4.11 STALL

Stalling of a stage will be defined as the aerodynamic stall, or the breakaway of the flow from the suction side of the blade airfoil. A multistage compressor may operate stably in the unsurged region with one or more of the stages stalled, and the rest of the stages unstalled. Stall, in general, is characterized by reverse flow near the blade tip, which disrupts the velocity distribution and hence adversely affects the performance of the succeeding stages.

Referring to the cascade of Fig. 4.10, it is supposed that some nonuniformity in the approaching flow or in a blade profile causes blade B to stall. The air now flows onto blade A at an increased angle of incidence due to blockage of channel AB. The blade A then stalls, but the flow on blade C is now at a lower incidence, and blade C may unstall. Therefore the stall may pass along the cascade in the direction of lift on the blades. Rotating stall may lead to vibrations resulting in fatigue failure in other parts of the gas turbine.

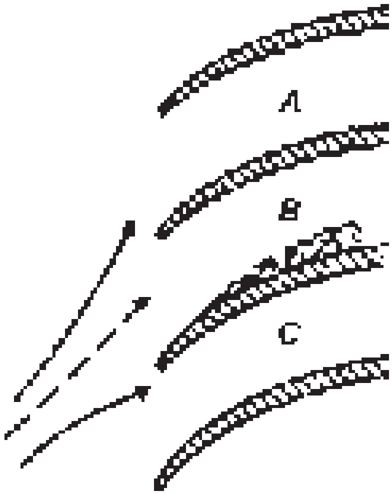


Figure 4.10 Mechanism of stall propagation.

4.12 SURGING

Surging is marked by a complete breakdown of the continuous steady flow throughout the whole compressor, resulting in large fluctuations of flow with time and also in subsequent mechanical damage to the compressor. The phenomenon of surging should not be confused with the stalling of a compressor stage.

Figure 4.11 shows typical overall pressure ratios and efficiencies η_c of a centrifugal compressor stage. The pressure ratio for a given speed, unlike the temperature ratio, is strongly dependent on mass flow rate, since the machine is usually at its peak value for a narrow range of mass flows. When the compressor is running at a particular speed and the discharge is gradually reduced, the pressure ratio will first increase, peaks at a maximum value, and then decreased. The pressure ratio is maximized when the isentropic efficiency has the maximum value. When the discharge is further reduced, the pressure ratio drops due to fall in the isentropic efficiency. If the downstream pressure does not drop quickly there will be backflow accompanied by further decrease in mass flow. In the mean time, if the downstream pressure drops below the compressor outlet pressure, there will be increase in mass flow. This phenomenon of sudden drop in delivery pressure accompanied by pulsating flow is called surging. The point on the curve where surging starts is called the surge point. When the discharge pipe of the compressor is completely choked (mass flow is zero) the pressure ratio will have some value due to the centrifugal head produced by the impeller.

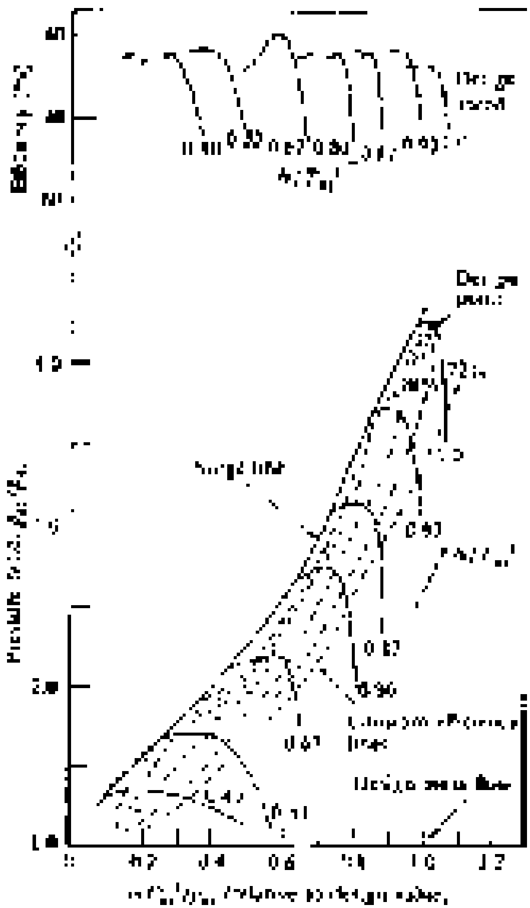


Figure 4.11 Centrifugal compressor characteristics.

Between the zero mass flow and the surge point mass flow, the operation of the compressor will be unstable. The line joining the surge points at different speeds gives the surge line.

4.13 CHOKING

When the velocity of fluid in a passage reaches the speed of sound at any cross-section, the flow becomes choked (air ceases to flow). In the case of inlet flow passages, mass flow is constant. The choking behavior of rotating passages differs from that of the stationary passages, and therefore it is necessary to make separate analysis for impeller and diffuser, assuming one dimensional, adiabatic flow, and that the fluid is a perfect gas.

4.13.1 Inlet

When the flow is choked, $C^2 = a^2 = \gamma RT$. Since $h_0 = h + \frac{1}{2}C^2$, then $C_p T_0 = C_p T + \frac{1}{2}\gamma\,RT$, and

$$\frac{T}{T_0} = \left(1 + \frac{\gamma R}{2C_p}\right)^{-1} = \frac{2}{\gamma + 1} \tag{4.8}$$

Assuming isentropic flow, we have:

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right) = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{(1-\gamma)/(\gamma-1)} \tag{4.9}$$

and when $C = a$, $M = 1$, so that:

$$\left(\frac{\rho}{\rho_0}\right) = \left[\frac{2}{(\gamma + 1)}\right]^{1/(\gamma-1)} \tag{4.10}$$

Using the continuity equation, $\left(\frac{\dot{m}}{A}\right) = \rho C = \rho\left[\gamma RT\right]^{1/2}$, we have

$$\left(\frac{\dot{m}}{A}\right) = \rho_0 a_0 \left[\frac{2}{\gamma + 1}\right]^{(\gamma+1)/2(\gamma-1)} \tag{4.11}$$

where (ρ_0 and a_0 refer to inlet stagnation conditions, which remain unchanged. The mass flow rate at choking is constant.

4.13.2 Impeller

When choking occurs in the impeller passages, the relative velocity equals the speed of sound at any section. The relative velocity is given by:

$$V^2 = a^2 = \left[\gamma RT\right] \text{ and } T_{01} = T + \left(\frac{\gamma RT}{2C_p}\right) - \frac{U^2}{2C_p}$$

Therefore,

$$\left(\frac{T}{T_{01}}\right) = \left(\frac{2}{\gamma + 1}\right)\left(1 + \frac{U^2}{2C_p T_{01}}\right) \tag{4.12}$$

Using isentropic conditions,

$$\left(\frac{\rho}{\rho_{01}}\right) = \left[\frac{T}{T_{01}}\right]^{1/(\gamma-1)} \text{ and, from the continuity equation:}$$

$$\begin{aligned}\left(\frac{\dot{m}}{\mathrm{A}}\right) &= \rho_0 a_{01}\left[\frac{T}{T_{01}}\right]^{(\gamma+1)/2(\gamma-1)} \\ &= \rho_{01} a_{01}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{U^2}{2C_\mathrm{p}T_{01}}\right)\right]^{(\gamma+1)/2(\gamma-1)} \\ &= \rho_{01} a_{01}\left(\frac{2+(\gamma-1)U^2/a_{01}^2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}\end{aligned} \tag{4.13}$$

Equation (4.13) indicates that for rotating passages, mass flow is dependent on the blade speed.

4.13.3 Diffuser

For choking in the diffuser, we use the stagnation conditions for the diffuser and not the inlet. Thus:

$$\left(\frac{\dot{m}}{\mathrm{A}}\right) = \rho_{02} a_{02}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \tag{4.14}$$

It is clear that stagnation conditions at the diffuser inlet are dependent on the impeller process.

Illustrative Example 4.1: Air leaving the impeller with radial velocity 110 m/s makes an angle of 25°30′ with the axial direction. The impeller tip speed is 475 m/s. The compressor efficiency is 0.80 and the mechanical efficiency is 0.96. Find the slip factor, overall pressure ratio, and power required to drive the compressor. Neglect power input factor and assume $\gamma = 1.4$, $T_{01} = 298$ K, and the mass flow rate is 3 kg/s.

Solution:

From the velocity triangle (Fig. 4.12),

$$\tan(\beta_2) = \frac{U_2 - C_{\mathrm{w}2}}{C_{\mathrm{r}2}}$$

$$\tan(25.5°) = \frac{475 - C_{\mathrm{w}2}}{110}$$

Therefore, $C_{\mathrm{w}2} = 422.54$ m/s.

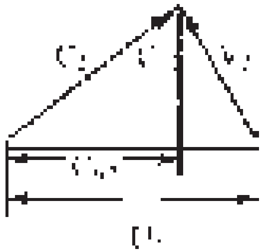


Figure 4.12 Velocity triangle at the impeller tip.

Now, $\sigma = \frac{C_{w2}}{U_2} = \frac{422.54}{475} = 0.89$

The overall pressure ratio of the compressor:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)} = \left[1 + \frac{(0.80)(0.89)(475^2)}{(1005)(298)}\right]^{3.5} = 4.5$$

The theoretical power required to drive the compressor:

$$P = \left[\frac{m\sigma\psi U_2^2}{1000}\right] \text{kW} = \left[\frac{(3)(0.89)(475^2)}{1000}\right] = 602.42\,\text{kW}$$

Using mechanical efficiency, the actual power required to drive the compressor is: $P = 602.42/0.96 = 627.52$ kW.

Illustrative Example 4.2: The impeller tip speed of a centrifugal compressor is 370 m/s, slip factor is 0.90, and the radial velocity component at the exit is 35 m/s. If the flow area at the exit is 0.18 m^2 and compressor efficiency is 0.88, determine the mass flow rate of air and the absolute Mach number at the impeller tip. Assume air density = 1.57 kg/m^3 and inlet stagnation temperature is 290 K. Neglect the work input factor. Also, find the overall pressure ratio of the compressor.

Solution:

Slip factor: $\sigma = \frac{C_{w2}}{U_2}$

Therefore: $C_{w2} = U_2\sigma = (0.90)(370) = 333$ m/s

The absolute velocity at the impeller exit:

$$C_2 = \sqrt{C_{r2}^2 + C_{\omega 2}^2} = \sqrt{333^2 + 35^2} = 334.8\,\text{m/s}$$

The mass flow rate of air: $\dot{m} = \rho_2 A_2 C_{r2} = 1.57*0.18*35 = 9.89$ kg/s

The temperature equivalent of work done (neglecting ψ):

$$T_{02} - T_{01} = \frac{\sigma U_2^2}{C_p}$$

Therefore, $T_{02} = T_{01} + \frac{\sigma U_2^2}{C_p} = 290 + \frac{(0.90)(370^2)}{1005} = 412.6\,\text{K}$

The static temperature at the impeller exit,

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 412.6 - \frac{334.8^2}{(2)(1005)} = 356.83\,\text{K}$$

The Mach number at the impeller tip:

$$\text{M}_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{334.8}{\sqrt{(1.4)(287)(356.83)}} = 0.884$$

The overall pressure ratio of the compressor (neglecting ψ):

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{3.5} = \left[1 + \frac{(0.88)(0.9)(370^2)}{(1005)(290)}\right]^{3.5} = 3.0$$

Illustrative Example 4.3: A centrifugal compressor is running at 16,000 rpm. The stagnation pressure ratio between the impeller inlet and outlet is 4.2. Air enters the compressor at stagnation temperature of 20°C and 1 bar. If the impeller has radial blades at the exit such that the radial velocity at the exit is 136 m/s and the isentropic efficiency of the compressor is 0.82. Draw the velocity triangle at the exit (Fig. 4.13) of the impeller and calculate slip. Assume axial entrance and rotor diameter at the outlet is 58 cm.

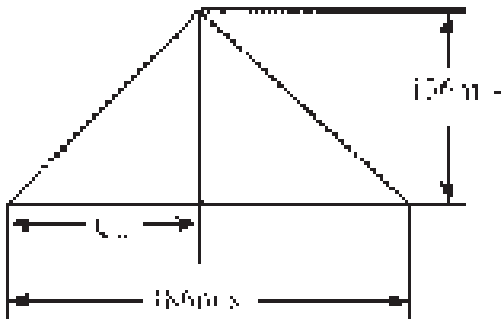


Figure 4.13 Velocity triangle at exit.

Solution:
Impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(0.58)(16000)}{60} = 486\,\text{m/s}$$

Assuming isentropic flow between impeller inlet and outlet, then

$$T_{02'} = T_{01}(4.2)^{0.286} = 441.69\,\text{K}$$

Using compressor efficiency, the actual temperature rise

$$T_{02} - T_{01} = \frac{\left(T_{02'} - T_{01}\right)}{\eta_c} = \frac{(441.69 - 293)}{0.82} = 181.33\text{K}$$

Since the flow at the inlet is axial, $C_{w1} = 0$

$$W = U_2 C_{w2} = C_p(T_{02} - T_{01}) = 1005(181.33)$$

Therefore: $C_{w2} = \dfrac{1005(181.33)}{486} = 375\,\text{m/s}$

Slip $= 486 - 375 = 111\,\text{m/s}$

Slip factor: $\sigma = \dfrac{C_{w2}}{U_2} = \dfrac{375}{486} = 0.772$

Illustrative Example 4.4: Determine the adiabatic efficiency, temperature of the air at the exit, and the power input of a centrifugal compressor from the following given data:

Impeller tip diameter = 1 m

Speed = 5945 rpm

Mass flow rate of air = 28 kg/s

Static pressure ratio $p_3/p_1 = 2.2$

Atmospheric pressure = 1 bar

Atmospheric temperature = 25°C

Slip factor = 0.90

Neglect the power input factor.

Solution:
The impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(1)(5945)}{60} = 311\,\text{m/s}$$

The work input: $W = \sigma U_2^2 = \dfrac{(0.9)(311^2)}{1000} = 87\,\text{kJ/kg}$

Using the isentropic P–T relation and denoting isentropic temperature by $T_{3'}$, we get:

$$T_{3'} = T_1 \left(\frac{P_3}{P_1}\right)^{0.286} = (298)(2.2)^{0.286} = 373.38\,\text{K}$$

Hence the isentropic temperature rise:

$$T_{3'} - T_1 = 373.38 - 298 = 75.38\,\text{K}$$

The temperature equivalent of work done:

$$T_3 - T_1 = \left(\frac{W}{C_\text{p}}\right) = 87/1.005 = 86.57\,\text{K}$$

The compressor adiabatic efficiency is given by:

$$\eta_\text{c} = \frac{(T_{3'} - T_1)}{(T_3 - T_1)} = \frac{75.38}{86.57} = 0.871 \text{ or } 87.1\%$$

The air temperature at the impeller exit is:

$$T_3 = T_1 + 86.57 = 384.57\,\text{K}$$

Power input:

$$P = \dot{m}W = (28)(87) = 2436\,\text{kW}$$

Illustrative Example 4.5: A centrifugal compressor impeller rotates at 9000 rpm. If the impeller tip diameter is 0.914 m and $\alpha_2 = 20°$, calculate the following for operation in standard sea level atmospheric conditions: (1) U_2, (2) C_{w2}, (3) C_{r2}, (4) β_2, and (5) C_2.

1. Impeller tip speed is given by $U_2 = \frac{\pi DN}{60} = \frac{(\pi)(0.914)(9000)}{60} = 431$ m/s
2. Since the exit is radial and no slip, $C_{w2} = U_2 = 431$ m/s
3. From the velocity triangle,
   $C_{r2} = U_2 \tan(\alpha_2) = (431)\ (0.364) = 156.87$ m/s
4. For radial exit, relative velocity is exactly perpendicular to rotational velocity U_2. Thus the angle β_2 is 90° for radial exit.
5. Using the velocity triangle (Fig. 4.14),

$$C_2 = \sqrt{U_2^2 + C_{r2}^2} = \sqrt{431^2 + 156.87^2} = 458.67\,\text{m/s}$$

Illustrative Example 4.6: A centrifugal compressor operates with no prewhirl is run with a rotor tip speed of 457 m/s. If C_{w2} is 95% of U_2 and $\eta_\text{c} = 0.88$,

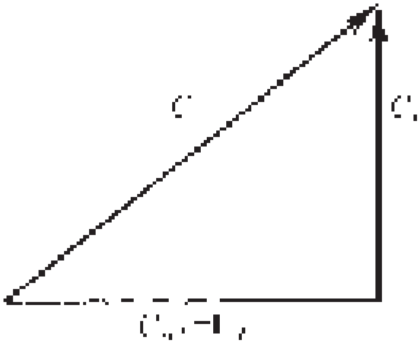


Figure 4.14 Velocity triangle at impeller exit.

calculate the following for operation in standard sea level air: (1) pressure ratio, (2) work input per kg of air, and (3) the power required for a flow of 29 k/s.

Solution:

1. The pressure ratio is given by (assuming $\sigma = \psi = 1$):

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{(0.88)(0.95)(457^2)}{(1005)(288)}\right]^{3.5} = 5.22$$

2. The work per kg of air

$$W = U_2 C_{w2} = (457)(0.95)(457) = 198.4\,\text{kJ/kg}$$

3. The power for 29 kg/s of air

$$P = \dot{m}W = (29)(198.4) = 5753.6\,\text{kW}$$

Illustrative Example 4.7: A centrifugal compressor is running at 10,000 rpm and air enters in the axial direction. The inlet stagnation temperature of air is 290 K and at the exit from the impeller tip the stagnation temperature is 440 K. The isentropic efficiency of the compressor is 0.85, work input factor $\psi = 1.04$, and the slip factor $\sigma = 0.88$. Calculate the impeller tip diameter, overall pressure ratio, and power required to drive the compressor per unit mass flow rate of air.

Solution:
Temperature equivalent of work done:

$$T_{02} - T_{01} = \frac{\sigma\psi U_2^2}{C_p} \text{ or } \frac{(0.88)(1.04)(U_2^2)}{1005}.$$

Therefore, $U_2 = 405.85\,\text{m/s}$

$$\text{and} \quad D = \frac{60U_2}{\pi N} = \frac{(60)(405.85)}{(\pi)(10,000)} = 0.775\,\text{m}$$

The overall pressure ratio is given by:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma\psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{(0.85)(0.88)(1.04)(405.85^2)}{(1005)(290)}\right]^{3.5} = 3.58$$

Power required to drive the compressor per unit mass flow:

$$P = m\psi\sigma U_2^2 = \frac{(1)(0.88)(1.04)(405.85^2)}{1000} = 150.75\,\text{kW}$$

Design Example 4.8: Air enters axially in a centrifugal compressor at a stagnation temperature of 20°C and is compressed from 1 to 4.5 bars. The impeller has 19 radial vanes and rotates at 17,000 rpm. Isentropic efficiency of the compressor is 0.84 and the work input factor is 1.04. Determine the overall diameter of the impeller and the power required to drive the compressor when the mass flow is 2.5 kg/s.

Solution:
Since the vanes are radial, using the Stanitz formula to find the slip factor:

$$\sigma = 1 - \frac{0.63\pi}{n} = 1 - \frac{0.63\pi}{19} = 0.8958$$

The overall pressure ratio

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma\psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}, \text{ or } 4.5$$

$$= \left[1 + \frac{(0.84)(0.8958)(1.04)(U_2^2)}{(1005)(293)}\right]^{3.5}, \text{ so } U_2 = 449.9\,\text{m/s}$$

The impeller diameter, $D = \frac{60U_2}{\pi N} = \frac{(60)(449.9)}{\pi(17,000)} = 0.5053\,\text{m} = 50.53\,\text{cm}.$

The work done on the air $W = \dfrac{\psi \sigma U_2^2}{1000} = \dfrac{(0.8958)(1.04)(449.9^2)}{1000} =$ 188.57 kJ/kg

Power required to drive the compressor: $P = \dot{m}W = (2.5)(188.57) =$ 471.43 kW

Design Example 4.9: Repeat problem 4.8, assuming the air density at the impeller tip is 1.8 kg/m^3 and the axial width at the entrance to the diffuser is 12 mm. Determine the radial velocity at the impeller exit and the absolute Mach number at the impeller tip.

Solution:

Slip factor: $\sigma = \dfrac{C_{w2}}{U_2}$, or $C_{w2} = (0.8958)(449.9) = 403$ m/s

Using the continuity equation,

$$\dot{m} = \rho_2 A_2 C_{r2} = \rho_2 2\pi r_2 b_2 C_{r2}$$

where:

$$b_2 = \text{axial width}$$
$$r_2 = \text{radius}$$

Therefore:

$$C_{r2} = \frac{2.5}{(1.8)(2\pi)(0.25)(0.012)} = 73.65 \text{ m/s}$$

Absolute velocity at the impeller exit

$$C_2 = \sqrt{C_{r2}^2 + C_{w2}^2} = \sqrt{73.65^2 + 403^2} = 409.67 \text{ m/s}$$

The temperature equivalent of work done:

$$T_{02} - T_{01} = 188.57/C_p = 188.57/1.005 = 187.63 \text{ K}$$

Therefore, $T_{02} = 293 + 187.63 = 480.63$ K

Hence the static temperature at the impeller exit is:

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 480.63 - \frac{409.67^2}{(2)(1005)} = 397 \text{ K}$$

Now, the Mach number at the impeller exit is:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{409.67}{\sqrt{(1.4)(287)(397)}} = 1.03$$

Design Example 4.10: A centrifugal compressor is required to deliver 8 kg/s of air with a stagnation pressure ratio of 4 rotating at 15,000 rpm. The air enters the compressor at 25°C and 1 bar. Assume that the air enters axially with velocity of 145 m/s and the slip factor is 0.89. If the compressor isentropic efficiency is 0.89, find the rise in stagnation temperature, impeller tip speed, diameter, work input, and area at the impeller eye.

Solution:

Inlet stagnation temperature:

$$T_{01} = T_a + \frac{C_1^2}{2C_p} = 298 + \frac{145^2}{(2)(1005)} = 308.46\,\mathrm{K}$$

Using the isentropic P–T relation for the compression process,

$$T_{03'} = T_{01}\left(\frac{P_{03}}{P_{01}}\right)^{(\gamma-1)/\gamma} = (308.46)(4)^{0.286} = 458.55K$$

Using the compressor efficiency,

$$T_{02} - T_{01} = \frac{\left(T_{02'} - T_{01}\right)}{\eta_c} = \frac{(458.55 - 308.46)}{0.89} = 168.64\,\mathrm{K}$$

Hence, work done on the air is given by:

$$W = C_p(T_{02} - T_{01}) = (1.005)(168.64) = 169.48\,\mathrm{kJ/kg}$$

But,

$$W = \sigma U_2^2 = \frac{(0.89)(U_2)}{1000}, \text{ or :}169.48 = 0.89U_2^2/1000$$

or:

$$U_2 = \sqrt{\frac{(1000)(169.48)}{0.89}} = 436.38\,\mathrm{m/s}$$

Hence, the impeller tip diameter

$$D = \frac{60U_2}{\pi N} = \frac{(60)(436.38)}{\pi(15,000)} = 0.555\,\mathrm{m}$$

The air density at the impeller eye is given by:

$$\rho_1 = \frac{\mathrm{P}_1}{\mathrm{R}T_1} = \frac{(1)(100)}{(0.287)(298)} = 1.17\,\mathrm{kg/m^3}$$

Using the continuity equation in order to find the area at the impeller eye,

$$A_1 = \frac{\dot{m}}{\rho_1 C_1} = \frac{8}{(1.17)(145)} = 0.047\,\mathrm{m^2}$$

The power input is:

$$P = \dot{m}\ W = (8)(169.48) = 1355.24\,\mathrm{kW}$$

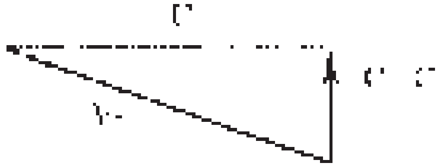


Figure 4.15 The velocity triangle at the impeller eye.

Design Example 4.11: The following data apply to a double-sided centrifugal compressor (Fig. 4.15):

Impeller eye tip diameter:	0.28 m
Impeller eye root diameter:	0.14 m
Impeller tip diameter:	0.48 m
Mass flow of air:	10 kg/s
Inlet stagnation temperature:	290 K
Inlet stagnation pressure:	1 bar
Air enters axially with velocity:	145 m/s
Slip factor:	0.89
Power input factor:	1.03
Rotational speed:	15, 000 rpm

Calculate (1) the impeller vane angles at the eye tip and eye root, (2) power input, and (3) the maximum Mach number at the eye.

Solution:

(1) Let U_{er} be the impeller speed at the eye root. Then the vane angle at the eye root is:

$$\alpha_{er} = \tan^{-1}\left(\frac{C_a}{U_{er}}\right)$$

and

$$U_{er} = \frac{\pi D_{er} N}{60} = \frac{\pi(0.14)(15,000)}{60} = 110\,\text{m/s}$$

Hence, the vane angle at the impeller eye root:

$$\alpha_{er} = \tan^{-1}\left(\frac{C_a}{U_{er}}\right) = \tan^{-1}\left(\frac{145}{110}\right) = 52^{\circ}48'$$

Impeller velocity at the eye tip:

$$U_{\mathrm{et}} = \frac{\pi D_{\mathrm{et}} N}{60} = \frac{\pi(0.28)(15,000)}{60} = 220\,\mathrm{m/s}$$

Therefore vane angle at the eye tip:

$$\alpha_{\mathrm{et}} = \tan^{-1}\left(\frac{C_{\mathrm{a}}}{U_{\mathrm{et}}}\right) = \tan^{-1}\left(\frac{145}{220}\right) = 33^{\circ}23'$$

(2) Work input:

$$W = \dot{m}\psi\sigma U_2^2 = (10)(0.819)(1.03U_2^2)$$

but:

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.48)(15,000)}{60} = 377.14\,\mathrm{m/s}$$

Hence,

$$W = \frac{(10)(0.89)(1.03)(377.14^2)}{1000} = 1303.86\,\mathrm{kW}$$

(3) The relative velocity at the eye tip:

$$V_1 = \sqrt{U_{\mathrm{et}}^2 + C_{\mathrm{a}}^2} = \sqrt{220^2 + 145^2} = 263.5\,\mathrm{m/s}$$

Hence, the maximum relative Mach number at the eye tip:

$$\mathrm{M}_1 = \frac{V_1}{\sqrt{\gamma \mathrm{R} T_1}},$$

where T_1 is the static temperature at the inlet

$$T_1 = T_{01} - \frac{C_1^2}{2C_{\mathrm{p}}} = 290 - \frac{145^2}{(2)(1005)} = 279.54\,\mathrm{K}$$

The Mach number at the inlet then is:

$$\mathrm{M}_1 = \frac{V_1}{\sqrt{\gamma \mathrm{R} T_1}} = \frac{263/5}{\sqrt{(1.4)(287)(279.54)}} = 0.786$$

Design Example 4.12: Recalculate the maximum Mach number at the impeller eye for the same data as in the previous question, assuming prewhirl angle of 20°.

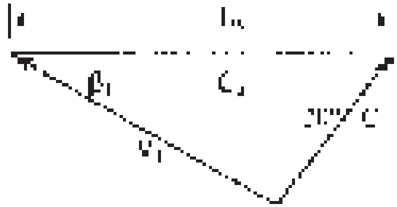


Figure 4.16 The velocity triangle at the impeller eye.

Solution:
Figure 4.16 shows the velocity triangle with the prewhirl angle.
From the velocity triangle:

$$C_1 = \frac{145}{\cos(20^\circ)} = 154.305\,\text{m/s}$$

Equivalent dynamic temperature:

$$\frac{C_1^2}{2C_p} = \frac{154.305^2}{(2)(1005)} = 11.846\,\text{K}$$

$$C_{w1} = \tan(20^\circ)\,C_{a1} = (0.36)(145) = 52.78\,\text{m/s}$$

Relative velocity at the inlet:

$$V_1^2 = C_a^2 + (U_e - C_{w1})^2 = 145^2 + (220 - 52.78)^2, \text{ or } V_1 = 221.3\,\text{m/s}$$

Therefore the static temperature at the inlet:

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - 11.846 = 278.2\,\text{K}$$

Hence,

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{221.3}{\sqrt{(1.4)(287)(278.2)}} = 0.662$$

Note the reduction in Mach number due to prewhirl.

Design Example 4.13: The following data refers to a single-sided centrifugal compressor:

Ambient Temperature:	288 K
Ambient Pressure:	1 bar
Hub diameter:	0.125 m
Eye tip diameter:	0.25 m
Mass flow:	5.5 kg/s
Speed:	16, 500 rpm

Assume zero whirl at the inlet and no losses in the intake duct. Calculate the blade inlet angle at the root and tip and the Mach number at the eye tip.

Solution:

Let: r_h = hub radius
r_t = tip radius

The flow area of the impeller inlet annulus is:

$$A_1 = \pi\left(r_t^2 - r_h^2\right) = \pi\left(0.125^2 - 0.0625^2\right) = 0.038\,\text{m}^2$$

Axial velocity can be determined from the continuity equation but since the inlet density (ρ_1) is unknown a trial and error method must be followed. Assuming a density based on the inlet stagnation condition,

$$\rho_1 = \frac{P_{01}}{RT_{01}} = \frac{(1)(10^5)}{(287)(288)} = 1.21\,\text{kg/m}^3$$

Using the continuity equation,

$$C_a = \frac{\dot{m}}{\rho_1 A_1} = \frac{5.5}{(1.21)(0.038)} = 119.6\,\text{m/s}$$

Since the whirl component at the inlet is zero, the absolute velocity at the inlet is $C_1 = C_a$.

The temperature equivalent of the velocity is:

$$\frac{C_1^2}{2C_p} = \frac{119.6^2}{(2)(1005)} = 7.12\,\text{K}$$

Therefore:

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 288 - 7.12 = 280.9\,\text{K}$$

Using isentropic P–T relationship,

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)}, \text{ or } P_1 = 10^5\left(\frac{280.9}{288}\right)^{3.5} = 92\,\text{kPa}$$

and:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(92)(10^3)}{(287)(280.9)} = 1.14\,\text{kg/m}^3, \text{ and}$$

$$C_a = \frac{5.5}{(1.14)(0.038)} = 126.96\,\text{m/s}$$

Therefore:

$$\frac{C_1^2}{2C_p} = \frac{(126.96)^2}{2(1005)} = 8.02\,\text{K}$$

$$T_1 = 288 - 8.02 = 279.98°\,\text{K}$$

$$P_1 = 10^5\left(\frac{279.98}{288}\right)^{3.5} = 90.58\,\text{kPa}$$

$$\rho_1 = \frac{(90.58)(10^3)}{(287)(279.98)} = 1.13\,\text{kg/m}^3$$

Further iterations are not required and the value of $\rho_1 = 1.13\,\text{kg/m}^3$ may be taken as the inlet density and $C_a = C_1$ as the inlet velocity. At the eye tip:

$$U_{et} = \frac{2\pi r_{et} N}{60} = \frac{2\pi(0.125)(16,500)}{60} = 216\,\text{m/s}$$

The blade angle at the eye tip:

$$\beta_{et} = \tan^{-1}\left(\frac{U_{et}}{C_a}\right) = \tan^{-1}\left(\frac{216}{126.96}\right) = 59.56°$$

At the hub,

$$U_{eh} = \frac{2\pi(0.0625)(16,500)}{60} = 108\,\text{m/s}$$

The blade angle at the hub:

$$\beta_{eh} = \tan^{-1}\left(\frac{108}{126.96}\right) = 40.39°$$

The Mach number based on the relative velocity at the eye tip using the inlet velocity triangle is:

$$V_1 = \sqrt{C_a^2 + U_1^2} = \sqrt{126.96^2 + 216^2}, \text{ or } V_1 = 250.6\,\text{m/s}$$

The relative Mach number

$$M = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{250.6}{\sqrt{(1.4)(287)(279.98)}} = 0.747$$

Design Example 4.14: A centrifugal compressor compresses air at ambient temperature and pressure of 288 K and 1 bar respectively. The impeller tip speed is 364 m/s, the radial velocity at the exit from the impeller is 28 m/s, and the slip factor is 0.89. Calculate the Mach number of the flow at the impeller tip. If the impeller total-to-total efficiency is 0.88 and the flow area from the impeller is 0.085 m^2, calculate the mass flow rate of air. Assume an axial entrance at the impeller eye and radial blades.

Solution:
The absolute Mach number of the air at the impeller tip is:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}}$$

where T_2 is the static temperature at the impeller tip. Let us first calculate C_2 and T_2.
Slip factor:

$$\sigma = \frac{C_{w2}}{U_2}$$

Or:

$$C_{w2} = \sigma U_2 = (0.89)(364) = 323.96\,\text{m/s}$$

From the velocity triangle,

$$C_2^2 = C_{r2}^2 + C_{w2}^2 = 28^2 + 323.96^2 = (1.06)(10^5)\,\text{m}^2/\text{s}^2$$

With zero whirl at the inlet

$$\frac{W}{m} = \sigma U_2^2 = C_p(T_{02} - T_{01})$$

Hence,

$$T_{02} = T_{01} + \frac{\sigma U_2^2}{C_p} = 288 + \frac{(0.89)(364^2)}{1005} = 405.33\,\text{K}$$

Static Temperature

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 405.33 - \frac{106000}{(2)(1005)} = 352.6\,\text{K}$$

Therefore,

$$\text{M}_2 = \left(\frac{(1.06)(10^5)}{(1.4)(287)(352.6)}\right)^{\frac{1}{2}} = 0.865$$

Using the isentropic P–T relation:

$$\left(\frac{P_{02}}{P_{01}}\right) = \left[1 + \eta_c\left(\frac{T_{02}}{T_{01}} - 1\right)\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + 0.88\left(\frac{405.33}{288} - 1\right)\right]^{3.5} = 2.922$$

$$\left(\frac{P_2}{P_{02}}\right) = \left(\frac{T_2}{T_{02}}\right)^{3.5} = \left(\frac{352.6}{405.33}\right)^{3.5} = 0.614$$

Therefore,

$$P_2 = \left(\frac{P_2}{P_{02}}\right)\left(\frac{P_{02}}{P_{01}}\right)P_{01}$$

$$= (0.614)(2.922)(1)(100)$$

$$= 179.4\,\text{kPa}$$

$$\rho_2 = \frac{179.4(1000)}{287(352.6)} = 1.773\,\text{kg/m}^3$$

Mass flow:

$$\dot{m} = (1.773)(0.085)(28) = 4.22\,\text{kg/s}$$

Design Example 4.15: The impeller of a centrifugal compressor rotates at 15,500 rpm, inlet stagnation temperature of air is 290 K, and stagnation pressure at inlet is 101 kPa. The isentropic efficiency of impeller is 0.88, diameter of the impellar is 0.56 m, axial depth of the vaneless space is 38 mm, and width of the vaneless space is 43 mm. Assume ship factor as 0.9, power input factor 1.04, mass flow rate as 16 kg/s. Calculate

1. Stagnation conditions at the impeller outlet, assume no fore whirl at the inlet,
2. Assume axial velocity approximately equal to 105 m/s at the impeller outlet, calculate the Mach number and air angle at the impeller outlet,

3. The angle of the diffuser vane leading edges and the Mach number at this radius if the diffusion in the vaneless space is isentropic.

Solution:

1. Impeller tip speed

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.56 \times 15500}{60}$$

$$U_2 = 454.67 \text{ m/s}$$

Overall stagnation temperature rise

$$T_{03} - T_{01} = \frac{\psi \sigma U_2^2}{1005} = \frac{1.04 \times 0.9 \times 454.67^2}{1005}$$
$$= 192.53\text{K}$$

Since $T_{03} = T_{02}$

Therefore, $T_{02} - T_{01} = 192.53\text{K}$ and $T_{02} = 192.53 + 290 = 482.53\text{K}$

Now pressure ratio for impeller

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{3.5} = \left(\frac{482.53}{290}\right)^{3.5} = 5.94$$

then, $p_{02} = 5.94 \times 101 = 600$ KPa

2.

$$\sigma = \frac{C_{w2}}{U_2}$$

$$C_{w2} = \sigma U_2$$

or

$$C_{w2} = 0.9 \times 454.67 = 409 \text{ m/s}$$

Let $C_{r2} = 105$ m/s

Outlet area normal to periphery

$$A_2 = \pi D_2 \times \text{impeller depth}$$
$$= \pi \times 0.56 \times 0.038$$
$$A_2 = 0.0669 \text{ m}^2$$

From outlet velocity triangle

$$C_2{}^2 = C_{r2}^2 + C_{w2}^2$$
$$= 105^2 + 409^2$$
$$C_2{}^2 = 178306$$

i.e. $C_2 = 422.26\,\text{m/s}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 482.53 - \frac{422.26^2}{2 \times 1005}$$

$$T_2 = 393.82\,\text{K}$$

Using isentropic P–T relations

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 600\left(\frac{393.82}{482.53}\right)^{3.5} = 294.69\,\text{kPa}$$

From equation of state

$$\rho_2 = \frac{P_2}{\text{R}T_2} = \frac{293.69 \times 10^3}{287 \times 393.82} = 2.61\,\text{kg/m}^3$$

The equation of continuity gives

$$C_{r2} = \frac{\dot{m}}{A_2 P_2} = \frac{16}{0.0669 \times 2.61} = 91.63\,\text{m/s}$$

Thus, impeller outlet radial velocity $= 91.63\,\text{m/s}$
Impeller outlet Mach number

$$\text{M}_2 = \frac{C_2}{\sqrt{\gamma \text{R} T_2}} = \frac{422.26}{(1.4 \times 287 \times 393.82)^{0.5}}$$

$$\text{M}_2 = 1.06$$

From outlet velocity triangle

$$\text{Cos}\alpha_2 = \frac{C_{r2}}{C_2} = \frac{91.63}{422.26} = 0.217$$

i.e., $\alpha_2 = 77.47°$

3. Assuming free vortex flow in the vaneless space and for convenience denoting conditions at the diffuser vane without a subscript ($r = 0.28 + 0.043 = 0.323$)

$$C_w = \frac{C_{w2} r_2}{r} = \frac{409 \times 0.28}{0.323}$$

$$C_w = 354.55\,\text{m/s}$$

The radial component of velocity can be found by trial and error. Choose as a first try, $C_r = 105$ m/s

$$\frac{C^2}{2C_p} = \frac{105^2 + 354.55^2}{2 \times 1005} = 68\,\text{K}$$

$T = 482.53 - 68$ (since $T = T_{02}$ in vaneless space)

$$T = 414.53\text{K}$$

$$p = p_{02}\left(\frac{T_2}{T_{02}}\right)^{3.5} = 600\left(\frac{419.53}{482.53}\right)^{3.5} = 352.58\,\text{kPa}$$

$$\rho = \frac{p_2}{RT_2} = \frac{294.69}{287 \times 393.82}$$

$$\rho = 2.61\,\text{kg/m}^3$$

The equation of continuity gives

$$\begin{aligned} A &= 2\pi r \times \text{depth of vanes} \\ &= 2\pi \times 0.323 \times 0.038 \\ &= 0.0772\,\text{m}^2 \end{aligned}$$

$$C_r = \frac{16}{2.61 \times 0.0772} = 79.41\ \text{m/s}$$

Next try $C_r = 79.41$ m/s

$$\frac{C^2}{2C_p} = \frac{79.41^2 + 354.55^2}{2 \times 1005} = 65.68$$

$$T = 482.53 - 65.68 = 416.85\text{K}$$

$$p = p_{02}\left(\frac{T}{T_{02}}\right)^{3.5} = 600\left(\frac{416.85}{482.53}\right)^{3.5}$$

$$p = 359.54\,\text{Pa}$$

$$\rho = \frac{359.54}{416.85 \times 287} = 3\,\text{kg/m}^3$$

$$C_r = \frac{16}{3.0 \times 0.772} = 69.08\ \text{m/s}$$

Try $C_r = 69.08$ m/s

$$\frac{C^2}{2C_p} = \frac{69.08^2 + 354.55^2}{2 \times 1005} = 64.9$$

$$T = 482.53 - 64.9 = 417.63\ \text{K}$$

$$p = p_{02}\left(\frac{T}{T_{02}}\right)^{3.5} = 600\left(\frac{417.63}{482.53}\right)^{3.5}$$

$$p = 361.9\ \text{Pa}$$

$$\rho = \frac{361.9}{417.63 \times 287} = 3.02\ \text{kg/m}^3$$

$$C_r = \frac{16}{3.02 \times 0.772} = 68.63\ \text{m/s}$$

Taking C_r as 62.63 m/s, the vane angle

$$\tan\alpha = \frac{C_w}{C_r}$$

$$= \frac{354.5}{68.63} = 5.17$$

i.e. $\alpha = 79°$

Mach number at vane

$$\text{M} = \left(\frac{65.68 \times 2 \times 1005}{1.4 \times 287 \times 417.63}\right)^{1/2} = 0.787$$

Design Example 4.16: The following design data apply to a double-sided centrifugal compressor:

Impeller eye root diameter:	18 cm
Impeller eye tip diameter:	31.75 cm
Mass flow:	18.5 kg/s
Impeller speed:	15500 rpm
Inlet stagnation pressure:	1.0 bar
Inlet stagnation temperature:	288 K
Axial velocity at inlet (constant):	150 m/s

Find suitable values for the impeller vane angles at root and tip of eye if the air is given 20° of prewhirl at all radii, and also find the maximum Mach number at the eye.

Solution:

At eye root, $C_a = 150$ m/s

$$\therefore C_1 = \frac{C_a}{\cos 20°} = \frac{150}{\cos 20°} = 159.63 \text{ m/s}$$

and $C_{w1} = 150 \tan 20° = 54.6$ m/s

Impeller speed at eye root

$$U_{er} = \frac{\pi D_{er} N}{60} = \frac{\pi \times 0.18 \times 15500}{60}$$

$$U_{er} = 146 \text{ m/s}$$

From velocity triangle

$$\tan \beta_{er} = \frac{C_a}{U_{er} - C_{w1}} = \frac{150}{146 - 54.6} = \frac{150}{91.4} = 1.641$$

$$\text{i.e., } \beta_{er} = 58.64°$$

At eye tip from Fig. 4.17(b)

$$U_{et} \frac{\pi D_{et} N}{60} = \frac{\pi \times 0.3175 \times 15500}{60}$$

$$U_{et} = 258 \text{ m/s}$$

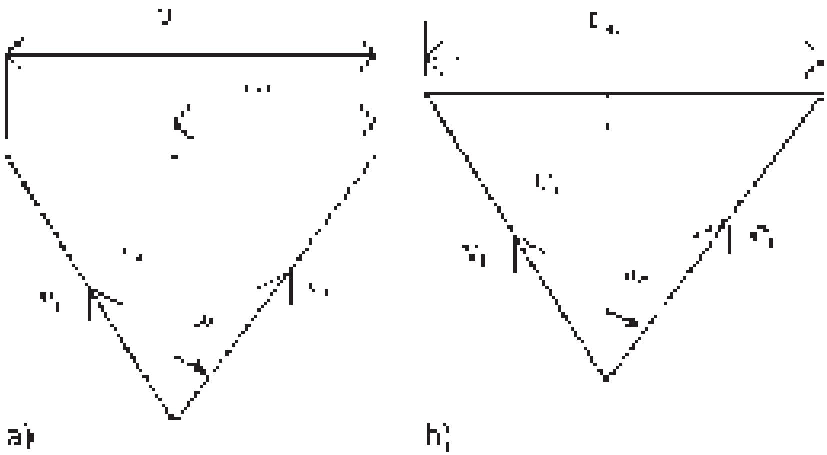


Figure 4.17 Velocity triangles at (a) eye root and (b) eye tip.

$$\tan\ \alpha_{et} = \frac{150}{258 - 54.6} = \frac{150}{203.4} = 0.7375$$

i.e. $\alpha_{et} = 36.41°$

Mach number will be maximum at the point where relative velocity is maximum.
Relative velocity at eye root is:

$$V_{er} = \frac{C_a}{\sin \beta_{er}} = \frac{150}{\sin 58.64°} = \frac{150}{0.8539}$$

$$V_{er} = 175.66\ \text{m/s}$$

Relative velocity at eye tip is:

$$V_{et} = \frac{C_a}{\sin \alpha_{et}} = \frac{150}{\sin 36.41°} = \frac{150}{0.5936}$$

$$V_{et} = 252.7\ \text{m/s}$$

Relative velocity at the tip is maximum.
Static temperature at inlet:

$$T_1 = T_{01} = \frac{V_{et}^2}{2C_p} = 288 - \frac{252.7^2}{2 \times 1005} = 288 - 31.77$$

$$T_1 = 256.23\ \text{K}$$

$$\text{M}_{max} = \frac{V_{et}}{(\gamma R T_1)^{1/2}} = \frac{252.7}{(1.4 \times 287 \times 256.23)^{1/2}} = \frac{252.7}{320.86}$$

$$\text{M}_{max} = 0.788$$

Design Example 4.17: In a centrifugal compressor air enters at a stagnation temperature of 288 K and stagnation pressure of 1.01 bar. The impeller has 17 radial vanes and no inlet guide vanes. The following data apply:

Mass flow rate:	2.5 kg/s
Impeller tip speed:	475 m/s
Mechanical efficiency:	96%
Absolute air velocity at diffuser exit:	90 m/s
Compressor isentropic efficiency:	84%
Absolute velocity at impeller inlet:	150 m/s

Diffuser efficiency:	82%
Axial depth of impeller:	6.5 mm
Power input factor:	1.04
γ for air:	1.4

Determine:

1. shaft power
2. stagnation and static pressure at diffuser outlet
3. radial velocity, absolute Mach number and stagnation and static pressures at the impeller exit, assume reaction ratio as 0.5, and
4. impeller efficiency and rotational speed

Solution:

1. Mechanical efficiency is

$$\eta_m = \frac{\text{Work transferred to air}}{\text{Work supplied to shaft}}$$

$$\text{or shaft power} = \frac{W}{\eta_m}$$

for vaned impeller, slip factor, by Stanitz formula is

$$\sigma = 1 - \frac{0.63\pi}{n} = 1 - \frac{0.63 \times \pi}{17}$$

$$\sigma = 0.884$$

Work input per unit mass flow

$$W = \psi \sigma U_2 C_{w2}$$

Since $C_{w1} = 0$

$$= \psi \sigma U_2^2$$

$$= 1.04 \times 0.884 \times 475^2$$

Work input for 2.5 kg/s

$$W = 1.04 \times 0.884 \times 2.5 \times 475^2$$

$$W = 518.58\text{K}$$

$$\text{Hence, Shaft Power} = \frac{518.58}{0.96} = 540.19\text{kW}$$

2. The overall pressure ratio is

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_c \psi \sigma U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{0.84 \times 1.04 \times 0.884 \times 475^2}{1005 \times 288}\right]^{3.5} = 5.2$$

Stagnation pressure at diffuser exit

$$P_{03} = p_{01} \times 5.20 = 1.01 \times 5.20$$

$$P_{03} = 5.25\,\text{bar}$$

$$\frac{p_3}{p_{03}} = \left(\frac{T_3}{T_{03}}\right)^{\gamma/\gamma-1}$$

$$W = m \times C_p(T_{03} - T_{01})$$

$$\therefore T_{03} = \frac{W}{mC_p} + T_{01} = \frac{518.58 \times 10^3}{2.5 \times 1005} + 288 = 494.4\,\text{K}$$

Static temperature at diffuser exit

$$T_3 = T_{03} - \frac{C_3^2}{2C_p} = 494.4 - \frac{90^2}{2 \times 1005}$$

$$T_3 = 490.37\,\text{K}$$

Static pressure at diffuser exit

$$p_3 = p_{03}\left(\frac{T_3}{T_{03}}\right)^{\gamma/\gamma-1} = 5.25\left(\frac{490.37}{494.4}\right)^{3.5}$$

$$p_3 = 5.10\,\text{bar}$$

3. The reaction is

$$0.5 = \frac{T_2 - T_1}{T_3 - T_1}$$

and

$$T_3 - T_1 = (T_{03} - T_{01}) + \left(\frac{C_1{}^2 - C_3{}^2}{2C_p}\right) = \frac{W}{mC_p} + \frac{150^2 - 90^2}{2 \times 1005}$$

$$= \frac{518.58 \times 10^3}{2.5 \times 1005} + 7.164 = 213.56\,K$$

Substituting

$$T_2 - T_1 = 0.5 \times 213.56$$
$$= 106.78\,\mathrm{K}$$

Now

$$T_2 = T_{01} - \frac{C_1^2}{2C_p} + (T_2 - T_1)$$
$$= 288 - 11.19 + 106.78$$
$$T_2 = 383.59\,\mathrm{K}$$

At the impeller exit

$$T_{02} = T_2 + \frac{C_2{}^2}{2C_p}$$

or

$$T_{03} = T_2 + \frac{C_2{}^2}{2C_p}\,(\text{Since } T_{02} = T_{03})$$

Therefore,

$$C_2{}^2 = 2C_p[(T_{03} - T_{01}) + (T_{01} - T_2)]$$
$$= 2 \times 1005(206.4 + 288 + 383.59)$$
$$C_2 = 471.94\,\mathrm{m/s}$$

Mach number at impeller outlet

$$\mathrm{M}_2 = \frac{C_2}{(1.4 \times 287 \times 383.59)^{1/2}}$$

$$\mathrm{M}_2 = 1.20$$

Radial velocity at impeller outlet

$$C_{r2}^2 = C_2{}^2 - C_{w2}^2$$
$$= (471.94)^2 - (0.884 \times 475)^2$$

$$C_{r2}^{\;2} = 215.43 \text{ m/s}$$

Diffuser efficiency is given by

$$\eta_D = \frac{h_{3'} - h_2}{h_3 - h_2} = \frac{\text{isentropic enthalpy increase}}{\text{actual enthalpy increase}} = \frac{T_{3'} - T_2}{T_3 - T_2}$$

$$= \frac{T_2\left(\frac{T_{3'}}{T_2} - 1\right)}{T_3 - T_2} = \frac{\left[T_2\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma} - 1\right]}{(T_3 - T_2)}$$

Therefore

$$\frac{p_3}{p_2} = \left[1 + \eta_D\left(\frac{T_3 - T_2}{T_2}\right)\right]^{3.5}$$

$$= \left(1 + \frac{0.821 \times 106.72}{383.59}\right)^{3.5}$$

$$= 2.05$$

$$\text{or } p_2 = \frac{5.10}{2.05} = 2.49 \text{ bar}$$

From isentropic P–T relations

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{3.5} = 2.49\left(\frac{494.4}{383.59}\right)^{3.5}$$

$$p_{02} = 6.05 \text{ bar}$$

4. Impeller efficiency is

$$\eta_i = \frac{T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{03} - T_{01}}$$

$$= \frac{288\left[\left(\frac{6.05}{1.01}\right)^{0.286} - 1\right]}{494.4 - 288}$$

$$= 0.938$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.49 \times 10^5}{287 \times 383.59}$$

$$\rho_2 = 2.27 \text{ kg/m}^3$$

$$\dot{m} = \rho_2 A_2 C_{r2}$$
$$= 2\pi r_2 \rho_2 b_2$$

But

$$U_2 = \frac{\pi N D_2}{60} = \frac{\pi N \dot{m}}{\rho_2 \pi C_{r2} b_2 \times 60}$$

$$N = \frac{475 \times 2.27 \times 246.58 \times 0.0065 \times 60}{2.5}$$

$$N = 41476 \text{ rpm}$$

PROBLEMS

4.1 The impeller tip speed of a centrifugal compressor is 450 m/s with no prewhirl. If the slip factor is 0.90 and the isentropic efficiency of the compressor is 0.86, calculate the pressure ratio, the work input per kg of air, and the power required for 25 kg/s of airflow. Assume that the compressor is operating at standard sea level and a power input factor of 1.

(4.5, 182.25 kJ/kg, 4556.3 kW)

4.2 Air with negligible velocity enters the impeller eye of a centrifugal compressor at 15°C and 1 bar. The impeller tip diameter is 0.45 m and rotates at 18,000 rpm. Find the pressure and temperature of the air at the compressor outlet. Neglect losses and assume $\gamma = 1.4$.

(5.434 bar, 467 K)

4.3 A centrifugal compressor running at 15,000 rpm, overall diameter of the impeller is 60 cm, isentropic efficiency is 0.84 and the inlet stagnation temperature at the impeller eye is 15°C. Calculate the overall pressure ratio, and neglect losses.

(6)

4.4 A centrifugal compressor that runs at 20,000 rpm has 20 radial vanes, power input factor of 1.04, and inlet temperature of air is 10°C. If the pressure ratio is 2 and the impeller tip diameter is 28 cm, calculate the isentropic efficiency of the compressor. Take $\gamma = 1.4$ (77.4%)

4.5 Derive the expression for the pressure ratio of a centrifugal compressor:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

4.6 Explain the terms "slip factor" and "power input factor."

4.7 What are the three main types of centrifugal compressor impellers? Draw the exit velocity diagrams for these three types.

4.8 Explain the phenomenon of stalling, surging and choking in centrifugal compressors.

4.9 A centrifugal compressor operates with no prewhirl and is run with a tip speed of 475 the slip factor is 0.89, the work input factor is 1.03, compressor efficiency is 0.88, calculate the pressure ratio, work input per kg of air and power for 29 airflow. Assume $T_{01} = 290$ K and $C_p = 1.005$ kJ/kg K.

(5.5, 232.4 kJ/kg, 6739 kW)

4.10 A centrifugal compressor impeller rotates at 17,000 rpm and compresses 32 kg of air per second. Assume an axial entrance, impeller trip radius is 0.3 m, relative velocity of air at the impeller tip is 105 m/s at an exit angle of 80°. Find the torque and power required to drive this machine.

(4954 Nm, 8821 kW)

4.11 A single-sided centrifugal compressor designed with no prewhirl has the following dimensions and data:

Total head/pressure ratio:	3.8:1
Speed:	12,000 rpm
Inlet stagnation temperature:	293 K
Inlet stagnation pressure:	1.03 bar
Slip factor:	0.9
Power input factor:	1.03
Isentropic efficiency:	0.76
Mass flow rate:	20 kg/s

Assume an axial entrance. Calculate the overall diameter of the impeller and the power required to drive the compressor.

(0.693 m, 3610 kW)

4.12 A double-entry centrifugal compressor designed with no prewhirl has the following dimensions and data:

Impeller root diameter:	0.15 m
Impeller tip diameter:	0.30 m
Rotational speed:	15,000 rpm
Mass flow rate:	18 kg/s

Ambient temperature: 25°C

Ambient pressure: 1.03 bar

Density of air at eye inlet: 1.19 kg/m^3

Assume the axial entrance and unit is stationary. Find the inlet angles of the vane at the root and tip radii of the impeller eye and the maximum Mach number at the eye.

(α_1 at root $= 50.7°$, $\alpha_1 = 31.4°$ at tip, 0.79)

4.13 In Example 4.12, air does not enter the impeller eye in an axial direction but it is given a prewhirl of 20° (from the axial direction). The remaining values are the same. Calculate the inlet angles of the impeller vane at the root and tip of the eye.

(α_1 at root $= 65.5°$, α_1 at tip $= 38.1°$, 0.697)

NOTATION

C	absolute velocity
r	radius
U	impeller speed
V	relative velocity
α	vane angle
σ	slip factor
ω	angular velocity
ψ	power input factor

SUFFIXES

1	inlet to rotor
2	outlet from the rotor
3	outlet from the diffuser
a	axial, ambient
r	radial
w	whirl

5

Axial Flow Compressors and Fans

5.1 INTRODUCTION

As mentioned in Chapter 4, the maximum pressure ratio achieved in centrifugal compressors is about 4:1 for simple machines (unless multi-staging is used) at an efficiency of about 70–80%. The axial flow compressor, however, can achieve higher pressures at a higher level of efficiency. There are two important characteristics of the axial flow compressor—high-pressure ratios at good efficiency and thrust per unit frontal area. Although in overall appearance, axial turbines are very similar, examination of the blade cross-section will indicate a big difference. In the turbine, inlet passage area is greater than the outlet. The opposite occurs in the compressor, as shown in Fig. 5.1.

Thus the process in turbine blades can be described as an accelerating flow, the increase in velocity being achieved by the nozzle. However, in the axial flow compressor, the flow is decelerating or diffusing and the pressure rise occurs when the fluid passes through the blades. As mentioned in the chapter on diffuser design (Chapter 4, Sec. 4.7), it is much more difficult to carry out efficient diffusion due to the breakaway of air molecules from the walls of the diverging passage. The air molecules that break away tend to reverse direction and flow back in the direction of the pressure gradient. If the divergence is too rapid, this may result in the formation of eddies and reduction in useful pressure rise. During acceleration in a nozzle, there is a natural tendency for the air to fill the passage

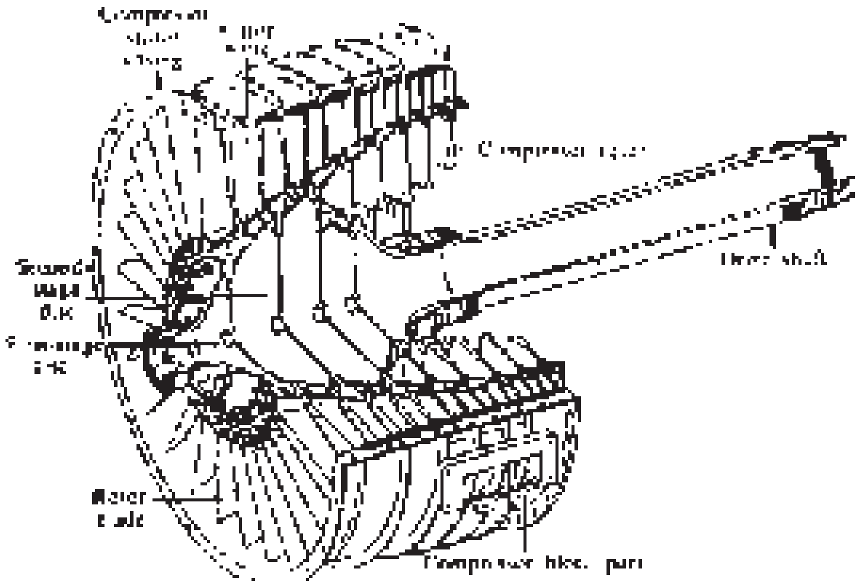


Figure 5.1 Cutaway sketch of a typical axial compressor assembly: the General Electric J85 compressor. (Courtesy of General Electric Co.)

walls closely (only the normal friction loss will be considered in this case). Typical blade sections are shown in Fig. 5.2. Modern axial flow compressors may give efficiencies of 86–90%—compressor design technology is a well-developed field. Axial flow compressors consist of a number of stages, each stage being formed by a stationary row and a rotating row of blades.

Figure 5.3 shows how a few compressor stages are built into the axial compressor. The rotating blades impart kinetic energy to the air while increasing air pressure and the stationary row of blades redirect the air in the proper direction and convert a part of the kinetic energy into pressure. The flow of air through the compressor is in the direction of the axis of the compressor and, therefore, it is called an axial flow compressor. The height of the blades is seen to decrease as the fluid moves through the compressor. As the pressure increases in the direction of flow, the volume of air decreases. To keep the air velocity the same for each stage, the blade height is decreased along the axis of the compressor. An extra row of fixed blades, called the inlet guide vanes, is fitted to the compressor inlet. These are provided to guide the air at the correct angle onto the first row of moving blades. In the analysis of the highly efficient axial flow compressor, the 2-D flow through the stage is very important due to cylindrical symmetry.

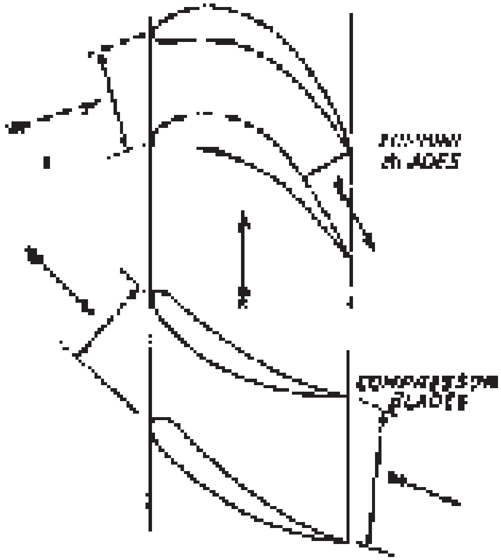


Figure 5.2 Compressor and turbine blade passages: turbine and compressor housing.

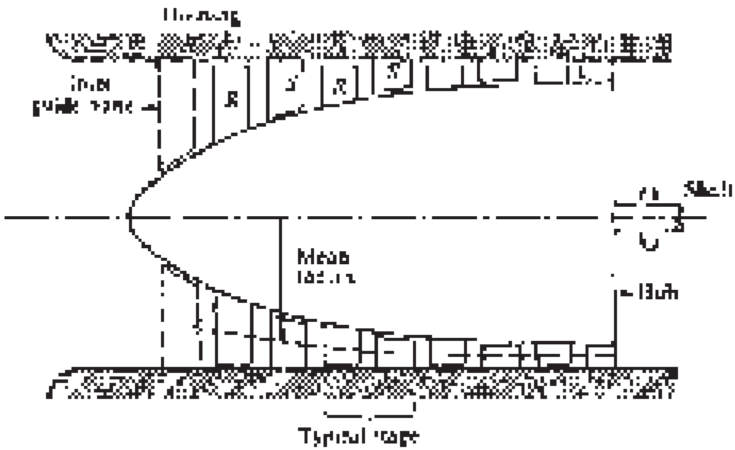


Figure 5.3 Schematic of an axial compressor section.

The flow is assumed to take place at a mean blade height, where the blade peripheral velocities at the inlet and outlet are the same. No flow is assumed in the radial direction.

5.2 VELOCITY DIAGRAM

The basic principle of axial compressor operation is that kinetic energy is imparted to the air in the rotating blade row, and then diffused through passages of both rotating and stationary blades. The process is carried out over multiple numbers of stages. As mentioned earlier, diffusion is a deceleration process. It is efficient only when the pressure rise per stage is very small. The blading diagram and the velocity triangle for an axial flow compressor stage are shown in Fig. 5.4.

Air enters the rotor blade with absolute velocity C_1 at an angle α_1 measured from the axial direction. Air leaves the rotor blade with absolute velocity C_2 at an angle α_2. Air passes through the diverging passages formed between the rotor blades. As work is done on the air in the rotor blades, C_2 is larger than C_1. The rotor row has tangential velocity U. Combining the two velocity vectors gives the relative velocity at inlet V_1 at an angle β_1. V_2 is the relative velocity at the rotor outlet. It is less than V_1, showing diffusion of the relative velocity has taken place with some static pressure rise across the rotor blades. Turning of the air towards the axial direction is brought about by the camber of the blades. Euler's equation

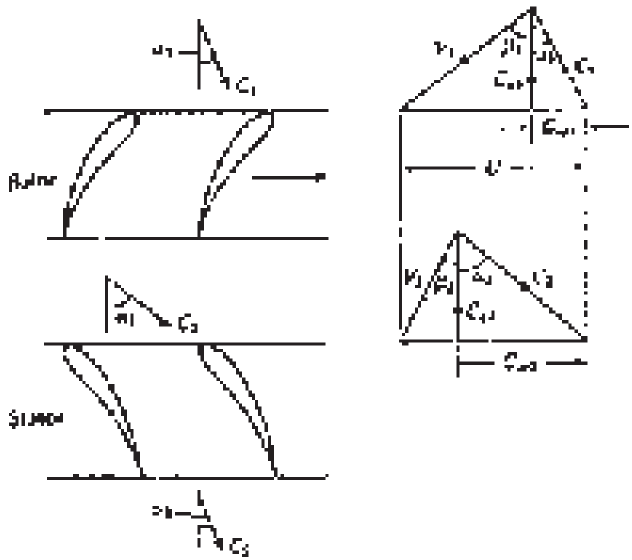


Figure 5.4 Velocity diagrams for a compressor stage.

provides the work done on the air:

$$W_c = U(C_{w2} - C_{w1}) \tag{5.1}$$

Using the velocity triangles, the following basic equations can be written:

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1 \tag{5.2}$$

$$\frac{U}{C_a} = \tan\alpha_2 + \tan\beta_2 \tag{5.3}$$

in which $C_a = C_{a1} = C_2$ is the axial velocity, assumed constant through the stage. The work done equation [Eq. (5.1)] may be written in terms of air angles:

$$W_c = UC_a(\tan\alpha_2 - \tan\alpha_1) \tag{5.4}$$

also,

$$W_c = UC_a(\tan\beta_1 - \tan\beta_2) \tag{5.5}$$

The whole of this input energy will be absorbed usefully in raising the pressure and velocity of the air and for overcoming various frictional losses. Regardless of the losses, all the energy is used to increase the stagnation temperature of the air, $\triangle T_{0s}$. If the velocity of air leaving the first stage C_3 is made equal to C_1, then the stagnation temperature rise will be equal to the static temperature rise, $\triangle T_s$. Hence:

$$T_{0s} = \Delta T_s = \frac{UC_a}{C_p}(\tan\beta_1 - \tan\beta_2) \tag{5.6}$$

Equation (5.6) is the theoretical temperature rise of the air in one stage. In reality, the stage temperature rise will be less than this value due to 3-D effects in the compressor annulus. To find the actual temperature rise of the air, a factor λ, which is between 0 and 100%, will be used. Thus the actual temperature rise of the air is given by:

$$T_{0s} = \frac{\lambda UC_a}{C_p}(\tan\beta_1 - \tan\beta_2) \tag{5.7}$$

If R_s is the stage pressure ratio and η_s is the stage isentropic efficiency, then:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/(\gamma-1)} \tag{5.8}$$

where T_{01} is the inlet stagnation temperature.

5.3 DEGREE OF REACTION

The degree of reaction, Λ, is defined as:

$$\Lambda = \frac{\text{Static enthalpy rise in the rotor}}{\text{Static enthalpy rise in the whole stage}} \tag{5.9}$$

The degree of reaction indicates the distribution of the total pressure rise into the two types of blades. The choice of a particular degree of reaction is important in that it affects the velocity triangles, the fluid friction and other losses.

Let:

ΔT_A = the static temperature rise in the rotor

ΔT_B = the static temperature rise in the stator

Using the work input equation [Eq. (5.4)], we get:

$$\left.\begin{aligned} W_c &= C_p(\Delta T_A + \Delta T_B) = \Delta T_S \\ &= UC_a(\tan\beta_1 - \tan\beta_2) \\ &= UC_a(\tan\alpha_2 - \tan\alpha_1) \end{aligned}\right\} \tag{5.10}$$

But since all the energy is transferred to the air in the rotor, using the steady flow energy equation, we have:

$$W_c = C_p \Delta T_A + \frac{1}{2}(C_2^2 - C_1^2) \tag{5.11}$$

Combining Eqs. (5.10) and (5.11), we get:

$$C_p \Delta T_A = UC_a(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}(C_2^2 - C_1^2)$$

from the velocity triangles,

$$C_2 = C_a \cos\alpha_2 \quad \text{and} \quad C_1 = C_a \cos\alpha_1$$

Therefore,

$$\begin{aligned} C_p \Delta T_A &= UC_a(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_a^2(\sec^2\alpha_2 - \sec^2\alpha_1) \\ &= UC_a(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_a^2(\tan^2\alpha_2 - \tan^2\alpha_1) \end{aligned}$$

Using the definition of degree of reaction,

$$\begin{aligned} \Lambda &= \frac{\Delta T_A}{\Delta T_A + \Delta T_B} \\ &= \frac{UC_a(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}C_a^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{UC_a(\tan\alpha_2 - \tan\alpha_1)} \\ &= 1 - \tfrac{C_a}{U}(\tan\alpha_2 + \tan\alpha_1) \end{aligned}$$

But from the velocity triangles, adding Eqs. (5.2) and (5.3),

$$\frac{2U}{C_a} = (\tan\alpha_1 + \tan\beta_1 + \tan\alpha_2 + \tan\beta_2)$$

Therefore,

$$\Lambda = \frac{C_a}{2U}\left(\frac{2U}{C_a} - \frac{2U}{C_a} + \tan\beta_1 + \tan\beta_2\right)$$

$$= \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2) \tag{5.12}$$

Usually the degree of reaction is set equal to 50%, which leads to this interesting result:

$$(\tan\beta_1 + \tan\beta_2) = \frac{U}{C_a}.$$

Again using Eqs. (5.1) and (5.2),

$$\tan\alpha_1 = \tan\beta_2, \quad \text{i.e.,} \quad \alpha_1 = \beta_2$$

$$\tan\beta_1 = \tan\alpha_2, \quad \text{i.e.,} \quad \alpha_2 = \beta_1$$

As we have assumed that C_a is constant through the stage,

$$C_a = C_1 \cos\alpha_1 = C_3 \cos\alpha_3.$$

Since we know $C_1 = C_3$, it follows that $\alpha_1 = \alpha_3$. Because the angles are equal, $\alpha_1 = \beta_2 = \alpha_3$, and $\beta_1 = \alpha_2$. Under these conditions, the velocity triangles become symmetric. In Eq. (5.12), the ratio of axial velocity to blade velocity is called the flow coefficient and denoted by Φ. For a reaction ratio of 50%, $(h_2 - h_1) = (h_3 - h_1)$, which implies the static enthalpy and the temperature increase in the rotor and stator are equal. If for a given value of C_a/U, β_2 is chosen to be greater than α_2 (Fig. 5.5), then the static pressure rise in the rotor is greater than the static pressure rise in the stator and the reaction is greater than 50%.

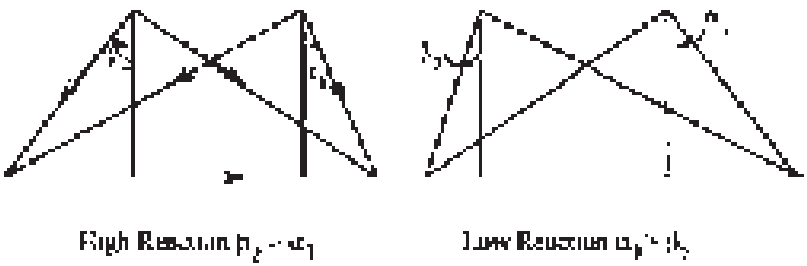


Figure 5.5 Stage reaction.

Conversely, if the designer chooses β_2 less than β_1, the stator pressure rise will be greater and the reaction is less than 50%.

5.4 STAGE LOADING

The stage-loading factor Ψ is defined as:

$$\Psi = \frac{W_c}{mU^2} = \frac{h_{03} - h_{01}}{U^2}$$
$$= \frac{\lambda(C_{w2} - C_{w1})}{U}$$
$$= \frac{\lambda C_a}{U}(\tan\alpha_2 - \tan\alpha_1)$$
$$\Psi = \lambda\Phi\,(\tan\alpha_2 - \tan\alpha_1) \tag{5.13}$$

5.5 LIFT-AND-DRAG COEFFICIENTS

The stage-loading factor Ψ may be expressed in terms of the lift-and-drag coefficients. Consider a rotor blade as shown in Fig. 5.6, with relative velocity vectors V_1 and V_2 at angles β_1 and β_2. Let $\tan(\beta_m) = (\tan(\beta_1) + \tan(\beta_2))/2$. The flow on the rotor blade is similar to flow over an airfoil, so lift-and-drag forces will be set up on the blade while the forces on the air will act on the opposite direction.

The tangential force on each moving blade is:

$$F_x = L\cos\beta_m + D\sin\beta_m$$

$$F_x = L\cos\beta_m\left[1 + \left(\frac{C_D}{C_L}\right)\tan\beta_m\right] \tag{5.14}$$

where: L = lift and D = drag.

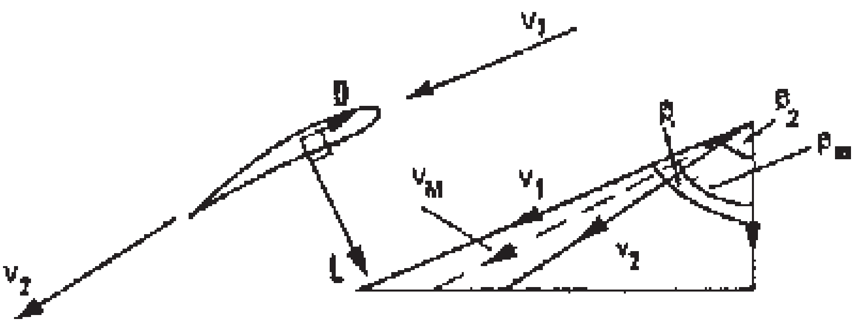


Figure 5.6 Lift-and-drag forces on a compressor rotor blade.

The lift coefficient is defined as:

$$C_L = \frac{L}{0.5\rho V_m^2 A} \tag{5.15}$$

where the blade area is the product of the chord c and the span l.

Substituting $V_m = \frac{C_a}{\cos\beta_m}$ into the above equation,

$$F_x = \frac{\rho C_a^2 c l C_L}{2} \sec\beta_m \left[1 + \left(\frac{C_D}{C_L}\right)\tan\beta_m\right] \tag{5.16}$$

The power delivered to the air is given by:

$$\begin{aligned} UF_x &= m(h_{03} - h_{01}) \\ &= \rho C_a l s(h_{03} - h_{01}) \end{aligned} \tag{5.17}$$

considering the flow through one blade passage of width s.

Therefore,

$$\begin{aligned} &= \frac{h_{03} - h_{01}}{U^2} \\ &= \frac{F_x}{\rho C_a l s U} \\ &= \frac{1}{2}\left(\frac{C_a}{U}\right)\left(\frac{c}{s}\right)\sec\beta_m(C_L + C_D\tan\beta_m) \\ &= \frac{1}{2}\left(\frac{c}{s}\right)\sec\beta_m(C_L + C_D\tan\beta_m) \end{aligned} \tag{5.18}$$

For a stage in which $\beta_m = 45°$, efficiency will be maximum. Substituting this back into Eq. (5.18), the optimal blade-loading factor is given by:

$$\Psi_{opt} = \frac{\varphi}{\sqrt{2}}\left(\frac{c}{s}\right)(C_L + C_D) \tag{5.19}$$

For a well-designed blade, C_D is much smaller than C_L, and therefore the optimal blade-loading factor is approximated by:

$$\Psi_{opt} = \frac{\varphi}{\sqrt{2}}\left(\frac{c}{s}\right)C_L \tag{5.20}$$

5.6 CASCADE NOMENCLATURE AND TERMINOLOGY

Studying the 2-D flow through cascades of airfoils facilitates designing highly efficient axial flow compressors. A cascade is a row of geometrically similar blades arranged at equal distance from each other and aligned to the flow direction. Figure 5.7, which is reproduced from Howell's early paper on cascade theory and performance, shows the standard nomenclature relating to airfoils in cascade.

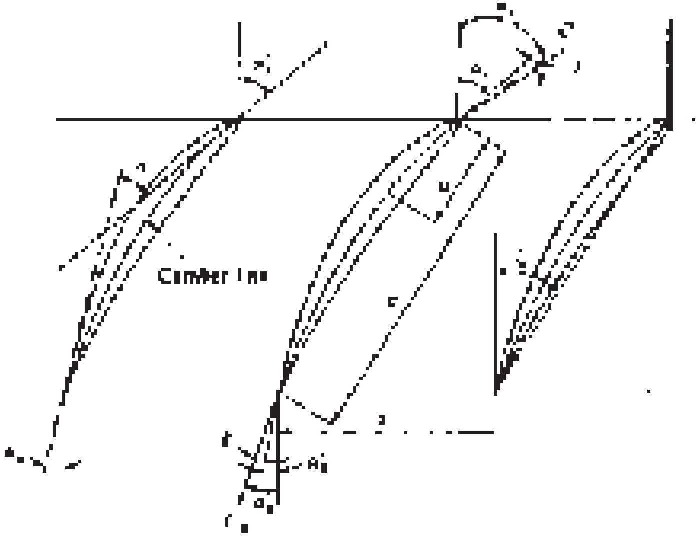


Figure 5.7 Cascade nomenclature.

$\alpha_1{}'$ and $\alpha_2{}'$ are the camber angles of the entry and exit tangents the camber line makes with the axial direction. The blade camber angle $\theta = \alpha_1' - \alpha_2'$. The chord c is the length of the perpendicular of the blade profile onto the chord line. It is approximately equal to the linear distance between the leading edge and the trailing edge. The stagger angle ξ is the angle between the chord line and the axial direction and represents the angle at which the blade is set in the cascade. The pitch s is the distance in the direction of rotation between corresponding points on adjacent blades. The incidence angle i is the difference between the air inlet angle (α_1) and the blade inlet angle $\left(\alpha_1'\right)$. That is, $i = \alpha_1 - \alpha_1'$. The deviation angle (δ) is the difference between the air outlet angle (α_2) and the blade outlet angle $\left(\alpha_2'\right)$. The air deflection angle, $\varepsilon = \alpha_1 - \alpha_2$, is the difference between the entry and exit air angles.

A cross-section of three blades forming part of a typical cascade is shown in Fig. 5.7. For any particular test, the blade camber angle θ, its chord c, and the pitch (or space) s will be fixed and the blade inlet and outlet angles α_1' and α_2' are determined by the chosen setting or stagger angle ξ. The angle of incidence, i, is then fixed by the choice of a suitable air inlet angle α_1, since $i = \alpha_1 - \alpha_1'$. An appropriate setting of the turntable on which the cascade is mounted can accomplish this. With the cascade in this position the pressure and direction measuring instruments are then traversed along the blade row in the upstream and downstream position. The results of the traverses are usually presented as shown

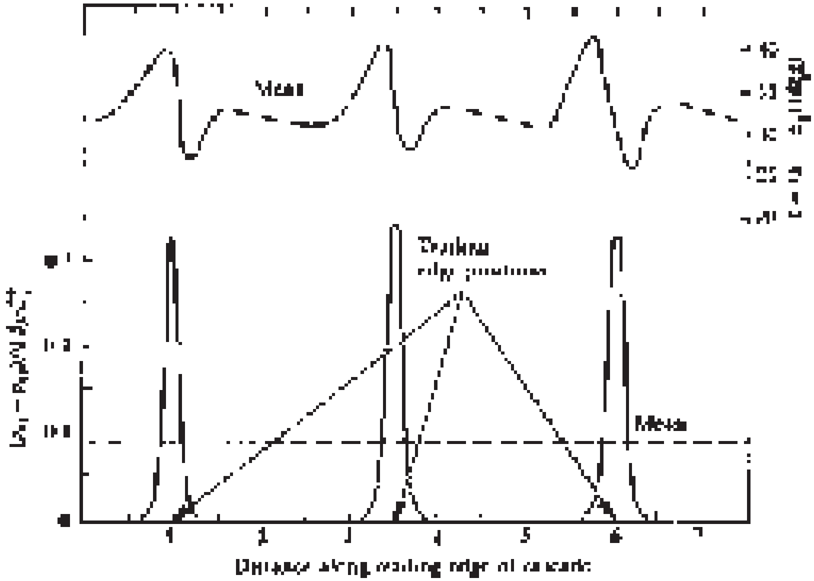


Figure 5.8 Variation of stagnation pressure loss and deflection for cascade at fixed incidence.

in Fig. 5.8. The stagnation pressure loss is plotted as a dimensionless number given by:

$$\text{Stagnation pressure loss coefficient} = \frac{P_{01} - P_{02}}{0.5\rho C_1^2} \tag{5.21}$$

This shows the variation of loss of stagnation pressure and the air deflection, $\varepsilon = \alpha_1 - \alpha_2$, covering two blades at the center of the cascade. The curves of Fig. 5.8 can now be repeated for different values of incidence angle, and the whole set of results condensed to the form shown in Fig. 5.9, in which the mean loss and mean deflection are plotted against incidence for a cascade of fixed geometrical form.

The total pressure loss owing to the increase in deflection angle of air is marked when i is increased beyond a particular value. The stalling incidence of the cascade is the angle at which the total pressure loss is twice the minimum cascade pressure loss. Reducing the incidence i generates a negative angle of incidence at which stalling will occur.

Knowing the limits for air deflection without very high (more than twice the minimum) total pressure loss is very useful for designers in the design of efficient compressors. Howell has defined nominal conditions of deflection for

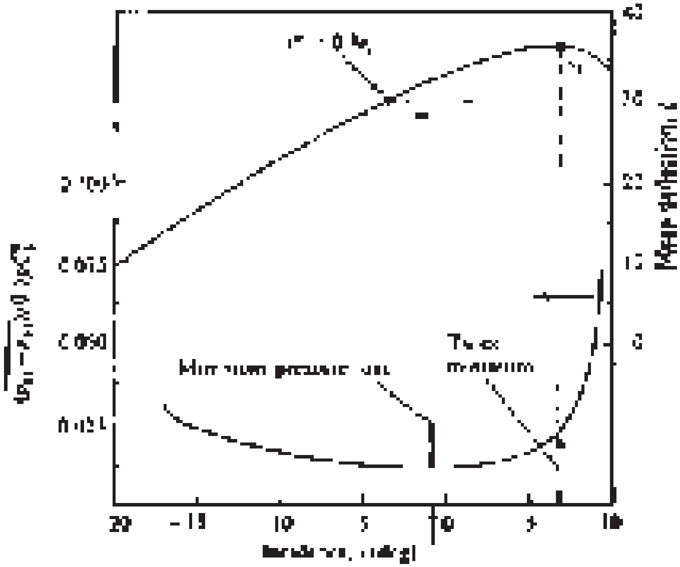


Figure 5.9 Cascade mean deflection and pressure loss curves.

a cascade as 80% of its stalling deflection, that is:

$$\varepsilon^* = 0.8\varepsilon_s \tag{5.22}$$

where ε_s is the stalling deflection and ε^* is the nominal deflection for the cascade.

Howell and Constant also introduced a relation correlating nominal deviation δ^* with pitch chord ratio and the camber of the blade. The relation is given by:

$$\delta^* = m\theta\left(\frac{s}{l}\right)^n \tag{5.23}$$

For compressor cascade, $n = \frac{1}{2}$, and for the inlet guide vane in front of the compressor, $n = 1$. Hence, for a compressor cascade, nominal deviation is given by:

$$\delta^* = m\theta\left(\frac{s}{l}\right)^{\frac{1}{2}} \tag{5.24}$$

The approximate value suggested by Constant is 0.26, and Howell suggested a modified value for m:

$$m = 0.23\left(\frac{2a}{l}\right)^2 + 0.1\left(\frac{\alpha_2^*}{50}\right) \tag{5.25}$$

where the maximum camber of the cascade airfoil is at a distance a from the leading edge and α_2^* is the nominal air outlet angle.

Then,

$$\alpha_2^* = \beta_2 + \delta^*$$

$$= \beta_2 + m\theta\left(\frac{s}{l}\right)^{\frac{1}{2}}$$

and,

$$\alpha_1^* - \alpha_2^* = \varepsilon^*$$

or:

$$\alpha_1^* = \alpha_2^* + \varepsilon^*$$

Also,

$$i^* = \alpha_1^* - \beta_1 = \alpha_2^* + \varepsilon^* - \beta_1$$

5.7 3-D CONSIDERATION

So far, all the above discussions were based on the velocity triangle at one particular radius of the blading. Actually, there is a considerable difference in the velocity diagram between the blade hub and tip sections, as shown in Fig. 5.10.

The shape of the velocity triangle will influence the blade geometry, and, therefore, it is important to consider this in the design. In the case of a compressor with high hub/tip ratio, there is little variation in blade speed from root to tip. The shape of the velocity diagram does not change much and, therefore, little variation in pressure occurs along the length of the blade. The blading is of the same section at all radii and the performance of the compressor stage is calculated from the performance of the blading at the mean radial section. The flow along the compressor is considered to be 2-D. That is, in 2-D flow only whirl and axial flow velocities exist with no radial velocity component. In an axial flow compressor in which high hub/tip radius ratio exists on the order of 0.8, 2-D flow in the compressor annulus is a fairly reasonable assumption. For hub/tip ratios lower than 0.8, the assumption of two-dimensional flow is no longer valid. Such compressors, having long blades relative to the mean diameter, have been used in aircraft applications in which a high mass flow requires a large annulus area but a small blade tip must be used to keep down the frontal area. Whenever the fluid has an angular velocity as well as velocity in the direction parallel to the axis of rotation, it is said to have "vorticity." The flow through an axial compressor is vortex flow in nature. The rotating fluid is subjected to a centrifugal force and to balance this force, a radial pressure gradient is necessary. Let us consider the pressure forces on a fluid element as shown in Fig. 5.10. Now, resolve

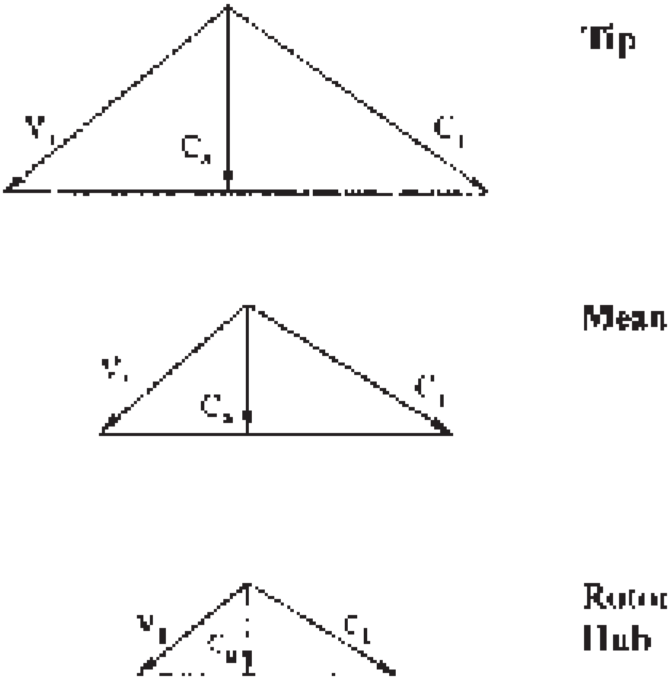


Figure 5.10 Variation of velocity diagram along blade.

the forces in the radial direction Fig. 5.11:

$$d\theta(P + dP)(r + dr) - Pr\, d\theta - 2\left(P + \frac{dP}{2}\right)dr\frac{d\theta}{2}$$

$$= \rho\, dr\, r\, d\theta\frac{C_w^2}{r} \tag{5.26}$$

or

$$(P + dP)(r + dr) - Pr - \left(P + \frac{dP}{2}\right)dr = \rho\, dr\, C_w^2$$

where: P is the pressure, ρ, the density, C_w, the whirl velocity, r, the radius.

After simplification, we get the following expression:

$$Pr + P\,dr + r\,dP + dP\,dr - Pr + \rho\,dr - \frac{1}{2}dP\,dr = \rho\,dr\,C_w^2$$

or:

$$r\,dP = \rho\,dr\,C_w^2$$

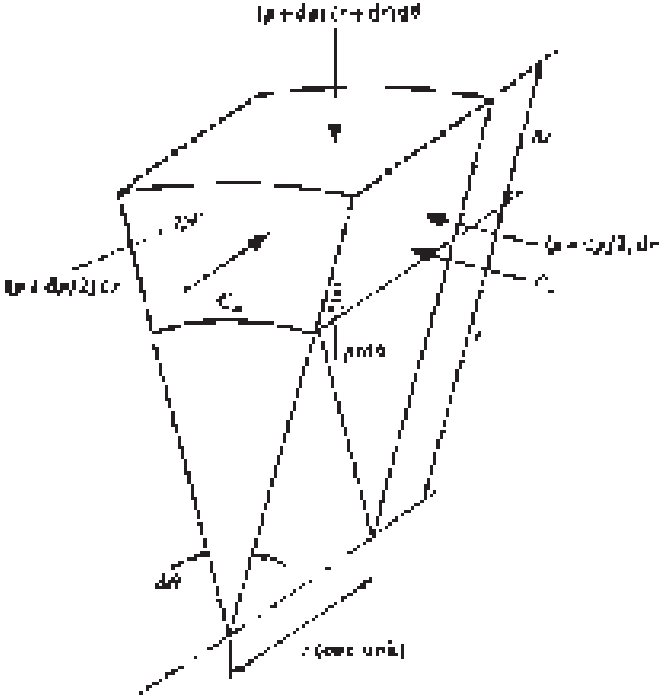


Figure 5.11 Pressure forces on a fluid element.

That is,

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{C_\mathrm{w}^2}{r} \tag{5.27}$$

The approximation represented by Eq. (5.27) has become known as radial equilibrium.

The stagnation enthalpy h_0 at any radius r where the absolute velocity is C may be rewritten as:

$$h_0 = h + \frac{1}{2}C_\mathrm{a}^2 + \frac{1}{2}C_\mathrm{w}^2; \left(h = c_\mathrm{p}T, \quad \text{and} \quad C^2 = C_\mathrm{a}^2 + C_\mathrm{w}^2\right)$$

Differentiating the above equation w.r.t. r and equating it to zero yields:

$$\frac{\mathrm{d}h_0}{\mathrm{d}r} = \frac{\gamma}{\gamma - 1} \times \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} + \frac{1}{2}\left(0 + 2C_\mathrm{w}\frac{\mathrm{d}C_\mathrm{w}}{\mathrm{d}r}\right)$$

or:

$$\frac{\gamma}{\gamma-1}\times\frac{1}{\rho}\frac{dP}{dr}+C_w\frac{dC_w}{dr}=0$$

Combining this with Eq. (5.27):

$$\frac{\gamma}{\gamma-1}\frac{C_w^2}{r}+C_w\frac{dC_w}{dr}=0$$

or:

$$\frac{dC_w}{dr}=-\frac{\gamma}{\gamma-1}\frac{C_w}{r}$$

Separating the variables,

$$\frac{dC_w}{C_w}=-\frac{\gamma}{\gamma-1}\frac{dr}{r}$$

Integrating the above equation

$$\int\frac{dC_w}{C_w}=-\frac{\gamma}{\gamma-1}\int\frac{dr}{r}$$

$$-\frac{\gamma}{\gamma-1}\ln C_w r=c \quad \text{where } c \text{ is a constant.}$$

Taking antilog on both sides,

$$\frac{\gamma}{\gamma-1}\times C_w\times r=e^c$$

Therefore, we have

$$C_w r=\text{constant} \tag{5.28}$$

Equation (5.28) indicates that the whirl velocity component of the flow varies inversely with the radius. This is commonly known as free vortex. The outlet blade angles would therefore be calculated using the free vortex distribution.

5.8 MULTI-STAGE PERFORMANCE

An axial flow compressor consists of a number of stages. If R is the overall pressure ratio, R_s is the stage pressure ratio, and N is the number of stages, then the total pressure ratio is given by:

$$R=(R_s)^N \tag{5.29}$$

Equation (5.29) gives only a rough value of R because as the air passes through the compressor the temperature rises continuously. The equation used to find stage pressure is given by:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{5.30}$$

The above equation indicates that the stage pressure ratio depends only on inlet stagnation temperature T_{01}, which goes on increasing in the successive stages. To find the value of R, the concept of polytropic or small stage efficiency is very useful. The polytropic or small stage efficiency of a compressor is given by:

$$\eta_{\infty,c} = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

or:

$$\left(\frac{n}{n-1}\right) = \eta_s\left(\frac{\gamma}{\gamma - 1}\right)$$

where $\eta_s = \eta_{\infty,c}$ = small stage efficiency.

The overall pressure ratio is given by:

$$R = \left[1 + \frac{N \Delta T_{0s}}{T_{01}}\right]^{\frac{n}{n-1}} \tag{5.31}$$

Although Eq. (5.31) is used to find the overall pressure ratio of a compressor, in actual practice the step-by-step method is used.

5.9 AXIAL FLOW COMPRESSOR CHARACTERISTICS

The forms of characteristic curves of axial flow compressors are shown in Fig. 5.12. These curves are quite similar to the centrifugal compressor. However, axial flow compressors cover a narrower range of mass flow than the centrifugal compressors, and the surge line is also steeper than that of a centrifugal compressor. Surging and choking limit the curves at the two ends. However, the surge points in the axial flow compressors are reached before the curves reach a maximum value. In practice, the design points is very close to the surge line. Therefore, the operating range of axial flow compressors is quite narrow.

Illustrative Example 5.1: In an axial flow compressor air enters the compressor at stagnation pressure and temperature of 1 bar and 292K, respectively. The pressure ratio of the compressor is 9.5. If isentropic efficiency of the compressor is 0.85, find the work of compression and the final temperature at the outlet. Assume $\gamma = 1.4$, and $C_p = 1.005$ kJ/kg K.

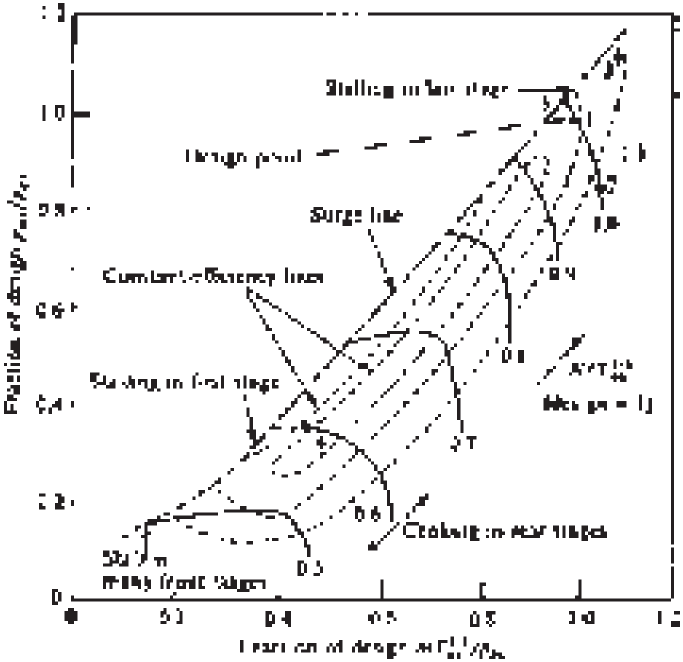


Figure 5.12 Axial flow compressor characteristics.

Solution:

$$T_{01} = 292\text{K}, \quad P_{01} = 1\,\text{bar}, \quad \eta_c = 0.85.$$

Using the isentropic $P{-}T$ relation for compression processes,

$$\frac{P_{02}}{P_{01}} = \left[\frac{T'_{02}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

where $T_{02'}$ is the isentropic temperature at the outlet.

Therefore,

$$T'_{02} = T_{01}\left[\frac{P_{02}}{P_{01}}\right]^{\frac{\gamma-1}{\gamma}} = 292(9.5)^{0.286} = 555.92\,\text{K}$$

Now, using isentropic efficiency of the compressor in order to find the actual temperature at the outlet,

$$T_{02} = T_{01} + \frac{\left(T'_{02} - T_{01}\right)}{\eta_c} = 292 + \frac{(555.92 - 292)}{0.85} = 602.49\,\text{K}$$

Work of compression:

$$W_c = C_p(T_{02} - T_{01}) = 1.005(602.49 - 292) = 312\,\text{kJ/kg}$$

Illustrative Example 5.2: In one stage of an axial flow compressor, the pressure ratio is to be 1.22 and the air inlet stagnation temperature is 288K. If the stagnation temperature rise of the stages is 21K, the rotor tip speed is 200 m/s, and the rotor rotates at 4500 rpm, calculate the stage efficiency and diameter of the rotor.

Solution:
The stage pressure ratio is given by:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

or

$$1.22 = \left[1 + \frac{\eta_s(21)}{288}\right]^{3.5}$$

that is,

$$\eta_s = 0.8026 \quad \text{or} \quad 80.26\%$$

The rotor speed is given by:

$$U = \frac{\pi DN}{60}, \quad \text{or} \quad D = \frac{(60)(200)}{\pi(4500)} = 0.85\ \text{m}$$

Illustrative Example 5.3: An axial flow compressor has a tip diameter of 0.95 m and a hub diameter of 0.85 m. The absolute velocity of air makes an angle of 28° measured from the axial direction and relative velocity angle is 56°. The absolute velocity outlet angle is 56° and the relative velocity outlet angle is 28°. The rotor rotates at 5000 rpm and the density of air is 1.2 kg/m^3. Determine:

1. The axial velocity.
2. The mass flow rate.
3. The power required.
4. The flow angles at the hub.
5. The degree of reaction at the hub.

Solution:
1. Rotor speed is given by:

$$U = \frac{\pi DN}{60} = \frac{\pi(0.95)(5000)}{60} = 249\,\text{m/s}$$

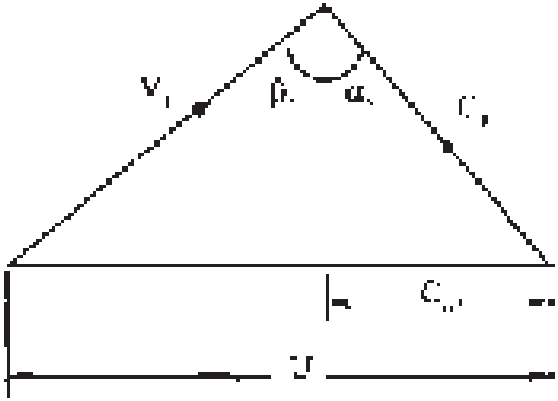


Figure 5.13 Inlet velocity triangle.

Blade speed at the hub:

$$U_h = \frac{\pi D_h N}{60} = \frac{\pi(0.85)(5000)}{60} = 223\,\text{m/s}$$

From the inlet velocity triangle (Fig. 5.13),

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} \qquad \text{and} \qquad \tan\beta_1 = \frac{(U - C_{w1})}{C_a}$$

Adding the above two equations:

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

or:

$$U = C_a(\tan 28^\circ + \tan 56^\circ) = C_a(2.0146)$$

Therefore, $C_a = 123.6\,\text{m/s}$ (constant at all radii)

2. The mass flow rate:

$$\dot{m} = \pi(r_t^2 - r_h^2)\rho C_a$$
$$= \pi(0.475^2 - 0.425^2)(1.2)(123.6) = 20.98\,\text{kg/s}$$

3. The power required per unit kg for compression is:

$$W_c = \lambda U C_a(\tan\beta_1 - \tan\beta_2)$$
$$= (1)(249)(123.6)(\tan 56^\circ - \tan 28^\circ)10^{-3}$$
$$= (249)(123.6)(1.483 - 0.53)$$
$$= 29.268\,\text{kJ/kg}$$

The total power required to drive the compressor is:

$$W_c = \dot{m}(29.268) = (20.98)(29.268) = 614\,\text{kW}$$

4. At the inlet to the rotor tip:

$$C_{w1} = C_a \tan\alpha_1 = 123.6 \tan 28^{\circ} = 65.72\,\text{m/s}$$

Using free vortex condition, i.e., $C_w r$ = constant, and using h as the subscript for the hub,

$$C_{w1h} = C_{w1t}\frac{r_t}{r_h} = (65.72)\frac{0.475}{0.425} = 73.452\,\text{m/s}$$

At the outlet to the rotor tip,

$$C_{w2t} = C_a \tan\alpha_2 = 123.6 \tan 56^{\circ} = 183.24\,\text{m/s}$$

Therefore,

$$C_{w2h} = C_{w2t}\frac{r_t}{r_h} = (183.24)\frac{0.475}{0.425} = 204.8\,\text{m/s}$$

Hence the flow angles at the hub:

$$\tan\alpha_1 = \frac{C_{w1h}}{C_a} = \frac{73.452}{123.6} = 0.594 \;\text{ or, }\; \alpha_1 = 30.72^{\circ}$$

$$\tan\beta_1 = \frac{(U_h)}{C_a} - \tan\alpha_1 = \frac{223}{123.6} - 0.5942 = 1.21$$

i.e., $\beta_1 = 50.43^{\circ}$

$$\tan\alpha_2 = \frac{C_{w2h}}{C_a} = \frac{204.8}{123.6} = 1.657$$

i.e., $\alpha_2 = 58.89^{\circ}$

$$\tan\beta_2 = \frac{(U_h)}{C_a} - \tan\alpha_2 = \frac{223}{123.6} - \tan 58.59^{\circ} = 0.1472$$

i.e., $\beta_2 = 8.37^{\circ}$

5. The degree of reaction at the hub is given by:

$$\Lambda_h = \frac{C_a}{2U_h}(\tan\beta_1 + \tan\beta_2) = \frac{123.6}{(2)(223)}(\tan 50.43^{\circ} + \tan 8.37^{\circ})$$

$$= \frac{123.6}{(2)(223)}(1.21 + 0.147) = 37.61\%$$

Illustrative Example 5.4: An axial flow compressor has the following data:

Blade velocity at root:	140 m/s
Blade velocity at mean radius:	185 m/s
Blade velocity at tip:	240 m/s
Stagnation temperature rise in this stage:	15K
Axial velocity (constant from root to tip):	140 m/s
Work done factor:	0.85
Degree of reaction at mean radius:	50%

Calculate the stage air angles at the root, mean, and tip for a free vortex design.

Solution:

Calculation at mean radius:

From Eq. (5.1), $W_c = U(C_{w2} - C_{w1}) = U\Delta C_w$

or:

$$C_p(T_{02} - T_{01}) = C_p\Delta T_{0s} = \lambda U \Delta C_w$$

So:

$$\Delta C_w = \frac{C_p \Delta T_{0s}}{\lambda U} = \frac{(1005)(15)}{(0.85)(185)} = 95.87 \text{ m/s}$$

Since the degree of reaction (Fig. 5.14) at the mean radius is 50%, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the velocity triangle at the mean,

$$U = \Delta C_w + 2C_{w1}$$

or:

$$C_{w1} = \frac{U - \Delta C_w}{2} = \frac{185 - 95.87}{2} = 44.57 \text{ m/s}$$

Hence,

$$\tan \alpha_1 = \frac{C_{w1}}{C_a} = \frac{44.57}{140} = 0.3184$$

that is,

$$\alpha_1 = 17.66^\circ = \beta_2$$

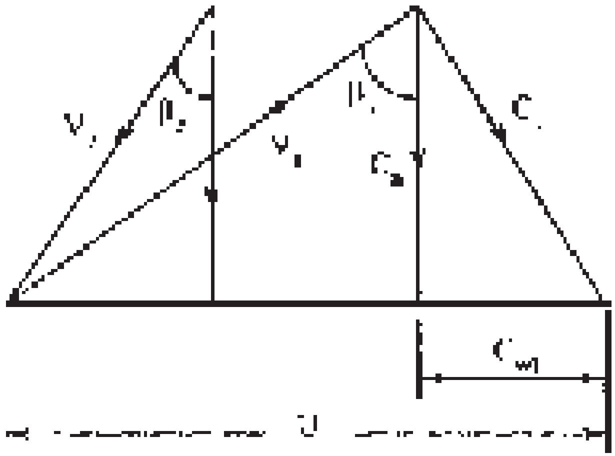


Figure 5.14 Velocity triangle at the mean radius.

and

$$\tan\beta_1 = \frac{(\Delta C_w + C_{w1})}{C_a} = \frac{(95.87 + 44.57)}{140} = 1.003$$

i.e., $\beta_1 = 45.09° = \alpha_2$

Calculation at the blade tip:
Using the free vortex diagram (Fig. 5.15),

$$(\Delta C_w \times U)_t = (\Delta C_w \times U)_m$$

Therefore,

$$\Delta C_w = \frac{(95.87)(185)}{240} = 73.9\,\text{m/s}$$

Whirl velocity component at the tip:

$$C_{w1} \times 240 = (44.57)(185)$$

Therefore:

$$C_{w1} = \frac{(44.57)(185)}{240} = 34.36\,\text{m/s}$$

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} = \frac{34.36}{140} = 0.245$$

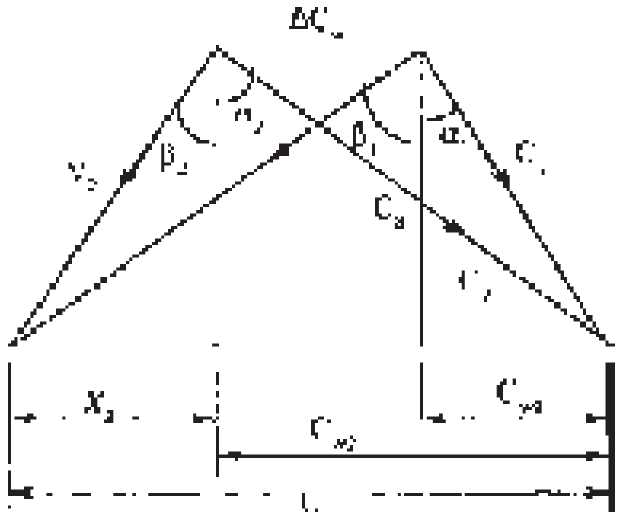


Figure 5.15 Velocity triangles at tip.

Therefore,

$$\alpha_1 = 13.79°$$

From the velocity triangle at the tip,

$$x_2 + \Delta C_w + C_{w1} = U$$

or:

$$x_2 = U - \Delta C_w - C_{w1} = 240 - 73.9 - 34.36 = 131.74$$

$$\tan \beta_1 = \frac{\Delta C_w + x_2}{C_a} = \frac{73.9 + 131.74}{140} = 1.469$$

i.e., $\beta_1 = 55.75°$

$$\tan \alpha_2 = \frac{(C_{w1} + \Delta C_w)}{C_a} = \frac{(34.36 + 73.9)}{140} = 0.7733$$

i.e., $\alpha_2 = 37.71°$

$$\tan \beta_2 = \frac{x_2}{C_a} = \frac{131.74}{140} = 0.941$$

i.e., $\beta_2 = 43.26°$

Calculation at the blade root:

$$(\Delta C_w \times U)_r = (\Delta C_w \times U)_m$$

or:

$$\Delta C_w \times 140 = (95.87)(185) \qquad \text{and} \qquad \Delta C_w = 126.69\,\text{m/s}$$

Also:

$$(C_{w1} \times U)_r = (C_{w1} \times U)_m$$

or:

$$C_{w1} \times 140 = (44.57)(185) \quad \text{and} \quad C_{w1} = 58.9\,\text{m/s}$$

and

$$(C_{w2} \times U)_t = (C_{w2} \times U)_r$$

so:

$$C_{w2,tip} = C_a \tan\alpha_2 = 140 \tan 37.71^\circ = 108.24\,\text{m/s}$$

Therefore:

$$C_{w2,root} = \frac{(108.24)(240)}{140} = 185.55\,\text{m/s}$$

$$\tan\alpha_1 = \frac{58.9}{140} = 0.421$$

i.e., $\alpha_1 = 22.82°$

From the velocity triangle at the blade root, (Fig. 5.16)

or:

$$x_2 = C_{w2} - U = 185.55 - 140 = 45.55$$

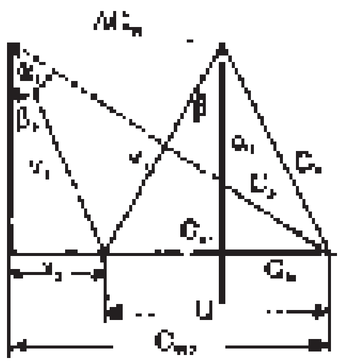


Figure 5.16 Velocity triangles at root.

Therefore:

$$\tan\beta_1 = \frac{U - C_{w1}}{C_a} = \frac{140 - 58.9}{140} = 0.579$$

i.e., $\beta_1 = 30.08°$

$$\tan\alpha_2 = \frac{C_{w2}}{C_a} = \frac{185.55}{140} = 1.325$$

i.e., $\alpha_2 = 52.96°$

$$\tan\beta_2 = -\frac{x_2}{C_a} = -\frac{45.55}{140} = -0.325$$

i.e., $\beta_2 = -18°$

Design Example 5.5: From the data given in the previous problem, calculate the degree of reaction at the blade root and tip.

Solution:

Reaction at the blade root:

$$\Lambda_{root} = \frac{C_a}{2U_r}(\tan\beta_{1r} + \tan\beta_{2r}) = \frac{140}{(2)(140)}(\tan 30.08^{\circ} + \tan(-18^{\circ}))$$

$$= \frac{140}{(2)(140)}(0.579 - 0.325) = 0.127, \quad \text{or} \quad 12.7\%$$

Reaction at the blade tip:

$$\Lambda_{tip} = \frac{C_a}{2U_t}(\tan\beta_{1t} + \tan\beta_{2t}) = \frac{140}{(2)(240)}(\tan 55.75^{\circ} + \tan 43.26^{\circ})$$

$$= \frac{140}{(2)(240)}(1.469 + 0.941) = 0.7029, \quad \text{or} \quad 70.29\%$$

Illustrative Example 5.6: An axial flow compressor stage has the following data:

Air inlet stagnation temperature:	295K
Blade angle at outlet measured from the axial direction:	32°
Flow coefficient:	0.56
Relative inlet Mach number:	0.78
Degree of reaction:	0.5

Find the stagnation temperature rise in the first stage of the compressor.

Solution:

Since the degree of reaction is 50%, the velocity triangle is symmetric as shown in Fig. 5.17. Using the degree of reaction equation [Eq. (5.12)]:

$$\Lambda = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2) \quad \text{and} \quad \varphi = \frac{C_a}{U} = 0.56$$

Therefore:

$$\tan\beta_1 = \frac{2\Lambda}{0.56} - \tan 32^\circ = 1.16$$

i.e., $\beta_1 = 49.24^\circ$

Now, for the relative Mach number at the inlet:

$$M_{r1} = \frac{V_1}{(\gamma R T_1)^{\frac{1}{2}}}$$

or:

$$V_1^2 = \gamma R M_{r1}^2 \left(T_{01} - \frac{C_1^2}{2C_p} \right)$$

From the velocity triangle,

$$V_1 = \frac{C_a}{\cos\beta_1}, \quad \text{and} \quad C_1 = \frac{C_a}{\cos\alpha_1}$$

and:

$$\alpha_1 = \beta_2 (\text{since } \Lambda = 0.5)$$

Therefore:

$$C_1 = \frac{C_a}{\cos 32^\circ} = \frac{C_a}{0.848}$$

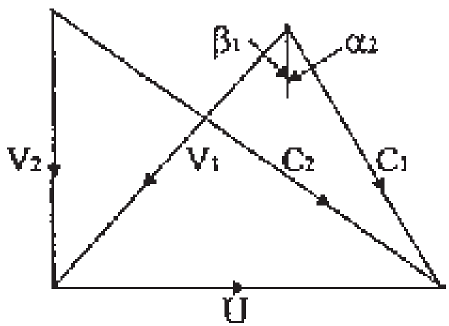


Figure 5.17 Combined velocity triangles for Example 5.6.

and:

$$V_1 = \frac{C_a}{\cos 49.24°} = \frac{C_a}{0.653}$$

Hence:

$$C_1^2 = \frac{C_a^2}{0.719}, \quad \text{and} \quad V_1^2 = \frac{C_a^2}{0.426}$$

Substituting for V_1 and C_1,

$$C_a^2 = 104.41\left(295 - \frac{C_a^2}{1445}\right), \quad \text{so :} \quad C_a = 169.51\,\text{m/s}$$

The stagnation temperature rise may be calculated as:

$$T_{02} - T_{01} = \frac{C_a^2}{C_p\varphi}(\tan\beta_1 - \tan\beta_2)$$

$$= \frac{169.51^2}{(1005)(0.56)}(\tan 49.24° - \tan 32°) = 27.31\text{K}$$

Design Example 5.7: An axial flow compressor has the following design data:

Inlet stagnation temperature:	290K
Inlet stagnation pressure:	1 bar
Stage stagnation temperature rise:	24K
Mass flow of air:	22kg/s
Axialvelocity through the stage:	155.5m/s
Rotational speed:	152rev/s
Work done factor:	0.93
Mean blade speed:	205m/s
Reaction at the mean radius:	50%

Determine: (1) the blade and air angles at the mean radius, (2) the mean radius, and (3) the blade height.

Solution:

(1) The following equation provides the relationship between the temperature rise and the desired angles:

$$T_{02} - T_{01} = \frac{\lambda U C_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

or:

$$24 = \frac{(0.93)(205)(155.5)}{1005}(\tan\beta_1 - \tan\beta_2)$$

so:

$$\tan\beta_1 - \tan\beta_2 = 0.814$$

Using the degree of reaction equation:

$$\Lambda = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

Hence:

$$\tan\beta_1 + \tan\beta_2 = \frac{(0.5)(2)(205)}{155.5} = 1.318$$

Solving the above two equations simultaneously for β_1 and β_2,

$$2\tan\beta_1 = 2.132,$$

$$\text{so}: \beta_1 = 46.83^\circ = \alpha_2 \text{ (since the degree of reaction is } 50\%)$$

and:

$$\tan\beta_2 = 1.318 - \tan 46.83^\circ = 1.318 - 1.066,$$

$$\text{so}: \beta_2 = 14.14^\circ = \alpha_1$$

(2) The mean radius, r_m, is given by:

$$r_m = \frac{U}{2\pi N} = \frac{205}{(2\pi)(152)} = 0.215\text{m}$$

(3) The blade height, h, is given by:
$m = \rho A C_{a,}$ where A is the annular area of the flow.

$$C_1 = \frac{C_a}{\cos\alpha_1} = \frac{155.5}{\cos 14.14^\circ} = 160.31 \text{ m/s}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{160.31^2}{(2)(1005)} = 277.21 \text{ K}$$

Using the isentropic $P\!-\!T$ relation:

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Static pressure:

$$P_1 = (1)\left(\frac{277.21}{290}\right)^{3.5} = 0.854\,\text{bar}$$

Then:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(0.854)(100)}{(0.287)(277.21)} = 1.073\,\text{kg/m}^3$$

From the continuity equation:

$$A = \frac{22}{(1.073)(155.5)} = 0.132\text{m}^2$$

and the blade height:

$$h = \frac{A}{2\pi r_m} = \frac{0.132}{(2\pi)(0.215)} = 0.098\text{m}$$

Illustrative Example 5.8: An axial flow compressor has an overall pressure ratio of 4.5:1, and a mean blade speed of 245 m/s. Each stage is of 50% reaction and the relative air angles are the same (30°) for each stage. The axial velocity is 158 m/s and is constant through the stage. If the polytropic efficiency is 87%, calculate the number of stages required. Assume $T_{01} = 290\text{K}$.

Solution:

Since the degree of reaction at the mean radius is 50%, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$. From the velocity triangles, the relative outlet velocity component in the x-direction is given by:

$$V_{x2} = C_a \tan\beta_2 = 158\tan 30^\circ = 91.22\,\text{m/s}$$

$$V_1 = C_2 = \left[(U - V_{x2})^2 + C_a^2\right]^{\frac{1}{2}}$$

$$= \left[(245 - 91.22)^2 + 158^2\right]^{\frac{1}{2}} = 220.48\,\text{m/s}$$

$$\cos\beta_1 = \frac{C_a}{V_1} = \frac{158}{220.48} = 0.7166$$

so: $\beta_1 = 44.23^\circ$

Stagnation temperature rise in the stage,

$$\Delta T_{0s} = \frac{UC_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$= \frac{(245)(158)}{1005}(\tan 44.23° - \tan 30°) = 15.21\text{K}$$

Number of stages

$$R = \left[1 + \frac{N\Delta T_{0s}}{T_{01}}\right]^{\frac{n}{n-1}}$$

$$\frac{n}{n-1} = \eta_\infty \frac{\gamma}{\gamma - 1} = 0.87\frac{1.4}{0.4} = 3.05$$

Substituting:

$$4.5 = \left[1 + \frac{N15.21}{290}\right]^{3.05}$$

Therefore,

$$N = 12 \text{ stages.}$$

Design Example 5.9: In an axial flow compressor, air enters at a stagnation temperature of 290K and 1 bar. The axial velocity of air is 180 m/s (constant throughout the stage), the absolute velocity at the inlet is 185 m/s, the work done factor is 0.86, and the degree of reaction is 50%. If the stage efficiency is 0.86, calculate the air angles at the rotor inlet and outlet and the static temperature at the inlet of the first stage and stage pressure ratio. Assume a rotor speed of 200 m/s.

Solution:

For 50% degree of reaction at the mean radius (Fig. 5.18), $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the inlet velocity triangle,

$$\cos\alpha_1 = \frac{C_a}{C_1} = \frac{180}{185} = 0.973$$

i.e., $\alpha_1 = 13.35° = \beta_2$

From the same velocity triangle,

$$C_{w1} = \left(C_1^2 - C_a^2\right)^{\frac{1}{2}} = \left(185^2 - 180^2\right)^{\frac{1}{2}} = 42.72 \text{ m/s}$$

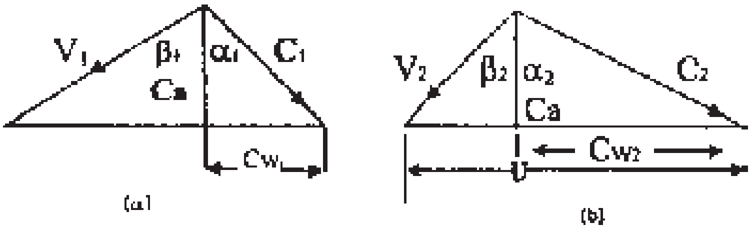


Figure 5.18 Velocity triangles (a) inlet, (b) outlet.

Therefore,

$$\tan\beta_1 = \frac{(U - C_{w1})}{C_a} = \frac{(200 - 42.72)}{180} = 0.874$$

i.e., $\beta_1 = 41.15° = \alpha_2$

Static temperature at stage inlet may be determined by using stagnation and static temperature relationship as given below:

$$T_1 = T_{01} - \frac{C_1}{2C_p} = 290 - \frac{185^2}{2(1005)} = 273\,\text{K}$$

Stagnation temperature rise of the stage is given by

$$\Delta T_{0s} = \frac{\lambda U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$= \frac{0.86(200)(180)}{1005}(0.874 - 0.237) = 19.62\,\text{K}$$

Stage pressure ratio is given by

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/\gamma-1} = \left[1 + \frac{0.86 \times 19.62}{290}\right]^{3.5} = 1.22$$

Illustrative Example 5.10: Find the isentropic efficiency of an axial flow compressor from the following data:

Pressure ratio:	6
Polytropic efficiency:	0.85
Inlet temperature:	285 K

Solution:

Using the isentropic P–T relation for the compression process,

$$T_{02'} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 285(6)^{0.286} = 475.77\,\text{K}$$

Using the polytropic P–T relation for the compression process:

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma\eta_{\infty,c}} = \frac{0.4}{1.4(0.85)} = 0.336$$

Actual temperature rise:

$$T_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n} = 285(6)^{0.336} = 520.36\,\text{K}$$

The compressor isentropic efficiency is given by:

$$\eta_c = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} = \frac{475.77 - 285}{520 - 285} = 0.8105, \quad \text{or} \quad 81.05\%$$

Design Example 5.11: In an axial flow compressor air enters the compressor at 1 bar and 290K. The first stage of the compressor is designed on free vortex principles, with no inlet guide vanes. The rotational speed is 5500 rpm and stagnation temperature rise is 22K. The hub tip ratio is 0.5, the work done factor is 0.92, and the isentropic efficiency of the stage is 0.90. Assuming an inlet velocity of 145 m/s, calculate

1. The tip radius and corresponding rotor air angles, if the Mach number relative to the tip is limited to 0.96.
2. The mass flow at compressor inlet.
3. The stagnation pressure ratio and power required to drive the compressor.
4. The rotor air angles at the root section.

Solution:

(1) As no inlet guide vanes

$$\alpha_1 = 0, C_{w1} = 0$$

Stagnation temperature, T_{01}, is given by

$$T_{01} = T_1 + \frac{C_1}{2C_{p2}}$$

or

$$T_1 = T_{01} - \frac{C_1}{2C_p} = 290 - \frac{145^2}{2 \times 1005} = 288.9\text{K}$$

The Mach number relative to tip is

$$M = \frac{V_1}{\sqrt{\gamma R T_1}}$$

or

$$V_1 = 0.96(1.4 \times 287 \times 288.9)^{0.5} = 340.7 \text{ m/s}$$

i.e., relative velocity at tip = 340.7 m/s

From velocity triangle at inlet (Fig. 5.3)

$$V_1^2 = U_t^2 + C_1^2 \text{ or } U_t = \left(340.7^2 - 145^2\right)^{0.5} = 308.3 \text{ m/s}$$

or tip speed,

$$U_t = \frac{2\pi r_t N}{60}$$

or

$$r_t = \frac{308.3 \times 60}{2\pi \times 5500} = 0.535\text{m}.$$

$$\tan\beta_1 = \frac{U_t}{C_a} = \frac{308.3}{145} = 2.126$$

$$\text{i.e., } \beta_1 = 64.81°$$

Stagnation temperature rise

$$\Delta T_{0s} = \frac{\tau U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

Substituting the values, we get

$$22 = \frac{0.92 \times 308.3 \times 145}{1005}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$\left(\tan\beta_1 - \tan\beta_2\right) = 0.538$$

(2) Therefore, $\tan\beta_2 = 1.588$ and $\beta_2 = 57.8°$

$$\text{root radius/tip radius} = \frac{r_m - h/2}{r_m + h/2} = 0.5$$

(where subscript m for mean and h for height)
or $r_m - h/2 = 0.5\, r_m + 0.25\, h$
$\therefore r_m = 1.5\, h$
but $r_t = r_m + h/2 = 1.5\, h + h/2$

or $0.535 = 2\ h$ or $h = 0.268$ m
and $r_m = 1.5\ h = 0.402$ m
Area, $A = 2\pi r_m h = 2\pi \times\ 0.402 \times 0.268 = 0.677\ \text{m}^2$
Now, using isentropic relationship for p–T

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)} \quad \text{or} \quad p_1 = 1 \times \left(\frac{288.9}{290}\right)^{3.5} = 0.987\ \text{bar}$$

and

$$\rho_1 = \frac{p_1}{RT_1} = \frac{0.987 \times 10^5}{287 \times 288.9} = 1.19\ \text{kg/m}^3$$

Therefore, the mass flow entering the stage

$$\dot{m} = \rho A C_a = 1.19 \times 0.677 \times 145 = 116.8\ \text{kg/s}$$

(3) Stage pressure ratio is

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{0.90 \times 22}{290}\right]^{3.5} = 1.26$$

Now,

$$W = C_p \Delta T_{0s} = 1005 \times 22 = 22110\text{J/kg}$$

Power required by the compressor

$$P = \dot{m}W = 116.8 \times 22110 = 2582.4\ \text{kW}$$

(4) In order to find out rotor air angles at the root section, radius at the root can be found as given below.

$$r_r = r_m - h/2$$
$$= 0.402 - 0.268/2 = 0.267\text{m.}$$

Impeller speed at root is

$$U_r = \frac{2\pi r_r N}{60}$$
$$= \frac{2 \times \pi \times 0.267 \times 5500}{60} = 153.843\ \text{m/s}$$

Therefore, from velocity triangle at root section

$$\tan\beta_1 = \frac{U_r}{C_a} = \frac{153.843}{145} = 1.061$$

i.e., $\beta_1 = 46.9^\circ$

For β_2 at the root section

$$\Delta T_{0s} = \frac{\tau U_r C_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

or

$$22 = \frac{0.92 \times 153.843 \times 145}{1005}(\tan\beta_1 - \tan\beta_2)$$

or

$$(\tan\beta_1 - \tan\beta_2) = 1.078$$

$$\therefore \beta_2 = -0.974^\circ$$

Design Example 5.12: The following design data apply to an axial flow compressor:

Overall pressure ratio:	4.5
Mass flow:	3.5kg/s
Polytropic efficiency:	0.87
Stagnation temperature rise per stage:	22k
Absolute velocity approaching the last rotor:	160m/s
Absolute velocity angle, measured from the axial direction:	20°
Work done factor:	0.85
Mean diameter of the last stage rotor is:	18.5cm
Ambient pressure:	1.0bar
Ambient temperature:	290K

Calculate the number of stages required, pressure ratio of the first and last stages, rotational speed, and the length of the last stage rotor blade at inlet to the stage. Assume equal temperature rise in all stages, and symmetrical velocity diagram.

Solution:

If N is the number of stages, then overall pressure rise is:

$$R = \left[1 + \frac{N \Delta T_{0s}}{T_{01}}\right]^{\frac{n-1}{n}}$$

where

$$\frac{n-1}{n} = \eta_{\alpha c} \frac{\gamma}{\gamma - 1}$$

(where $\eta_{\alpha c}$ is the polytropic efficiency)
substituting values

$$\frac{n-1}{n} = 0.87 \times \frac{1.4}{0.4} = 3.05$$

overall pressure ratio, R is

$$R = \left[1 + \frac{N \times 22}{290}\right]^{3.05}$$

or

$$(4.5)^{\frac{1}{3.05}} = \left[1 + \frac{N \times 22}{290}\right]$$

$$\therefore \quad N = 8.4$$

Hence number of stages = 8

Stagnation temperature rise, ΔT_{0s}, per stage = 22K, as we took 8 stages, therefore

$$\Delta T_{0s} = \frac{22 \times 8.4}{8} = 23.1$$

From velocity triangle

$$\cos a_8 = \frac{C_{a8}}{C_8}$$

or

$$C_{a8} = 160 \times \cos 20 = 150.35\,\text{m/s}$$

Using degree of reaction, $\Lambda = 0.5$

Then,

$$0.5 = \frac{C_{a8}}{2U}\left(\tan\beta_8 + \tan\beta_9\right)$$

or

$$0.5 = \frac{150.35}{2U}\left(\tan\beta_8 + \tan\beta_9\right) \quad \text{(A)}$$

Also,

$$\Delta T_{0s} = \frac{\tau U C_{a8}}{C_p}\left(\tan\beta_8 - \tan\beta_9\right)$$

Now, $\Delta T_{0s} = 22K$ for one stage.

As we took 8 stages, therefore;

$$\Delta T_{0s} = \frac{22 \times 8.4}{8} = 23.1\,\text{K}$$

$$\therefore 23.1 = \frac{0.85 \times \text{U} \times 150.35}{1005}\left(\tan\beta_8 - \tan 20\right) \quad \text{(B)}$$

Because of symmetry, $\alpha_8 = \beta_9 = 20°$

From Eq. (A)

$$U = 150.35\left(\tan\beta_8 + 0.364\right) \quad \text{(C)}$$

From Eq. (B)

$$U = \frac{181.66}{\tan\beta_8 - 0.364} \quad \text{(D)}$$

Comparing Eqs. (C) and (D), we have

$$150.35\left(\tan\beta_8 + 0.364\right) = \frac{181.66}{\left(\tan\beta_8 - 0.364\right)}$$

or

$$\left(\tan^2\beta_8 - 0.364^2\right) = \frac{181.66}{150.35} = 1.21$$

or

$$\tan^2\beta_8 = 1.21 + 0.1325 = 1.342$$

$$\therefore \tan\beta_8 = \sqrt{1.342} = 1.159$$

i.e., $\beta_8 = 49.20^{\circ}$

Substituting in Eq. (C)

$$U = 150.35(\tan 49.20° + 0.364)$$

$$= 228.9\,\text{m/s}$$

The rotational speed is given by

$$N = \frac{228.9}{2\pi \times 0.0925} = 393.69\text{rps}$$

In order to find the length of the last stage rotor blade at inlet to the stage, it is necessary to calculate stagnation temperature and pressure ratio of the last stage.
Stagnation temperature of last stage: Fig. 5.19

$$T_{o8} = T_{01} + 7 \times T_{0s}$$
$$= 290 + 7 \times 23.1 = 451.7\,\text{K}$$

Pressure ratio of the first stage is:

$$R = \left[\frac{1 + 1 \times 23.1}{451.7}\right]^{3.05}$$

Now,

$$p_{08}/p_{09} = 1.1643$$

$$\frac{p_{09}}{p_{01}} = 4, \quad \text{and} \quad p_{09} = 4\text{bar}$$

$$p_{08} = \frac{4}{1.1643} = 3.44\text{bar}$$

and

$$T_{08} = T_8 + \frac{C_8^2}{2C_p}$$

or

$$T_8 = T_{08} - \frac{C_8^2}{2C_p}$$
$$= 451.7 - \frac{160^2}{2 \times 1005}$$
$$= 438.96\,\text{K}$$

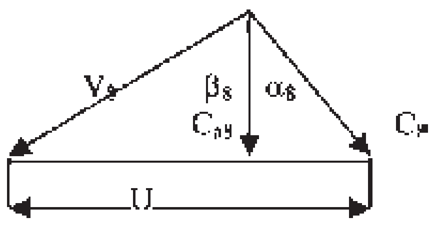


Figure 5.19 Velocity diagram of last stage.

Using stagnation and static isentropic temperature relationship for the last stage, we have

$$\frac{p_8}{p_{08}} = \left(\frac{T_8}{T_{08}}\right)^{1.4/0.4}$$

Therefore,

$$p_8 = 3.44\left(\frac{438.96}{451.7}\right)^{3.5} = 3.112\text{bar}$$

and

$$\rho_8 = \frac{p_8}{RT_8}$$

$$= \frac{3.112 \times 10^5}{287 \times 438.9} = 2.471\ \text{kg/m}^3$$

Using mass flow rate

$$\dot{m} = \rho_8 A_8 C_{a8}$$

or

$$3.5 = 2.471 \times A_8 \times 150.35$$

$$\therefore A_8 = 0.0094m^2$$

$$= 2\pi rh$$

or

$$h = \frac{0.0094}{2\pi \times 0.0925} = 0.0162m$$

i.e., length of the last stage rotor blade at inlet to the stage,
$h = 16.17$ mm.

Design Example 5.13: A 10-stage axial flow compressor is designed for stagnation pressure ratio of 4.5:1. The overall isentropic efficiency of the compressor is 88% and stagnation temperature at inlet is 290K. Assume equal temperature rise in all stages, and work done factor is 0.87. Determine the air angles of a stage at the design radius where the blade speed is 218 m/s. Assume a constant axial velocity of 165 m/s, and the degree of reaction is 76%.

Solution:

No. of stages = 10
The overall stagnation temperature rise is:

$$T_0 = \frac{T_{01}\left(R^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_c} = \frac{290\left(4.5^{0.286} - 1\right)}{0.88}$$

$$= 155.879\ \text{K}$$

The stagnation temperature rise of a stage

$$T_{0s} = \frac{155.879}{10} = 15.588\,\mathrm{K}$$

The stagnation temperature rise in terms of air angles is:

$$T_{0s} = \frac{\tau U C_a}{C_p}(\tan\alpha_2 - \tan\alpha_1)$$

or

$$(\tan\alpha_2 - \tan\alpha_1) = \frac{T_{0s} \times C_p}{\tau U C_a} = \frac{15.588 \times 1005}{0.87 \times 218 \times 165} = 0.501 \qquad \text{(A)}$$

From degree of reaction

$$\wedge = \left[1 - \frac{C_a}{2U}(\tan\alpha_2 + \tan\alpha_1)\right]$$

or

$$0.76 = \left[1 - \frac{165}{2 \times 218}(\tan\alpha_2 + \tan\alpha_1)\right]$$

$$\therefore (\tan\alpha_2 + \tan\alpha_1) = \frac{0.24 \times 2 \times 218}{165} = 0.634 \qquad \text{(B)}$$

Adding (A) and (B), we get

$2\tan\alpha_2 = 1.135$

or $\tan\alpha_2 = 0.5675$

i.e., $\alpha_2 = 29.57°$

and $\tan\alpha_1 = 0.634 - 0.5675 = 0.0665$

i.e., $\alpha_1 = 3.80°$

Similarly, for β_1 and β_2, degree of reaction

$\tan\beta_1 + \tan\beta_2 = 2.01$

and $\tan\beta_1 - \tan\beta_2 = 0.501$

$\therefore 2\tan\beta_1 = 2.511$

i.e., $\beta_1 = 51.46°$

and $\tan\beta_2 = 1.1256 - 0.501 = 0.755$

i.e., $\beta_2 = 37.03°$

Design Example 5.14: An axial flow compressor has a tip diameter of 0.9 m, hub diameter of 0.42 m, work done factor is 0.93, and runs at 5400 rpm. Angles of absolute velocities at inlet and exit are 28 and 58°, respectively and velocity diagram is symmetrical. Assume air density of 1.5 kg/m^3, calculate mass flow rate, work absorbed by the compressor, flow angles and degree of reaction at the hub for a free vortex design.

Solution:

Impeller speed is

$$U = \frac{2\pi rN}{60} = \frac{2\pi \times 0.45 \times 5400}{60} = 254.57 \text{ m/s}$$

From velocity triangle

$$U = C_a(\tan \alpha_1 + \tan \beta_1)$$

$$C_a = \frac{U}{\tan \alpha_1 + \tan \beta_1} = \frac{254.57}{(\tan 28^\circ + \tan 58^\circ)} = 119.47 \text{ m/s}$$

Flow area is

$$A = \pi[r_{tip} - r_{root}]$$
$$= \pi[0.45^2 - 0.42^2] = 0.0833 \text{ m}^2$$

Mass flow rate is

$$\dot{m} = \rho A C_a = 1.5 \times 0.0833 \times 119.47 = 14.928 \text{ kg/s}$$

Power absorbed by the compressor

$$= \tau U(C_{w2} - C_{w1})$$
$$= \tau U C_a(\tan \alpha_2 - \tan \alpha_1)$$
$$= 0.93 \times 254.57 \times 119.47(\tan 58^\circ - \tan 28^\circ)$$
$$= 30213.7 \text{ Nm}$$

$$\text{Total Power, } P = \frac{\dot{m} \times 30213.7}{1000} \text{ kW}$$
$$= 451 \text{ kW}$$

and whirl velocity at impeller tip $C_{wt} = C_a \tan \alpha_1 = 119.47 \times \tan 28^\circ = 63.52$ m/s

Now using free vortex condition
$r\,C_w$ = constant
$\therefore r_h C_{w1h} = r_t C_{w1t}$ (where subscripts h for hub and t for tip)

or

$$C_{w1h} = \frac{r_t C_{w1t}}{r_h} = \frac{0.45 \times 63.52}{0.4} = 71.46\,\text{m/s}$$

Similarly

$$C_{w2t} = C_a \tan\alpha_2 = 119.47 \tan 58° = 191.2\,\text{m/s}$$

and

$$r_h C_{w2h} = r_t C_{w2t}$$

or

$$C_{w2h} = \frac{r_t C_{w2t}}{r_h} = \frac{0.45 \times 191.2}{0.4} = 215.09\,\text{m/s}$$

Therefore, the flow angles at the hub are

$$\tan\alpha_1 = \frac{C_{w1h}}{C_a} \quad \text{(where } C_a \text{ is constant)}$$

$$= \frac{71.46}{119.47} = 0.598$$

i.e., $\alpha_1 = 30.88°$

$$\tan\beta_1 = \frac{U_h - C_a \tan\alpha_1}{C_a}$$

where U_h at the hub is given by

$$U_h = 2\pi r_h N = \frac{2 \times \pi \times 0.4 \times 5400}{60} = 226.29\,\text{m/s}$$

$$\therefore \tan\beta_1 = \frac{226.29 - 119.47 \tan 30.88°}{119.47}$$

i.e., $\beta_1 = 52.34°$

$$\tan\alpha_2 = \frac{C_{w2h}}{C_a} = \frac{215.09}{119.47} = 1.80$$

i.e., $\alpha_2 = 60.95°$
Similarly,

$$\tan\beta_2 = \frac{U_h - C_a \tan\alpha_2}{C_a} = \frac{226.29 - 119.47 \tan 60.95°}{119.47}$$

i.e., $\beta_2 = 5.36°$

Finally, the degree of reaction at the hub is

$$\wedge = \frac{C_a}{2U_h}\left(\tan\beta_1 + \tan\beta_2\right) = \frac{119.47}{2 \times 226.29}(\tan 52.34° + \tan 5.36°)$$

$$= 0.367 \text{ or } 36.7\%.$$

Design Example 5.15: An axial flow compressor is to deliver 22 kg of air per second at a speed of 8000 rpm. The stagnation temperature rise of the first stage is 20 K. The axial velocity is constant at 155 m/s, and work done factor is 0.94. The mean blade speed is 200 m/s, and reaction at the mean radius is 50%. The rotor blade aspect ratio is 3, inlet stagnation temperature and pressure are 290 K and 1.0 bar, respectively. Assume C_p for air as 1005 J/kg K and $\gamma = 1.4$. Determine:

1. The blade and air angles at the mean radius.
2. The mean radius.
3. The blade height.
4. The pitch and chord.

Solution:

1. Using Eq. (5.10) at the mean radius

$$T_{02} - T_{01} = \frac{\tau U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$20 = \frac{0.94 \times 200 \times 155}{1005}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$\left(\tan\beta_1 - \tan\beta_2\right) = 0.6898$$

Using Eq. (5.12), the degree of reaction is

$$\wedge = \frac{C_a}{2U}\left(\tan\beta_1 + \tan\beta_2\right)$$

or

$$\left(\tan\beta_1 + \tan\beta_2\right) = \frac{0.5 \times 2 \times 200}{155} = 1.29$$

Solving above two equations simultaneously

$$2\tan\beta_1 = 1.98$$

$$\therefore \beta_1 = 44.71° = \alpha_2 \text{ (as the diagram is symmetrical)}$$

$$\tan\beta_2 = 1.29 - \tan 44.71°$$

i.e.,

$$\beta_2 = 16.70° = \alpha_1$$

2. Let r_m be the mean radius

$$r_m = \frac{U}{2\pi N} = \frac{200 \times 60}{2\pi \times 8000} = 0.239\text{m}$$

3. Using continuity equation in order to find the annulus area of flow

$$C_1 = \frac{C_a}{\cos\alpha_1} = \frac{155}{\cos 16.70°} = 162\ \text{m/s}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{162^2}{2 \times 1005} = 276.94\ \text{K}$$

Using isentropic relationship at inlet

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Static pressure is

$$p_1 = 1.0\left(\frac{276.94}{290}\right)^{3.5} = 0.851\text{bars}$$

Density is

$$\rho_1 = \frac{p_1}{RT_1} = \frac{0.851 \times 100}{0.287 \times 276.94} = 1.07\ \text{kg/m}^3$$

From the continuity equation,

$$A = \frac{22}{1.07 \times 155} = 0.133\text{m}^2$$

Blade height is

$$h = \frac{A}{2\pi r_m} = \frac{0.133}{2 \times \pi \times 0.239} = 0.089\text{m}.$$

4. At mean radius, and noting that blades β, an equivalent to cascade, α, nominal air deflection is

$$\varepsilon = \beta_1 - \beta_2$$
$$= 44.71° - 16.70° = 28.01°$$

Using Fig. 5.20 for cascade nominal deflection vs. air outlet angle, the solidity,

$$\frac{s}{c} = 0.5$$

$$\text{Blade aspect ratio} = \frac{\text{span}}{\text{chord}}$$

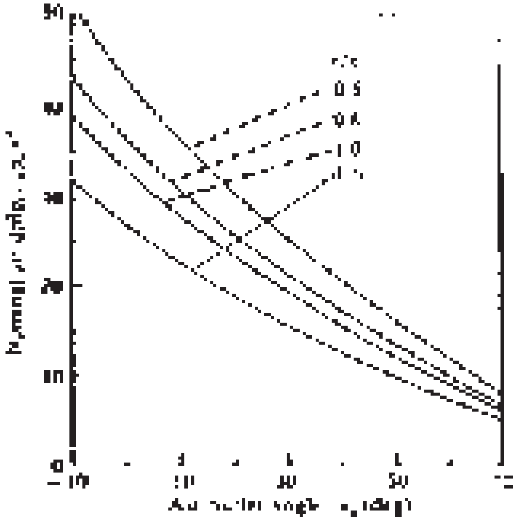


Figure 5.20 Cascade nominal deflection versus air outlet angle.

Blade chord,

$$C = \frac{0.089}{3} = 0.03\text{m}$$

Blade pitch,

$$s = 0.5 \times 0.03 = 0.015\,\text{m}.$$

PROBLEMS

5.1 An axial flow compressor has constant axial velocity throughout the compressor of 152 m/s, a mean blade speed of 162 m/s, and delivers 10.5 kg of air per second at a speed of 10,500 rpm. Each stage is of 50% reaction and the work done factor is 0.92. If the static temperature and pressure at the inlet to the first stage are 288K and 1 bar, respectively, and the stagnation stage temperature rise is 15K, calculate: 1 the mean diameter of the blade row, (2) the blade height, (3) the air exit angle from the rotating blades, and (4) the stagnation pressure ratio of the stage with stage efficiency 0.84.

(0.295 m, 0.062 m, 11.37°, 1.15)

5.2 The following design data apply to an axial flow compressor:

Stagnation temperature rise of the stage:	20 K
Work done factor:	0.90
Blade velocity at root:	155 m/s

Blade velocity at mean radius:	208 m/s
Blade velocity at tip:	255 m/s
Axial velocity (constant through the stage):	155 m/s
Degree of reaction at mean radius:	0.5

Calculate the inlet and outlet air and blade angles at the root, mean radius and tip for a free vortex design.

(18°, 45.5°, 14.84°, 54.07°, 39.71°, 39.18°, 23.56°, 29.42°, 53.75°, −20°)

5.3 Calculate the degree of reaction at the tip and root for the same data as Prob. 5.2.

(66.7%, 10%)

5.4 Calculate the air and blade angles at the root, mean and tip for 50% degree of reaction at all radii for the same data as in Prob. [5.2].

(47.86°, 28.37°, 43.98°, 1.72°)

5.5 Show that for vortex flow,

$$C_w \times r = \text{constant}$$

that is, the whirl velocity component of the flow varies inversely with the radius.

5.6 The inlet and outlet angles of an axial flow compressor rotor are 50 and 15°, respectively. The blades are symmetrical; mean blade speed and axial velocity remain constant throughout the compressor. If the mean blade speed is 200 m/s, work done factor is 0.86, pressure ratio is 4, inlet stagnation temperature is equal to 290 K, and polytropic efficiency of the compressor is 0.85, find the number of stages required.

(8 stages)

5.7 In an axial flow compressor air enters at 1 bar and 15°C. It is compressed through a pressure ratio of four. Find the actual work of compression and temperature at the outlet from the compressor. Take the isentropic efficiency of the compressor to be equal to 0.84

. (167.66 kJ/kg, 454.83 K)

5.8 Determine the number of stages required to drive the compressor for an axial flow compressor having the following data: difference between the tangents of the angles at outlet and inlet, i.e., $\tan \beta_1 - \tan \beta_2 = 0.55$. The isentropic efficiency of the stage is 0.84, the stagnation temperature at the compressor inlet is 288K, stagnation pressure at compressor inlet is 1 bar, the overall stagnation pressure rise is 3.5 bar, and the mass flow rate is 15 kg/s. Assume $C_p = 1.005$ kJ/kg K, $\gamma = 1.4$, $\lambda = 0.86$, $\eta_m = 0.99$

(10 stages, 287.5 kW)

5.9 From the data given below, calculate the power required to drive the compressor and stage air angles for an axial flow compressor.

Stagnation temperature at the inlet:	288 K
Overall pressure ratio:	4
Isentropic efficiency of the compressor:	0.88
Mean blade speed:	170 m/s
Axial velocity:	120 m/s
Degree of reaction:	0.5

(639.4 kW, $\beta_1 = 77.8°$, $\beta_2 = -72.69°$)

5.10 Calculate the number of stages from the data given below for an axial flow compressor:

Air stagnation temperature at the inlet:	288 K
Stage isentropic efficiency:	0.85
Degree of reaction:	0.5
Air angles at rotor inlet:	40°
Air angle at the rotor outlet:	10°
Meanblade speed:	180 m/s
Work done factor:	0.85
Overall pressure ratio:	6

(14 stages)

5.11 Derive the expression for polytropic efficiency of an axial flow compressor in terms of:

(a) n and γ
(b) inlet and exit stagnation temperatures and pressures.

5.12 Sketch the velocity diagrams for an axial flow compressor and derive the expression:

$$\frac{P_{02}}{P_{01}} = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

5.13 Explain the term "degree of reaction". Why is the degree of reaction generally kept at 50%?

5.14 Derive an expression for the degree of reaction and show that for 50% reaction, the blades are symmetrical; i.e., $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

5.16 What is vortex theory? Derive an expression for vortex flow.

5.17 What is an airfoil? Define, with a sketch, the various terms used in airfoil geometry.

NOTATION

C	absolute velocity
C_L	lift coefficient
C_p	specific heat at constant pressure
D	drag
F_x	tangential force on moving blade
h	blade height, specific enthalpy
L	lift
N	number of stage, rpm
n	number of blades
R	overall pressure ratio, gas constant
R_s	stage pressure ratio
U	tangential velocity
V	relative velocity
α	angle with absolute velocity, measured from the axial direction
α_2^*	nominal air outlet angle
β	angle with relative velocity, measure from the axial direction
ΔT_A	static temperature rise in the rotor
ΔT_B	static temperature rise in the stator
ΔT_{0s}	stagnation temperature rise
ΔT_s	static temperature rise
Δ^*	nominal deviation
ϵ^*	nominal deflection
ϵ_s	stalling deflection
φ	flow coefficient
Λ	degree of reaction
λ	work done factor
ψ	stage loading factor

SUFFIXES

1	inlet to rotor
2	outlet from the rotor
3	inlet to second stage
a	axial, ambient
m	mean
r	radial, root
t	tip
w	whirl

3. Combustion Chambers

Requirements: ① A g.t c.c. is required to release the chemical energy of the fuel in the smallest chamber volume and with minimum loss of total pressure.

② The gases leaving the c.c must have uniform velocity & temperature

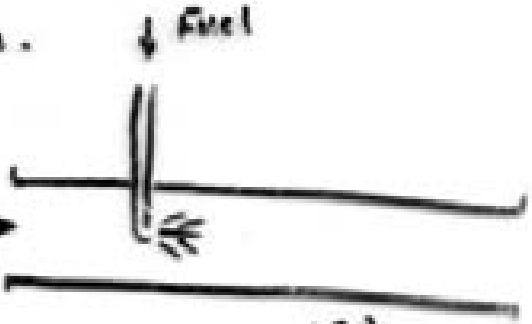
③ The c. process must be efficient.

④ must be reliable & long life.

⑤ re-light capability.

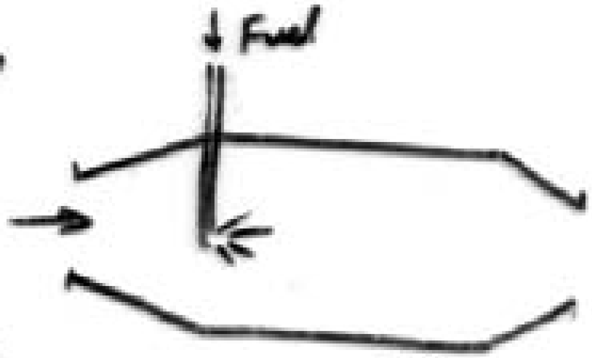
⑥ must have a wide range of operation.

a) plant type / not practical due to the high speed (160 m/s) at compressor exit ⇒ blow off ⇒ pressure loss = 25%



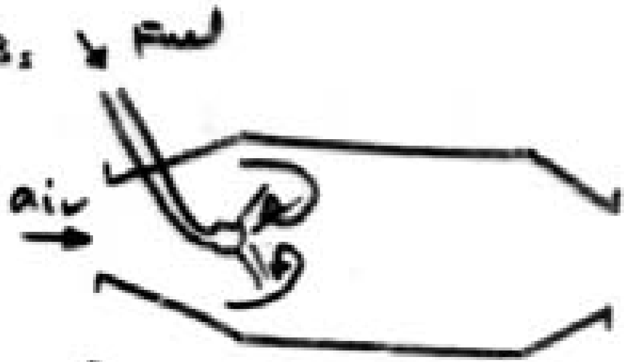
(a)

b) a diffuser is fitted ahead of the c.c. ⇒ Velocity still too high for flame stabilization.



(b)

c) A v-gutter is fitted to provide circulation, this is good for A.Bs (rich mixtures) but not c.cs.



(c)

d) A Flame tube is incorporated in the c.c. to divide the air & unify the temperature distribution & reduce TET.

(d)

5.1.1 Combustion Process

The C. process[es] occur with the vaporized fuel & air mixed on a molecular scale. The rate of this reaction depends on the static P & T in a very complex way. For many situations, the reaction rate can be approximated by a form of the Arrhenius equation, written for the mass rate of reaction as:-

$$\text{Reaction Rate} \propto P^{n} f(T) \exp(-E/RT) \quad \ldots 5.1$$

where n = no. of molecules involved in a reacting collision, $f(T)$ is a function that relates the reaction rate to forms of energy (translation, rotation & vibration).

E.. activation energy, R.. universal gas constant.

In most combustors, the fuel is injected as an atomized liquid droplet spray into the hot reaction zone where it mixes with air & hot combustion gases. The atomized fuel vaporizes & mixes with air. If P & T are high enough, the reaction rate will be fast & the fuel vapor will react as it comes in contact with sufficient oxygen.

The stoichiometric f/A ratio for the typical hydrocarbon fuel can be estimated by assuming octane as a representative hydrocarbon & writing the stoichiometric ~~reaction~~ chemical reaction:-

$$2C_8H_{18} + 25\left(O_2 + \frac{79}{21} N_2\right) \rightarrow 16CO_2 + 18H_2O + 25\left(\frac{79}{21}\right) N_2$$

For this reaction, the stoichiometric F/A is

$$f_{st} = \frac{2(96+18)}{25\left(32 + (79/21)28\right)} = 0.0664$$

The equivalence ratio (ϕ) is defined as the actual F/A divided by [redacted] f_{st}.

$$\phi = f / f_{st} \quad \ldots 5.2$$

$\phi > 1$ for rich fuel/air & $\phi < 1$ for lean f/a mixture. To protect the C.C & afterburner components from excessive temp.s, $\phi < 1.0$.

As an example, an engine being flown at $M = 0.9$/ 10 km & full throttle with $\pi_c = 20$, $\eta_c = 85\%$ will have a comp outlet temp of 780 K, if TET is limited to 2000 K, lower heating value h_{PR} is 41867.37 kJ/kg & C_{pc} & C_{pt} are 0.996 & 1.235 kJ/kg.K, then the (f) for 100% efficient combustion in the C.C. is :-

$$f = \frac{\dot m_f}{\dot m_{3a}} = \frac{\text{burner fuel flow}}{\text{" inlet air flow}} \quad \ldots 5.3$$

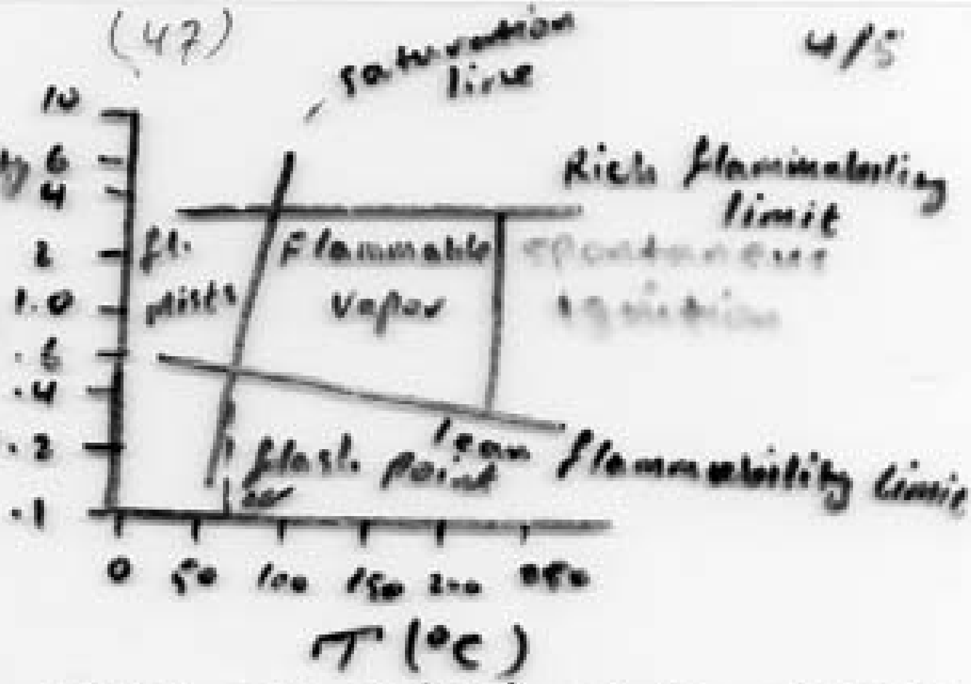
&

$$\eta_b = \frac{\dot m_4 C_{pt} T_{t4} - \dot m_{3a} C_{pc} T_{t3a}}{\dot m_f h_{PR}} \quad \ldots 5.4$$

$$\therefore f = \frac{C_{pt} T_{t4} - C_{pc} T_{t3a}}{\eta_b h_{PR}} = \frac{1.235 \times 2000 - 0.996 \times 780}{41867.37}$$

$f = 0.04$ which corresponds to ϕ of 0.56 for C.C using JP-4 fuel ($f_{st} = 0.0667$)

Fig 5.1 Flammability characteristics for a kerosene-Type fuel in air at atmospheric pressure.



The 0.56 φ of the above example is at the lower limit of the flammability shown in the fig 5.1. This presents a design problem at the full throttle value of φ & at the partial throttle value of φ. The problem of lean mixtures in a burner can be overcome by mixing & burning a rich (f/A) ratio in a small region where the local φ ~ 1.0. By using only a portion of the total air in a region, a locally rich mixture can be efficiently burnt & then the products of combustion diluted & cooled to an acceptable TET by the remaining air.

9.1.2 Ignition

Ignition of fuel/air mixture in a turbine engine combustion system requires inlet air & fuel conditions within flammability limits, sufficient residence time of a combustion mixture & the location of an effective ignition source in the vicinity of the combustion mixture.

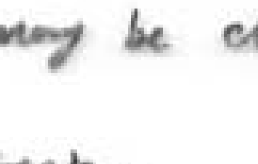
5.2 Types of combustion Chambers

Turbine engine burners have undergone continuing developpment over the past 40 years resulting in a variety of basic combustor configurations. However they may be classified into one of three types:

5.2.1 Can System

A can system consists of one or more cylindrical burners each contained in a burner case. Because of its modular design the can system was used during the early development of the turbojet engine.

CANNULAR



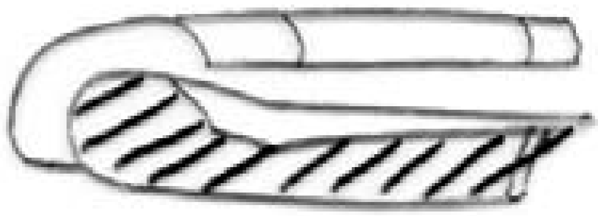
CAN

5.2.2. Cannular System: This system consists of a series of cylindrical burners arranged within a common annulus ... hence the cannular name. This type of burners was the most common in the A/c T. engine population. but has been replaced with the annular type in most modern engines.

5.2.3 Annular System

Most modern M.B. systems employ the annular design where in a single burner having an annular X-section supplies gas to the turbine. The improved combustion zone uniformity, design simplicity, reduced liner surface area & shorter system length provided by the common engine annulus has made the annular burner the leading contender for all future propulsion systems.

Annular

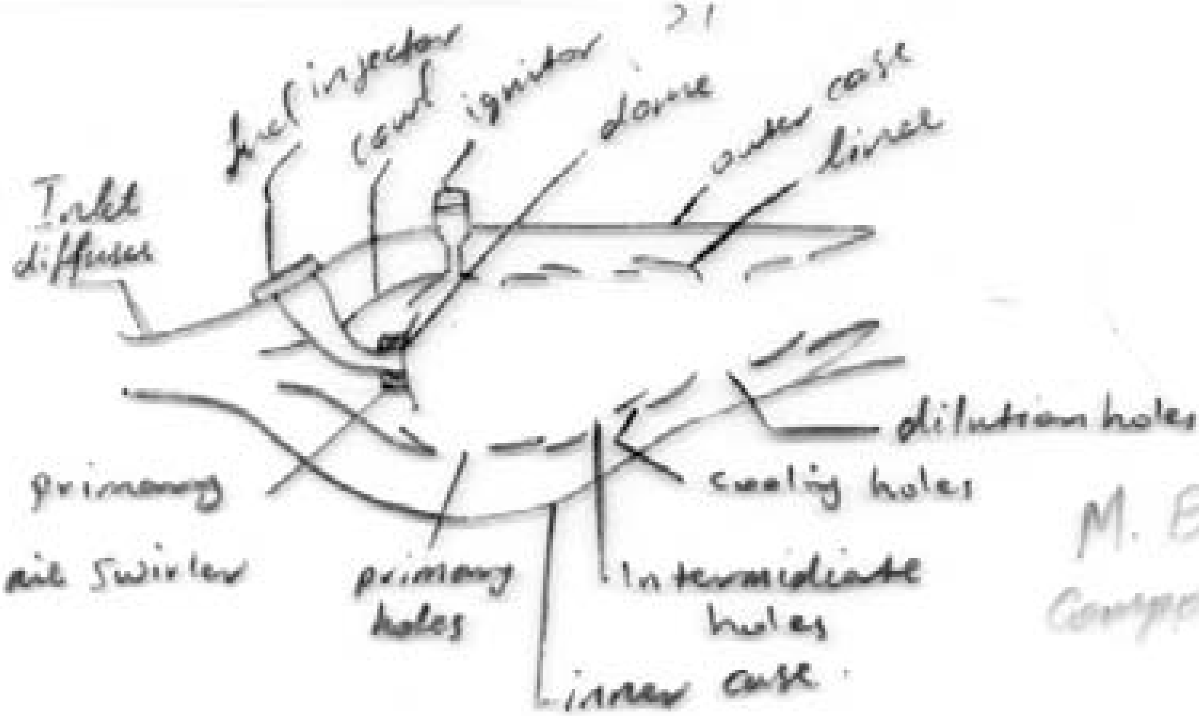


Main Burner Components:-

The turbine engine M.B system consists of three principle elements .. the inlet diffuser, dome & cowl and the liner.

In addition, important subcomponents are necessary the fuel injector, the igniter, the burner case & the primary air swirler if used. The term comb. zone is used to designate that portion of the M.B. within the dome & liner. The purpose of the inlet diffuser is to reduce the velocity of the air exiting the comp. & deliver it to the comb. zone as a stable uniform flowfield while recovering as much of the dynamic pres. as possible. The inlet diffuser represents a design & performance compromise relative to required compactness, low press. loss & good flow uniformity. The cowl devides the incoming air into two streams, primary air & the other air flows (intermediate, dilution & cooling air). The cowl streamlines the combustor dome & permits a large diffuser divergence angle & reduced overall diffuser length. The combustor dome is designed to produce an area of high turbulance & flow shear in the visinity of the fuel nozzle to finely atomise the fuel spray & promote rapid fuel-air mixing. There are two basic types of comb. domes: bluff body & swirl atomised. The bluff-body domes were used in the early main burners but swirl atomised domes are used in most modern A.Bs.

The combustion process is contained in the liner. The liner also allows introduction of intermidiate & dilution air flow & the liner cooling air flow. The liner must be designed to support forces resulting from press. drop & must have high thermal resistance capable of continuous & cyclic high-temp operation. This requires use of high strength, high temp, oxidation resistant materials & cooling air.



M.B Components

5.3 Airflow distribution & Cooling Air

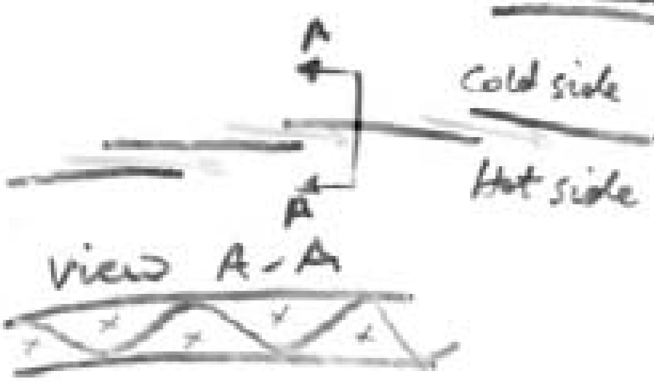
The effective control of air distribution is vital to the attainment of complete combustion, stable operation, correct burner exit temp. profile & acceptable liner temp. for long life.

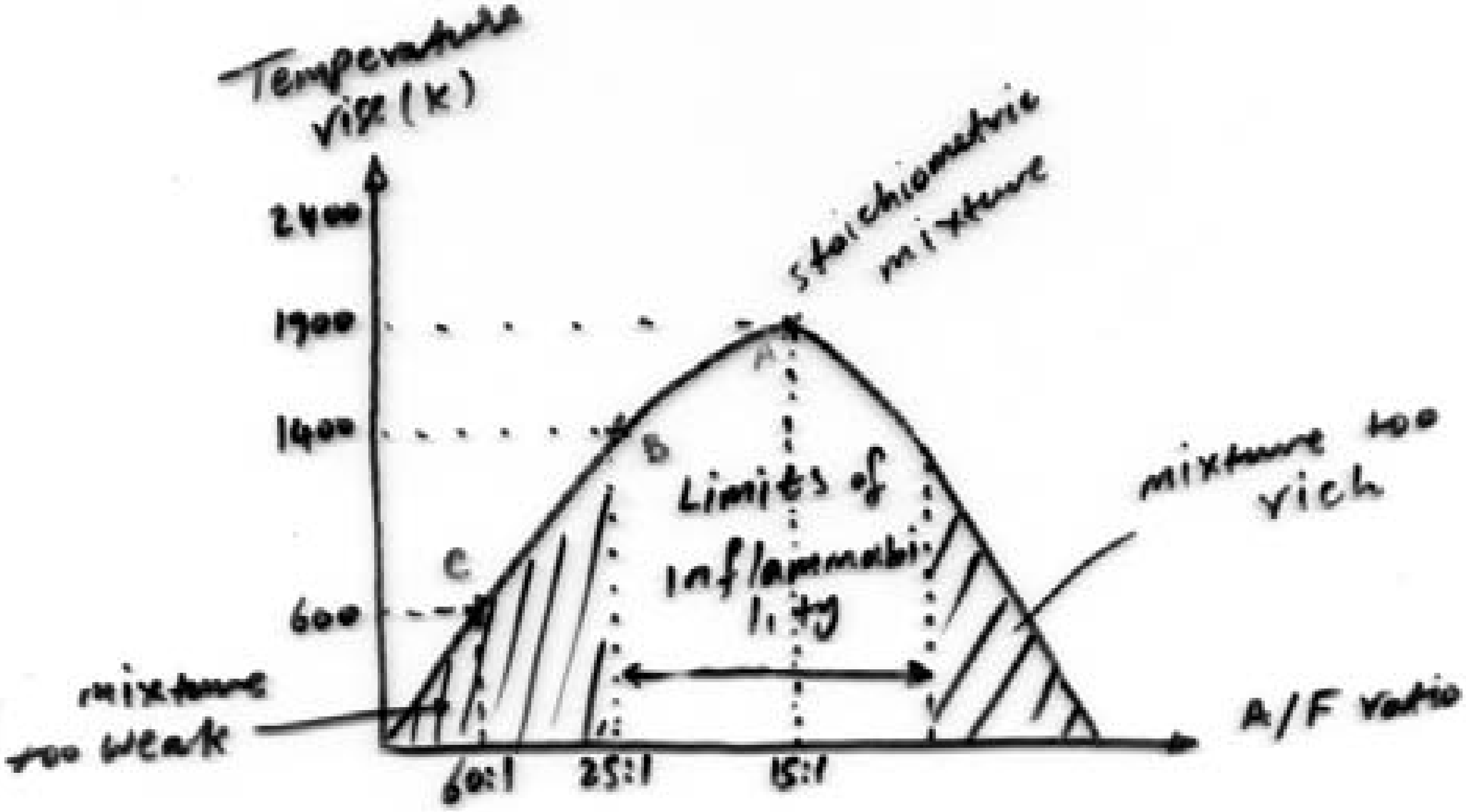
- Primary air is the combustion air introduced through the dome of the burner & through the 1st row of liner air holes. This air mixes with the incoming fuel producing the locally near stoichiometric mixture necessary for optimum flame stabilization & operation.

- To complete the reaction process & consume the high levels of primary zone CO, H & unburnt fuel, the intermediate air is introduced through a 2nd row of liner holes.

The dilution air is then introduced at the rear of the burner to reduce the high temp of the comb. gases. The dilution air is used to carefully tailor exit temp. radial & circumferential profiles to ensure acceptable turbine durability & performance. This requires min. temps at the root (where stresses are highest), and at tips to protect seal materials.

Cooling air must be used to protect the liner & dome from the high ~~temps~~. radiative & convective heat loads produced within the burner. This air is normally introduced through the liner, such that a protective blanket or film of air is formed between the comb. gases & the liner hardware



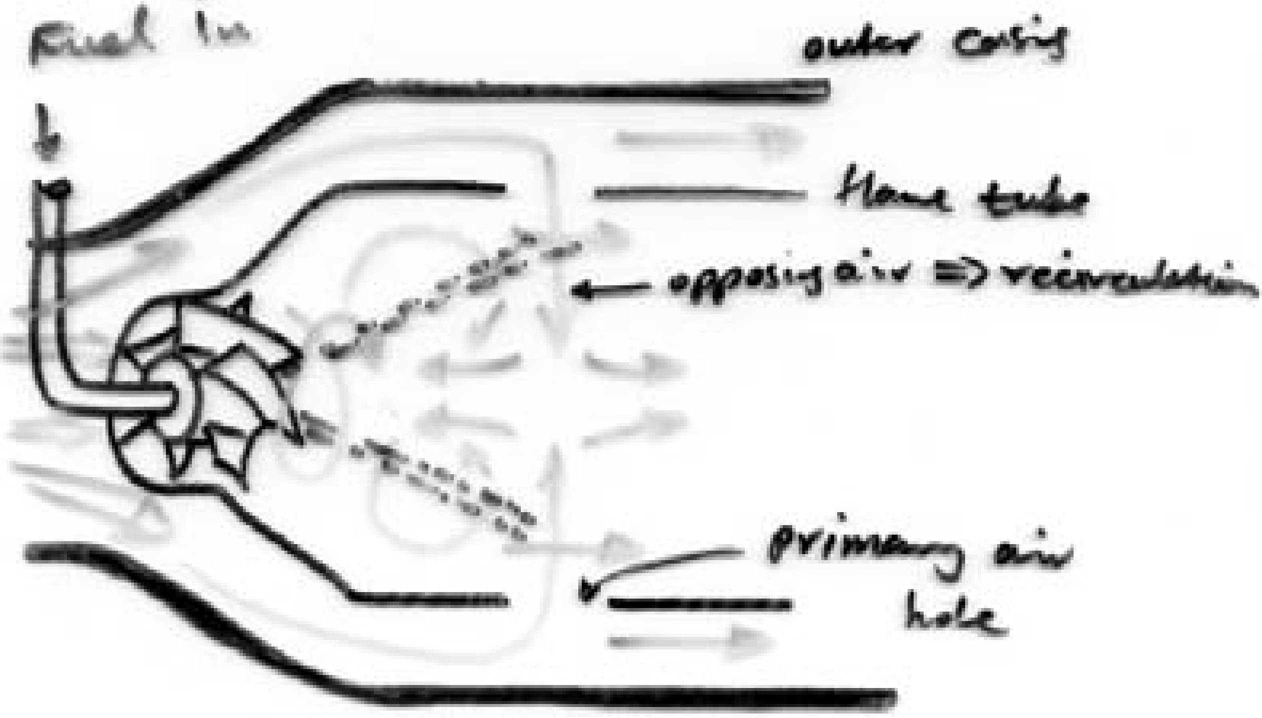


Limits of combustion for kerosine

Temp rise for stoichiometric mixture (kerosine) is approximately 1900 K, if the compressor exit is 600 K ⇒ the flame temp would be 2500 K, for high π_c engines, not only the compressor exit temp is high but also the temp rise as the combustion η is high ($\eta_c \propto \pi_c$): This means that the TET would be too high to be acceptable (TET should be ~ 1600 K). In practice Temp rise required is 600 K — 1000 K which falls in the weak region (out side the flammability range).

The solution to this problem is to devide the air into roughly 1 to 3, i.e 25% of the total air is burnt stoichiometrically in the primary zone ⇒ to give the highest stability, η & Temp rise.

The remaining 75% of the air flow is used in the secondary & Tertiary zones to complete the process & cool the products.



primary zone:

point A

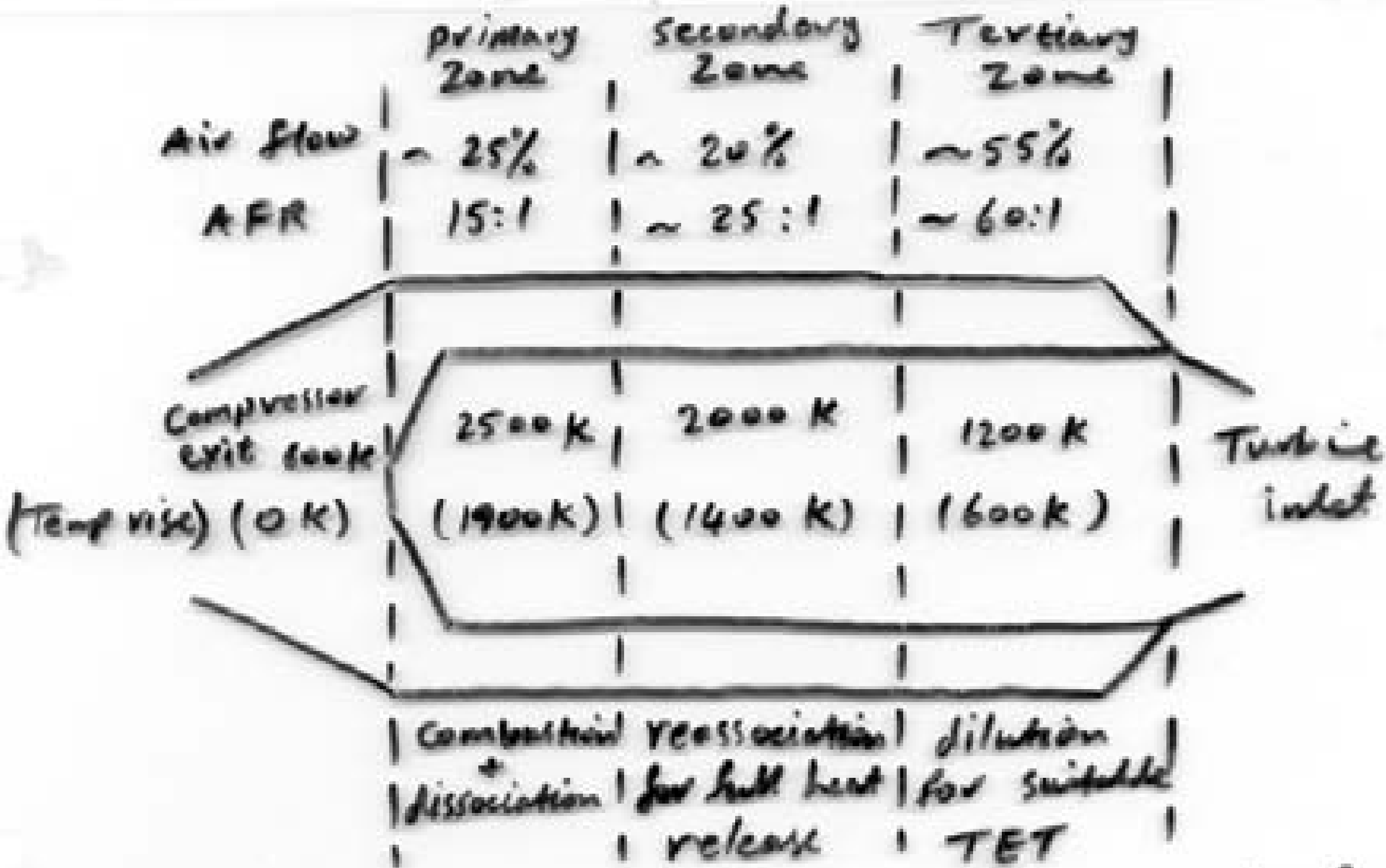
The air flow is devided into two halves 50% of the 75% is passed through the swirler to mix with the fuel this air has a helical motion. The other 50% passes through the primary holes these opposing air flows form a recirculating vortex. The fuel is sprayed as a conical jet. Once ignited the flame is stabilized by turbulent mixing of air.

Secondary zone: The combustion process is completed in the secondary zone. A → B

Tertiary zone: In this zone, rapid dilution and cooling occurs this reduces TET to an acceptable level.

B → C

(5U)



$CO_2 \quad CO + \frac{1}{2}O_2$ $\longrightarrow$ dissociation at high temp.

$H_2O \quad H_2 + \frac{1}{2}O_2$ $\longleftarrow$ reassociation at reduced temp.

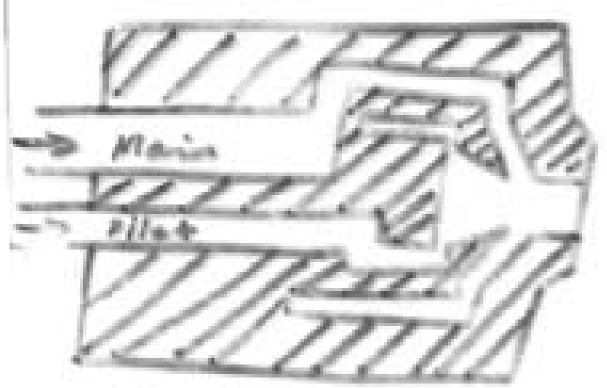
$H_2O \quad OH + \frac{1}{2}H_2$

5-4 Fuel injection

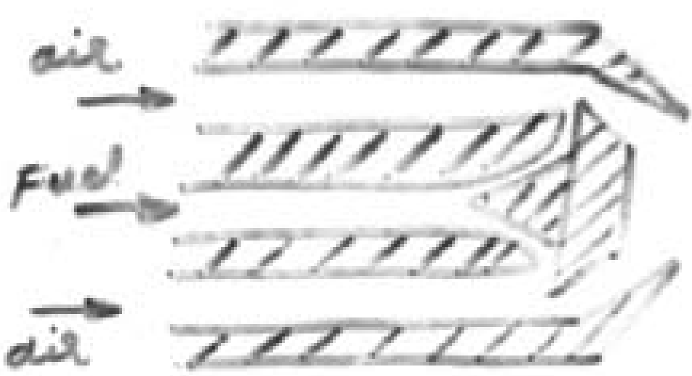
Fuel injectors can be classified into four basic types according to the injection method utilized: Pressure atomizing, airblast vaporizing & premix/prevaporizing.

1- Pressure atomizers: provide large flow range with excellent fuel atomization when system pressures are high (500 psi above M.B. pressure)
types: plain orifice atomizers, Simplex atomizers, Duplex atomizers & Dual orifice atomizers

2- Twin fluid atomizers:
The basic drawback of the simplex press. atomizers is that if the swirl ports are sized to pass the max. fuel flow at max. fuel injection press. then the fuel press. is too low to give good atomization at the lowest fuel flow. to overcome this problem an other method is used to accomplish fuel atomization, that is by exposing a relatively slow moving liquid to a high velocity air stream.

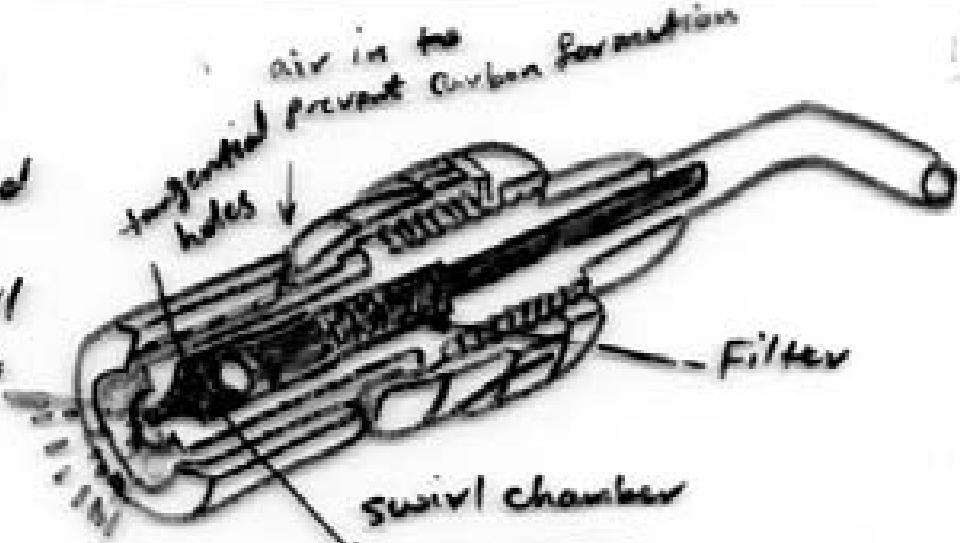


Dual orifice



pre-filming atomizer

Simplex Atomizer



The fuel is forced through tangential holes into the swirl chamber before it sprays out from the orifice

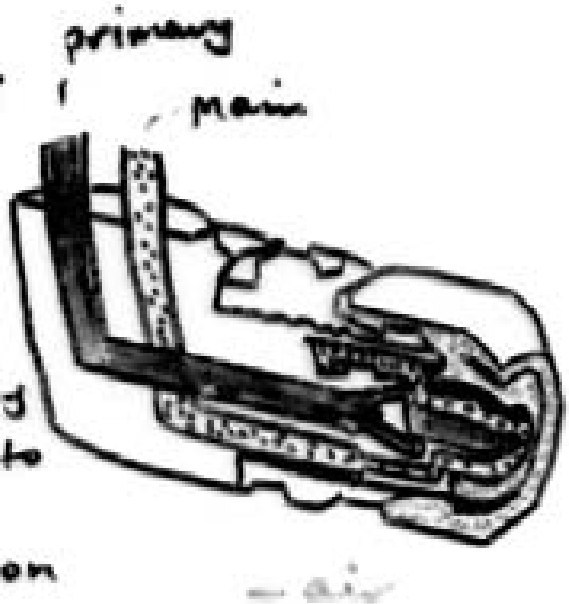
The air blasts the orifice face to prevent carbon formation and assist atomization

Limitation: It can cope with limited range of fuel flows as the fuel flow rate is determined by the pressure drop across the nozzle.

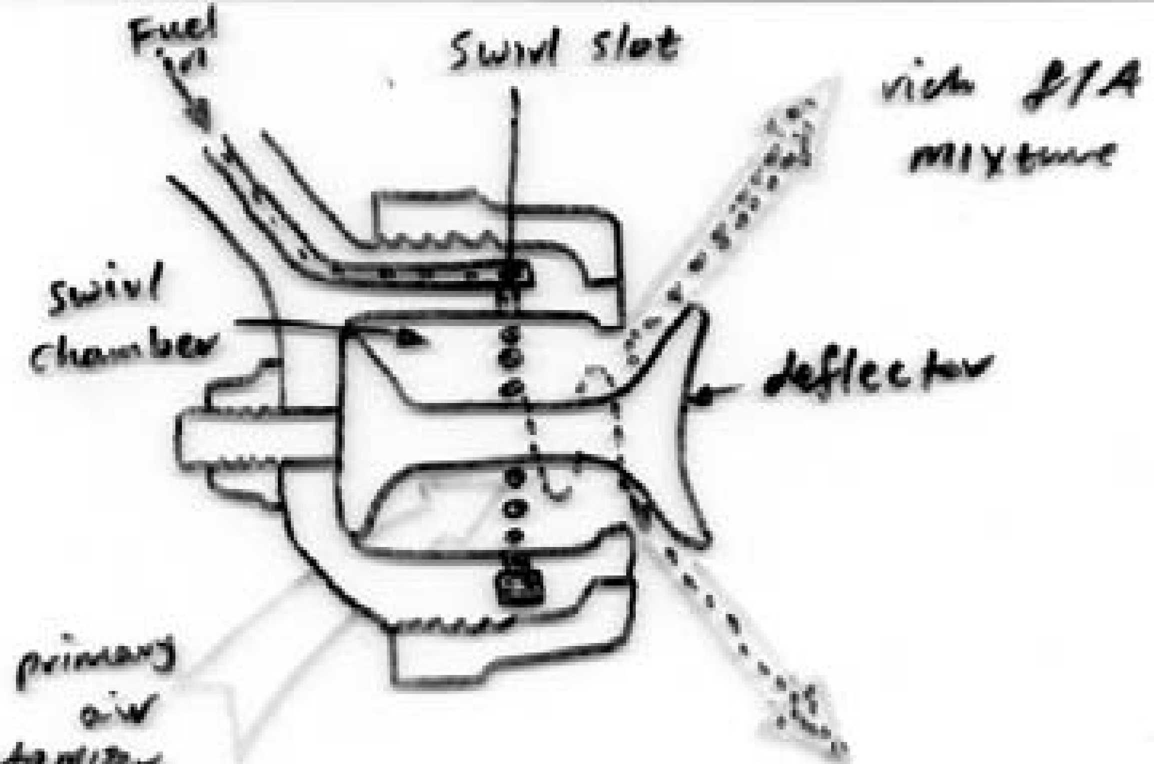
$$\dot{m}_f = KA\sqrt{Pd}$$

where $\dot{m}_f$ ... fuel flow
K ... constant
A ... Nozzle area
Pd ... Pressure difference across Nozzle.

Duplex Atomisers



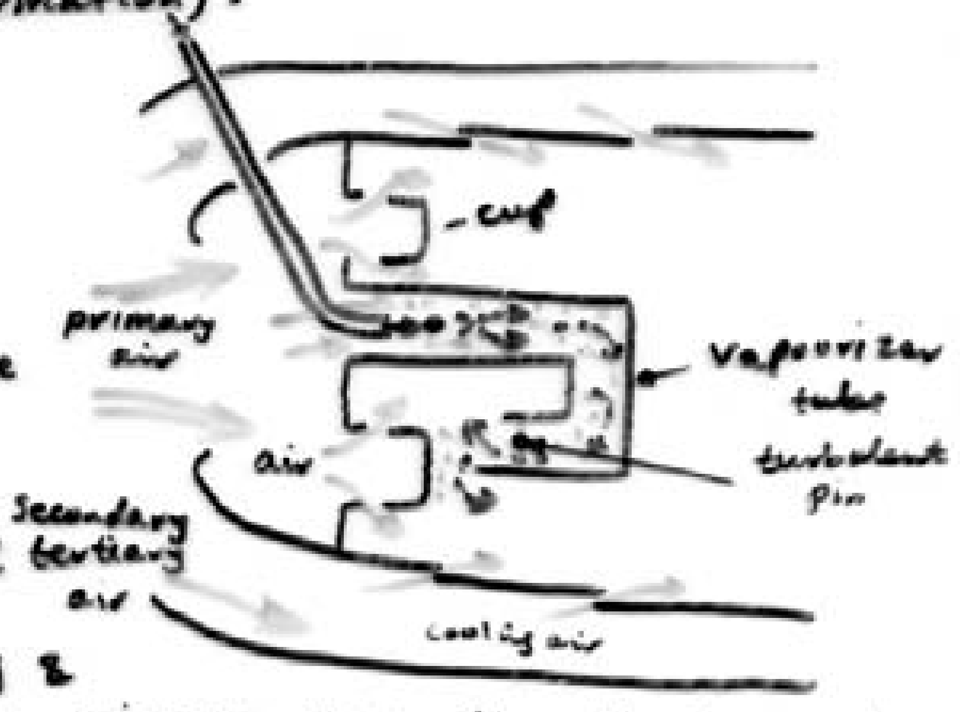
The limited range of the simplex atomizers is solved by using the dublex, which has a primary & main fuel flows with two independant orifices. As the pressure ([illegible]) increases a pressurising valve moves to admit fuel into the main manifold progressively. This type is used on the Avon Grey & Conway engines.



Air spray atomizer

The fuel passes through swirl slots into a swirl chamber, A small proportion of primary air passes into the swirl chamber. The deflector ensures that all the fuel breaks up into an atomised spray - This type works over the full range of operating conditions & (has less smoke formation).

Vapourisers



The fuel is sprayed into a U shaped tube with air to give a rich mixture of approximately 6:1 Turbulence pins are used to ensure mixing & additional air enters the primary zone through cups to give an overall A/f ratio of 15:1. The reverse flow serves to stabilize the flame, as the flame points forward, the C.C is shorter.

Centrifugal Injection Fuel is discharged into the chamber

by an injection wheel through radial holes. The centrifugal force increases the injection pressures from an inlet pressure of about 75 psi up to 5000 psi. This system is simple, cheap but,

a. Torch ignitors are required for starting
b. Sealing problem
c. The holes must be accurately machined for uniform fuel distribution

Engine	Burner type	Burner fuel pressure at max thrust SLS
Spey	Duplex	453 psi
RB 211	Air-spray Atomiser	578 psi
Olympus 593	Vapouriser	600 psi
Viper	Vapouriser	540 psi

7

Axial Flow and Radial Flow Gas Turbines

7.1 INTRODUCTION TO AXIAL FLOW TURBINES

The axial flow gas turbine is used in almost all applications of gas turbine power plant. Development of the axial flow gas turbine was hindered by the need to obtain both a high-enough flow rate and compression ratio from a compressor to maintain the air requirement for the combustion process and subsequent expansion of the exhaust gases. There are two basic types of turbines: the axial flow type and the radial or centrifugal flow type. The axial flow type has been used exclusively in aircraft gas turbine engines to date and will be discussed in detail in this chapter. Axial flow turbines are also normally employed in industrial and shipboard applications. Figure 7.1 shows a rotating assembly of the Rolls-Royce Nene engine, showing a typical single-stage turbine installation. On this particular engine, the single-stage turbine is directly connected to the main and cooling compressors. The axial flow turbine consists of one or more stages located immediately to the rear of the engine combustion chamber. The turbine extracts kinetic energy from the expanding gases as the gases come from the burner, converting this kinetic energy into shaft power to drive the compressor and the engine accessories. The turbines can be classified as (1) impulse and (2) reaction. In the impulse turbine, the gases will be expanded in the nozzle and passed over to the moving blades. The moving blades convert this kinetic energy into mechanical energy and also direct the gas flow to the next stage

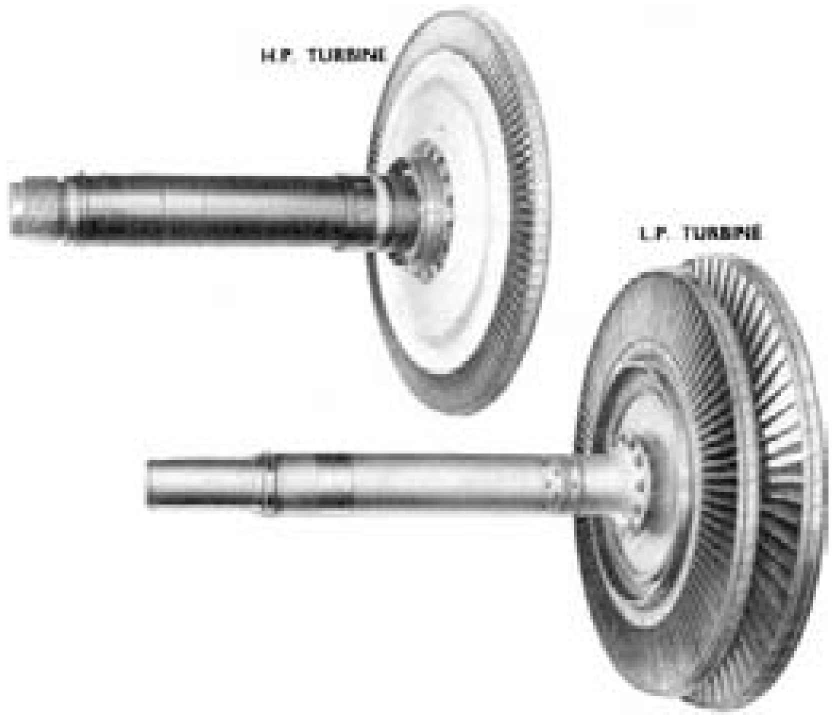


Figure 7.1 Axial flow turbine rotors. (Courtesy Rolls-Royce.)

(multi-stage turbine) or to exit (single-stage turbine). Fig. 7.1 shows the axial flow turbine rotors.

In the case of reaction turbine, pressure drop of expansion takes place in the stator as well as in the rotor-blades. The blade passage area varies continuously to allow for the continued expansion of the gas stream over the rotor-blades. The efficiency of a well-designed turbine is higher than the efficiency of a compressor, and the design process is often much simpler. The main reason for this fact, as discussed in compressor design, is that the fluid undergoes a pressure rise in the compressor. It is much more difficult to arrange for an efficient deceleration of flow than it is to obtain an efficient acceleration. The pressure drop in the turbine is sufficient to keep the boundary layer fluid well behaved, and separation problems, or breakaway of the molecules from the surface, which often can be serious in compressors, can be easily avoided. However, the turbine designer will face much more critical stress problem because the turbine rotors must operate in very high-temperature gases. Since the design principle and concepts of gas turbines are essentially the same as steam turbines, additional information on turbines in general already discussed in Chapter 6 on steam turbines.

7.2 VELOCITY TRIANGLES AND WORK OUTPUT

The velocity diagram at inlet and outlet from the rotor is shown in Fig. 7.2. Gas with an absolute velocity C_1 making an angle α_1, (angle measured from the axial direction) enters the nozzle (in impulse turbine) or stator blades (in reaction turbine). Gas leaves the nozzles or stator blades with an absolute velocity C_2, which makes and an α_2 with axial direction. The rotor-blade inlet angle will be chosen to suit the direction β_2 of the gas velocity V_2 relative to the blade at inlet. β_2 and V_2 are found by subtracting the blade velocity vector U from the absolute velocity C_2.

It is seen that the nozzles accelerate the flow, imparting an increased tangential velocity component. After expansion in the rotor-blade passages, the gas leaves with relative velocity V_3 at angle β_3. The magnitude and direction of the absolute velocity at exit from the rotor C_3 at an angle α_3 are found by vectorial addition of U to the relative velocity V_3. α_3 is known as the swirl angle.

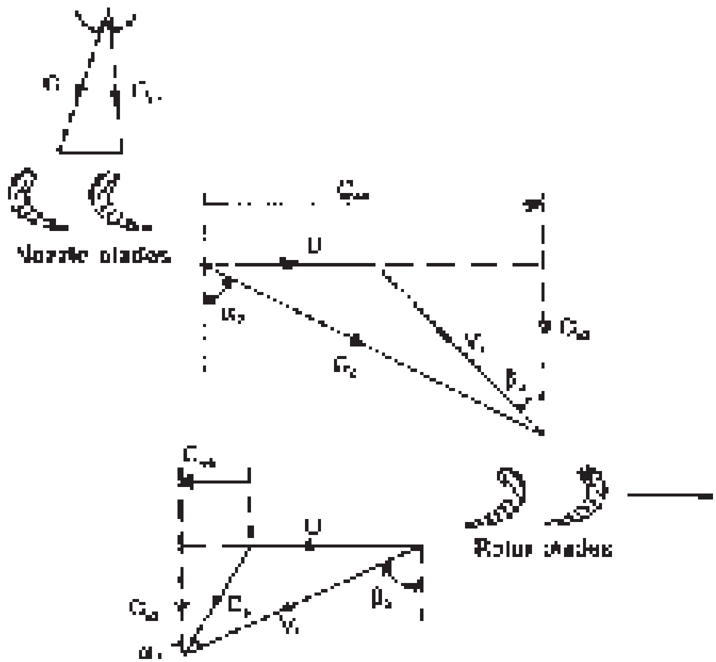


Figure 7.2 Velocity triangles for an axial flow gas turbine.

The gas enters the nozzle with a static pressure p_1 and temperature T_1. After expansion, the gas pressure is p_2 and temperature T_2. The gas leaves the rotor-blade passages at pressure p_3 and temperature T_3. Note that the velocity diagram of the turbine differs from that of the compressor, in that the change in tangential velocity in the rotor, ΔC_w, is in the direction opposite to the blade speed U. The reaction to this change in the tangential momentum of the fluid is a torque on the rotor in the direction of motion. V_3 is either slightly less than V_2 (due to friction) or equal to V_2. But in reaction stage, V_3 will always be greater than V_2 because part of pressure drop will be converted into kinetic energy in the moving blade. The blade speed U increases from root to tip and hence velocity diagrams will be different for root, tip, and other radii points. For short blades, 2-D approach in design is valid but for long blades, 3-D approach in the designing must be considered. We shall assume in this section that we are talking about conditions at the mean diameter of the annulus. Just as with the compressor blading diagram, it is more convenient to construct the velocity diagrams in combined form, as shown in Fig. 7.3. Assuming unit mass flow, work done by the gas is given by

$$W = U(C_{w2} + C_{w3}) \tag{7.1}$$

From velocity triangle

$$\frac{U}{Ca} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3 \tag{7.2}$$

In single-stage turbine, $\alpha_1 = 0$ and $C_1 = Ca_1$. In multi-stage turbine, $\alpha_1 = \alpha_3$ and $C_1 = C_3$ so that the same blade shape can be used. In terms of air angles, the stage

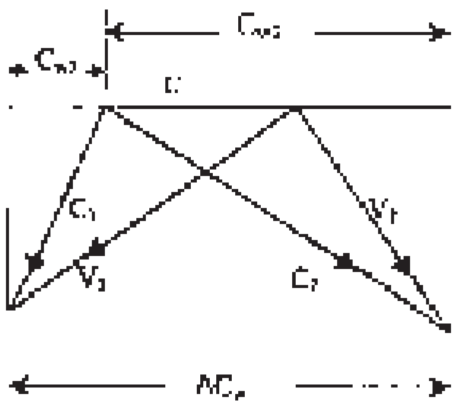


Figure 7.3 Combined velocity diagram.

work output per unit mass flow is given by

$$W = U(C_{w2} + C_{w3}) = UCa(\tan\alpha_2 + \tan\alpha_3) \tag{7.3}$$

or

$$W = UCa\left(\tan\beta_2 + \tan\beta_3\right) \tag{7.4}$$

Work done factor used in the designing of axial flow compressor is not required because in the turbine, flow is accelerating and molecules will not break away from the surface and growth of the boundary layer along the annulus walls is negligible. The stagnation pressure ratio of the stage p_{01}/p_{03} can be found from

$$\Delta T_{0s} = \eta_s T_{01}\left[1 - \left(\frac{1}{p_{01}/p_{03}}\right)^{(\gamma-1)/\gamma}\right] \tag{7.5}$$

where η_s is the isentropic efficiency given by

$$\eta_s = \frac{T_{01} - T_{03}}{T_{01} - T'_{03}} \tag{7.6}$$

The efficiency given by Eq. (7.6) is based on stagnation (or total) temperature, and it is known as total-to-total stage efficiency. Total-to-total stage efficiency term is used when the leaving kinetics energy is utilized either in the next stage of the turbine or in propelling nozzle. If the leaving kinetic energy from the exhaust is wasted, then total-to-static efficiency term is used. Thus total-to-static efficiency,

$$\eta_{ts} = \frac{T_{01} - T_{03}}{T_{01} - T'_3} \tag{7.7}$$

where T'_3 in Eq. (7.7) is the static temperature after an isentropic expansion from p_{01} to p_3.

7.3 DEGREE OF REACTION (Λ)

Degree of reaction is defined as

$$\Lambda = \frac{\text{Enthalpy drop in the moving blades}}{\text{Enthalpy drop in the stage}}$$

$$= \frac{h_2 - h_3}{h_1 - h_3} = \frac{Ca}{2U}\left(\tan\beta_1 - \tan\beta_2\right) \tag{7.8}$$

This shows the fraction of the stage expansion, which occurs in the rotor, and it is usual to define in terms of the static temperature drops, namely

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3} \tag{7.9}$$

Assuming that the axial velocity is constant throughout the stage, then

$$Ca_2 = Ca_3 = Ca_1, \quad \text{and} \quad C_3 = C_1$$

From Eq. (7.4)

$$C_p(T_1 - T_3) = C_p(T_{01} - T_{03}) = UCa(\tan\beta_2 + \tan\beta_3) \tag{7.10}$$

Temperature drop across the rotor-blades is equal to the change in relative velocity, that is

$$\begin{aligned} C_p(T_2 - T_3) &= \frac{1}{2}\left(V_3^2 - V_2^2\right) \\ &= \frac{1}{2}Ca^2\left(\sec^2\beta_3 - \sec^2\beta_2\right) \\ &= \frac{1}{2}Ca^2\left(\tan^2\beta_3 - \tan^2\beta_2\right) \end{aligned}$$

Thus

$$\Lambda = \frac{Ca}{2U}(\tan\beta_3 - \tan\beta_2) \tag{7.11}$$

7.4 BLADE-LOADING COEFFICIENT

The blade-loading coefficient is used to express work capacity of the stage. It is defined as the ratio of the specific work of the stage to the square of the blade velocity—that is, the blade-loading coefficient or temperature-drop coefficient ψ is given by

$$\psi = \frac{W}{\frac{1}{2}U^2} = \frac{2C_p\Delta T_{os}}{U^2} = \frac{2Ca}{U}(\tan\beta_2 + \tan\beta_3) \tag{7.12}$$

Flow Coefficient (ϕ)

The flow coefficient, ϕ, is defined as the ratio of the inlet velocity Ca to the blade velocity U, i.e.,

$$\phi = \frac{Ca}{U} \tag{7.13}$$

This parameter plays the same part as the blade-speed ratio U/C_1 used in the design of steam turbine. The two parameters, ψ and ϕ, are dimensionless and useful to plot the design charts. The gas angles in terms of ψ, Λ, and ϕ can be obtained easily as given below:

Eqs. (7.11) and (7.12) can be written as

$$\psi = 2\phi(\tan\beta_2 + \tan\beta_3) \tag{7.14}$$

$$\Lambda = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2) \tag{7.15}$$

Now, we may express gas angles β_2 and β_3 in terms of ψ, Λ, and ϕ as follows:

Adding and subtracting Eqs. (7.14) and (7.15), we get

$$\tan\beta_3 = \frac{1}{2\phi}\left(\frac{1}{2}\psi + 2\Lambda\right) \tag{7.16}$$

$$\tan\beta_2 = \frac{1}{2\phi}\left(\frac{1}{2}\psi - 2\Lambda\right) \tag{7.17}$$

Using Eq. (7.2)

$$\tan\alpha_3 = \tan\beta_3 - \frac{1}{\phi} \tag{7.18}$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\phi} \tag{7.19}$$

It has been discussed in Chapter 6 that steam turbines are usually impulse or a mixture of impulse and reaction stages but the turbine for a gas-turbine power plant is a reaction type. In the case of steam turbine, pressure ratio can be of the order of 1000:1 but for a gas turbine it is in the region of 10:1. Now it is clear that a very long steam turbine with many reaction stages would be required to reduce the pressure by a ratio of 1000:1. Therefore the reaction stages are used where pressure drop per stage is low and also where the overall pressure ratio of the turbine is low, especially in the case of aircraft engine, which may have only three or four reaction stages.

Let us consider 50% reaction at mean radius. Substituting $\Lambda = 0.5$ in Eq. (7.11), we have

$$\frac{1}{\phi} = \tan\beta_3 - \tan\beta_2 \tag{7.20}$$

Comparing this with Eq. (7.2), $\beta_3 = \alpha_2$ and $\beta_2 = \alpha_3$, and hence the velocity diagram becomes symmetrical. Now considering $C_1 = C_3$, we have $\alpha_1 = \alpha_3 = \beta_2$, and the stator and rotor-blades then have the same inlet and outlet angles. Finally, for $\Lambda = 0.5$, we can prove that

$$\psi = 4\phi\tan\beta_3 - 2 = 4\phi\tan\alpha_2 - 2 \tag{7.21}$$

and

$$\psi = 4\phi\tan\beta_2 + 2 = 4\phi\tan\alpha_3 + 2 \tag{7.22}$$

and hence all the gas angles can be obtained in terms of ψ and ϕ.

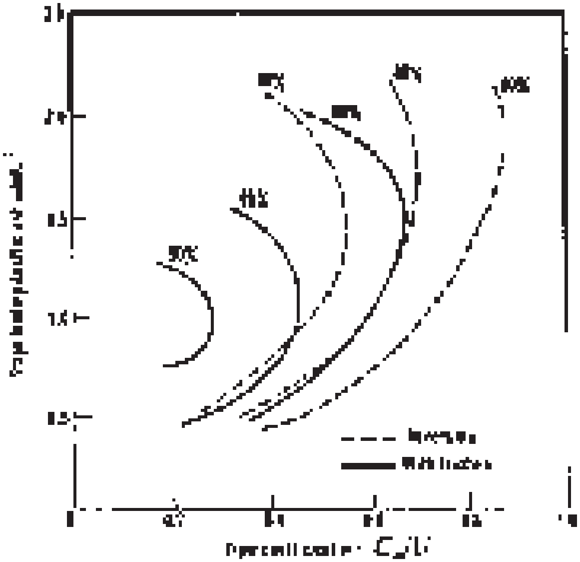


Figure 7.4 Total-to-static efficiency of a 50% reaction axial flow turbine stage.

The low values of ϕ and ψ imply low gas velocities and hence reduced friction losses. But a low value of ψ means more stages for a given overall turbine output, and low ϕ means larger turbine annulus area for a given mass flow. In industrial gas turbine plants, where low sfc is required, a large diameter, relatively long turbine, of low flow coefficient and low blade loading, would be accepted. However, for the gas turbine used in an aircraft engine, the primary consideration is to have minimum weight, and a small frontal area. Therefore it is necessary to use higher values of ψ and ϕ but at the expense of efficiency (see Fig. 7.4).

7.5 STATOR (NOZZLE) AND ROTOR LOSSES

A T–s diagram showing the change of state through a complete turbine stage, including the effects of irreversibility, is given in Fig. 7.5.

In Fig. 7.5, $T_{02} = T_{01}$ because no work is done in the nozzle, $(p_{01} - p_{02})$ represents the pressure drop due to friction in the nozzle. $(T_{01} - T_2^{'})$ represents the ideal expansion in the nozzle, T_2 is the temperature at the nozzle exit due to friction. Temperature, T_2 at the nozzle exit is higher than $T_2^{'}$. The nozzle loss coefficient, λ_N, in terms of temperature may be defined as

$$\lambda_N = \frac{T_2 - T_2^{'}}{C_2^2/2C_p} \tag{7.23}$$

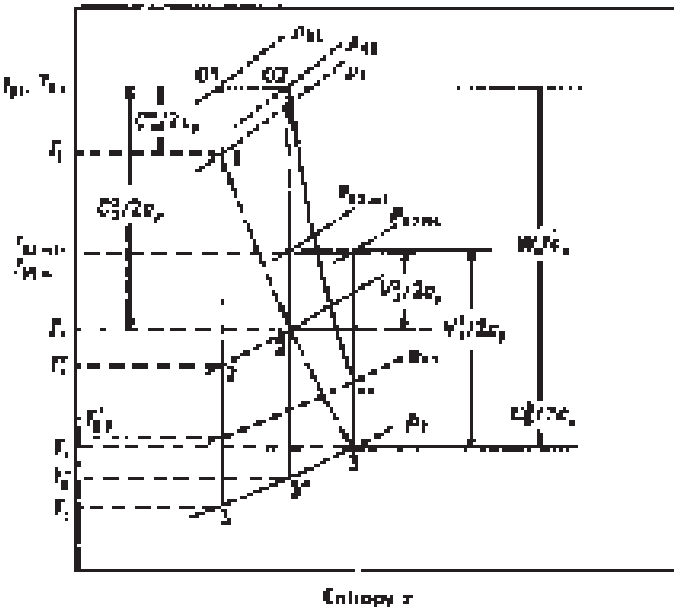


Figure 7.5 *T–s* diagram for a reaction stage.

Nozzle loss coefficient in term of pressure

$$y_N = \frac{p_{01} - p_{02}}{p_{01} - p_2} \tag{7.24}$$

λ_N and y_N are not very different numerically. From Fig. 7.5, further expansion in the rotor-blade passages reduces the pressure to p_3. $T_3^{'}$ is the final temperature after isentropic expansion in the whole stage, and $T_3^{''}$ is the temperature after expansion in the rotor-blade passages alone. Temperature T_3 represents the temperature due to friction in the rotor-blade passages. The rotor-blade loss can be expressed by

$$\lambda_R = \frac{T_3 - T_3^{''}}{V_3^2/2C_p} \tag{7.25}$$

As we know that no work is done by the gas relative to the blades, that is, $T_{03rel} = T_{02rel}$. The loss coefficient in terms of pressure drop for the rotor-blades is defined by

$$\lambda_R = \frac{p_{02\,rel} - p_{03\,rel}}{p_{03\,rel} - p_3} \tag{7.26}$$

The loss coefficient in the stator and rotor represents the percentage drop of energy due to friction in the blades, which results in a total pressure and static enthalpy drop across the blades. These losses are of the order of 10–15% but can be lower for very low values of flow coefficient.

Nozzle loss coefficients obtained from a large number of turbine tests are typically 0.09 and 0.05 for the rotor and stator rows, respectively. Figure 7.4 shows the effect of blade losses, determined with Soderberg's correlation, on the total-to-total efficiency of turbine stage for the constant reaction of 50%. It is the evident that exit losses become increasingly dominant as the flow coefficient is increased.

7.6 FREE VORTEX DESIGN

As pointed out earlier, velocity triangles vary from root to tip of the blade because the blade speed U is not constant and varies from root to tip. Twisted blading designed to take account of the changing gas angles is called vortex blading. As discussed in axial flow compressor (Chapter 5) the momentum equation is

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{C_\mathrm{w}^2}{r} \tag{7.27}$$

For constant enthalpy and entropy, the equation takes the form

$$\frac{\mathrm{d}h_0}{\mathrm{d}r} = Ca\frac{\mathrm{d}Ca}{\mathrm{d}r} + C_\mathrm{w}\frac{\mathrm{d}C_\mathrm{w}}{\mathrm{d}r} + \frac{C_\mathrm{w}^2}{r} \tag{7.28}$$

For constant stagnation enthalpy across the annulus ($\mathrm{d}h_0/\mathrm{d}r = 0$) and constant axial velocity ($\mathrm{d}Ca/\mathrm{d}r = 0$) then the whirl component of velocity C_w is inversely proportional to the radius and radial equilibrium is satisfied. That is,

$$C_\mathrm{w} \times r = \text{constant} \tag{7.29}$$

The flow, which follows Eq. (7.29), is called a "free vortex."

Now using subscript m to denote condition at mean diameter, the free vortex variation of nozzle angle α_2 may be found as given below:

$$C_{\mathrm{w}2}r = rCa_2 \tan\alpha_2 = \text{constant}$$

$$Ca_2 = \text{constant}$$

Therefore α_2 at any radius r is related to $\alpha_{2\mathrm{m}}$ at the mean radius r_m by

$$\tan\alpha_2 = \left(\frac{r_\mathrm{m}}{r}\right)_2 \tan\alpha_{2\mathrm{m}} \tag{7.30}$$

Similarly, α_3 at outlet is given by

$$\tan\alpha_3 = \left(\frac{r_\mathrm{m}}{r}\right)_3 \tan\alpha_{3\mathrm{m}} \tag{7.31}$$

The gas angles at inlet to the rotor-blade, from velocity triangle,

$$\tan \beta_3 = \tan \alpha_2 - \frac{U}{Ca}$$

$$= \left(\frac{r_m}{r}\right)_2 \tan \alpha_{2m} - \left(\frac{r}{r_m}\right)_2 \frac{U_m}{Ca_2} \tag{7.32}$$

and β_3 is given by

$$\tan \beta_2 = \left(\frac{r_m}{r}\right)_3 \tan \alpha_{3m} + \left(\frac{r}{r_m}\right)_3 \frac{U_m}{Ca_3} \tag{7.33}$$

7.7 CONSTANT NOZZLE ANGLE DESIGN

As before, we assume that the stagnation enthalpy at outlet is constant, that is, $dh_0/dr = 0$. If α_2 is constant, this leads to the axial velocity distribution given by

$$C_{w2} r^{\sin^2 \alpha_2} = \text{constant} \tag{7.34}$$

and since α_2 is constant, then Ca_2 is proportional to C_{w1}. Therefore

$$C_{a2} r^{\sin^2 \alpha_2} = \text{constant} \tag{7.35}$$

Normally the change in vortex design has only a small effect on the performance of the blade while secondary losses may actually increase.

Illustrative Example 7.1 Consider an impulse gas turbine in which gas enters at pressure = 5.2 bar and leaves at 1.03 bar. The turbine inlet temperature is 1000 K and isentropic efficiency of the turbine is 0.88. If mass flow rate of air is 28 kg/s, nozzle angle at outlet is 57°, and absolute velocity of gas at inlet is 140 m/s, determine the gas velocity at nozzle outlet, whirl component at rotor inlet and turbine work output. Take, $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK (see Fig. 7.6).

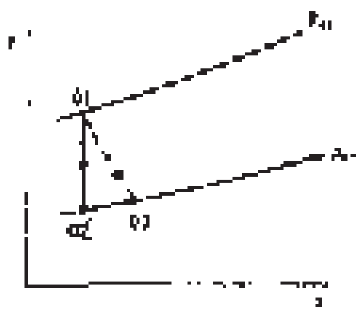


Figure 7.6 *T-s* diagram for Example 7.1.

Solution

From isentropic p–T relation for expansion process

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

or $$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 1000\left(\frac{1.03}{5.2}\right)^{(0.248)} = 669\,\text{K}$$

Using isentropic efficiency of turbine

$$T_{02} = T_{01} - \eta_t\left(T_{01} - T'_{02}\right) = 1000 - 0.88(1000 - 669)$$

$$= 708.72\,\text{K}$$

Using steady-flow energy equation

$$\frac{1}{2}\left(C_2^2 - C_1^2\right) = C_p(T_{01} - T_{02})$$

Therefore, $C_2 = \sqrt{[(2)(1147)(1000 - 708.72) + 19600]} = 829.33\,\text{m/s}$

From velocity triangle, velocity of whirl at rotor inlet

$$C_{w2} = 829.33\ \sin 57° = 695.5\ \ \text{m/s}$$

Turbine work output is given by

$$W_t = mC_{pg}(T_{01} - T_{02}) = (28)(1.147)(1000 - 708.72)$$

$$= 9354.8\ \text{kW}$$

Design Example 7.2 In a single-stage gas turbine, gas enters and leaves in axial direction. The nozzle efflux angle is 68°, the stagnation temperature and stagnation pressure at stage inlet are 800°C and 4 bar, respectively. The exhaust static pressure is 1 bar, total-to-static efficiency is 0.85, and mean blade speed is 480 m/s, determine (1) the work done, (2) the axial velocity which is constant through the stage, (3) the total-to-total efficiency, and (4) the degree of reaction. Assume $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

(1) The specific work output

$$\begin{aligned} W &= C_{pg}(T_{01} - T_{03}) \\ &= \eta_{ts} C_{pg} T_{01}\left[1 - (1/4)^{0.33/1.33}\right] \\ &= (0.85)(1.147)(1073)\left[1 - (0.25)^{0.248}\right] = 304.42\,\text{kJ/kg} \end{aligned}$$

(2) Since $\alpha_1 = 0$, $\alpha_3 = 0$, $C_{w1} = 0$ and specific work output is given by

$$W = UC_{w2} \quad \text{or} \quad C_{w2} = \frac{W}{U} = \frac{304.42 \times 1000}{480} = 634.21 \text{ m/s}$$

From velocity triangle

$$\sin \alpha_2 = \frac{C_{w2}}{C_2}$$

or

$$C_2 = \frac{C_{w2}}{\sin \alpha_2} = \frac{634.21}{\sin 68^\circ} = 684 \text{ m/s}$$

Axial velocity is given by

$$Ca_2 = 684 \cos 68^\circ = 256.23 \text{ m/s}$$

(3) Total-to-total efficiency, η_{tt}, is

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T'_{03}}$$

$$= \frac{w_s}{T_{01} - \left(T_3 + \frac{C_3^2}{2C_{pg}}\right)} = \frac{w_s}{\frac{w_s}{\eta_{ts}} - \frac{C_3^2}{2C_{pg}}}$$

$$= \frac{304.42}{\frac{304.42}{0.85} - \frac{(256.23)^2}{2 \times 1147}} = 92.4\%$$

(4) The degree of reaction

$$\Lambda = \frac{Ca}{2U}\left(\tan \beta_3 - \tan \beta_2\right)$$

$$= \left(\frac{Ca}{2U} \times \frac{U}{Ca}\right) - \left(\frac{Ca}{2U} \tan \alpha_2\right) + \left(\frac{U}{Ca} \times \frac{Ca}{2U}\right)$$

(from velocity triangle)

$$\Lambda = 1 - \frac{Ca}{2U} \tan \alpha_2 = 1 - \frac{256.23}{(2)(480)} \tan 68^\circ = 33.94\%$$

Design Example 7.3 In a single-stage axial flow gas turbine gas enters at stagnation temperature of 1100 K and stagnation pressure of 5 bar. Axial velocity is constant through the stage and equal to 250 m/s. Mean blade speed is 350 m/s. Mass flow rate of gas is 15 kg/s and assume equal inlet and outlet velocities. Nozzle efflux angle is 63°, stage exit swirl angle equal to 9°. Determine the rotor-blade gas angles, degree of reaction, and power output.

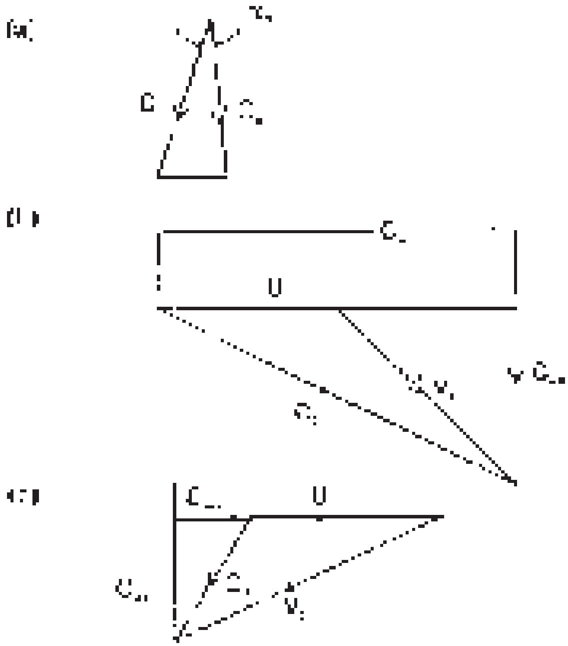


Figure 7.7 Velocity triangles for Example 7.3.

Solution

Refer to Fig. 7.7.

$$Ca_1 = Ca_2 = Ca_3 = Ca = 250 \text{ m/s}$$

From velocity triangle (b)

$$C_2 = \frac{Ca_2}{\cos\alpha_2} = \frac{250}{\cos 63^\circ} = 550.67 \text{ m/s}$$

From figure (c)

$$C_3 = \frac{Ca_3}{\cos\alpha_3} = \frac{250}{\cos 9^\circ} = 253 \text{ m/s}$$

$$C_{w3} = Ca_3 \tan\alpha_3 = 250 \tan 9^\circ = 39.596 \text{ m/s}$$

$$\tan\beta_3 = \frac{U + C_{w3}}{Ca_3} = \frac{350 + 39.596}{250} = 1.5584$$

i.e., $\beta_3 = 57.31^\circ$

From figure (b)

$$C_{w2} = Ca_2 \tan\alpha_2 = 250 \tan 63^\circ = 490.65 \text{ m/s}$$

and

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{490.65 - 350}{250} = 0.5626$$

$$\therefore \beta_2 = 29°21'$$

Power output

$$W = mUCa(\tan\beta_2 + \tan\beta_3)$$

$$= (15)(350)(250)(0.5626 + 1.5584)/1000$$

$$= 2784 \text{ kW}$$

The degree of reaction is given by

$$\Lambda = \frac{Ca}{2U}\left(\tan\beta_3 - \tan\beta_2\right)$$

$$= \frac{250}{2 \times 350}(1.5584 - 0.5626)$$

$$= 35.56\%$$

Design Example 7.4 Calculate the nozzle throat area for the same data as in the precious question, assuming nozzle loss coefficient, $T_N = 0.05$. Take $\gamma = 1.333$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

Nozzle throat area, $A = m/\rho_2 Ca_2$

and $\rho_2 = \dfrac{p_2}{RT_2}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1100 - \frac{(550.67)^2}{(2)(1.147)(1000)} \quad (T_{01} = T_{02})$$

i.e., $T_2 = 967.81$ K

From nozzle loss coefficient

$$T_2' = T_2 - \lambda_N \frac{C_2^2}{2C_p} = 967.81 - \frac{0.05 \times (550.67)^2}{(2)(1.147)(1000)} = 961.2 \text{ K}$$

Using isentropic p–T relation for nozzle expansion

$$p_2 = p_{01}/\left(T_{01}/T_2'\right)^{\gamma/(\gamma-1)} = 5/(1100/961.2)^4 = 2.915 \text{ bar}$$

Critical pressure ratio

$$p_{01}/p_c = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} = \left(\frac{2.333}{2}\right)^4 = 1.852$$

or $\quad p_{01}/p_2 = 5/2.915 = 1.715$

Since $\frac{p_{01}}{p_2} < \frac{p_{01}}{p_c}$, and therefore nozzle is unchoked.

Hence nozzle gas velocity at nozzle exit

$$C_2 = \sqrt{[2C_{pg}(T_{01} - T_2)]}$$

$$= \sqrt{[(2)(1.147)(1000)(1100 - 967.81)]} = 550.68 \text{ m/s}$$

Therefore, nozzle throat area

$$A = \frac{m}{\rho_2 C_2}, \text{ and } \rho_2 = \frac{p_2}{RT_2} = \frac{(2.915)(10^2)}{(0.287)(967.81)} = 1.05 \text{ kg/m}^3$$

Thus

$$A = \frac{15}{(1.05)(550.68)} = 0.026 \text{ m}^2$$

Design Example 7.5 In a single-stage turbine, gas enters and leaves the turbine axially. Inlet stagnation temperature is 1000 K, and pressure ratio is 1.8 bar. Gas leaving the stage with velocity 270 m/s and blade speed at root is 290 m/s. Stage isentropic efficiency is 0.85 and degree of reaction is zero. Find the nozzle efflux angle and blade inlet angle at the root radius.

Solution

Since $\Lambda = 0$, therefore

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3},$$

hence

$$T_2 = T_3$$

From isentropic p–T relation for expansion

$$T'_{03} = \frac{T_{01}}{(p_{01}/p_{03})^{(\gamma-1)/\gamma}} = \frac{1000}{(1.8)^{0.249}} = 863.558 \text{ K}$$

Using turbine efficiency

$$T_{03} = T_{01} - \eta_t\left(T_{01} - T'_{03}\right)$$

$$= 1000 - 0.85(1000 - 863.558) = 884\,\mathrm{K}$$

In order to find static temperature at turbine outlet, using static and stagnation temperature relation

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 884 - \frac{270^2}{(2)(1.147)(1000)} = 852\,\mathrm{K} = T_2$$

Dynamic temperature

$$\frac{C_2^2}{2C_{pg}} = 1000 - T_2 = 1000 - 852 = 148\,\mathrm{K}$$

$$C_2 = \sqrt{[(2)(1.147)(148)(1000)]} = 582.677\ \mathrm{m/s}$$

Since, $C_{pg}\Delta T_{os} = U(C_{w3} + C_{w2}) = UC_{w2}\ \ (C_{w3} = 0)$

Therefore, $C_{w2} = \dfrac{(1.147)(1000)(1000 - 884)}{290} = 458.8\ \mathrm{m/s}$

From velocity triangle

$$\sin\alpha_2 = \frac{C_{w2}}{C_2} = \frac{458.8}{582.677} = 0.787$$

That is, $\alpha_2 = 51°54'$

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{458.8 - 290}{C_2\cos\alpha_2}$$

$$= \frac{458.8 - 290}{582.677\cos 51.90°} = 0.47$$

i.e., $\beta_2 = 25°9'$

Design Example 7.6 In a single-stage axial flow gas turbine, gas enters the turbine at a stagnation temperature and pressure of 1150 K and 8 bar, respectively. Isentropic efficiency of stage is equal to 0.88, mean blade speed is 300 m/s, and rotational speed is 240 rps. The gas leaves the stage with velocity 390 m/s. Assuming inlet and outlet velocities are same and axial, find the blade height at the outlet conditions when the mass flow of gas is 34 kg/s, and temperature drop in the stage is 145 K.

Solution

Annulus area A is given by

$$A = 2\pi r_m h$$

where $h =$ blade height

$r_m =$ mean radius

As we have to find the blade height from the outlet conditions, in this case annulus area is A_3.

$$\therefore \quad h = \frac{A_3}{2\pi r_m}$$

$$U_m = \pi D_m N$$

or $D_m = \dfrac{(U_m)}{\pi N} = \dfrac{300}{(\pi)(240)} = 0.398$

i.e., $r_m = 0.199\,\text{m}$

Temperature drop in the stage is given by

$$T_{01} - T_{03} = 145\,\text{K}$$

Hence $T_{03} = 1150 - 145 = 1005\,\text{K}$

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 1005 - \frac{390^2}{(2)(1.147)(1000)} = 938.697\ \text{K}$$

Using turbine efficiency to find isentropic temperature drop

$$T'_{03} = 1150 - \frac{145}{0.88} = 985.23\ \text{K}$$

Using isentropic p–T relation for expansion process

$$p_{03} = \frac{p_{01}}{\left(T_{01}/T'_{03}\right)^{\gamma/(\gamma-1)}} = \frac{8}{(1150/985.23)^4} = \frac{8}{1.856}$$

i.e., $p_{03} = 4.31$ bar

Also from isentropic relation

$$p_3 = \frac{p_{03}}{\left(T'_{03}/T_3\right)^{\gamma/(\gamma-1)}} = \frac{4.31}{(985.23/938.697)^4} = \frac{4.31}{1.214} = 3.55\ \text{bar}$$

$$\rho_3 = \frac{p_3}{RT_3} = \frac{(3.55)(100)}{(0.287)(938.697)} = 1.32\ \text{kg/m}^3$$

$$A_3 = \frac{m}{\rho_3 Ca_3} = \frac{34}{(1.32)(390)} = 0.066\ \text{m}^2$$

Finally,

$$h = \frac{A_3}{2\pi r_m} = \frac{0.066}{(2\pi)(0.199)} = 0.053\,\text{m}$$

Design Example 7.7 The following data refer to a single-stage axial flow gas turbine with convergent nozzle:

Inlet stagnation temperature, T_{01}	1100 K
Inlet stagnation pressure, p_{01}	4 bar
Pressure ratio, p_{01}/p_{03}	1.9
Stagnation temperature drop	145 K
Mean blade speed	345 m/s
Mass flow, m	24 kg/s
Rotational speed	14, 500 rpm
Flow coefficient, Φ	0.75
Angle of gas leaving the stage	12°

$C_{pg} = 1147$ J/kg K, $\gamma = 1.333$, $\lambda_N = 0.05$

Assuming the axial velocity remains constant and the gas velocity at inlet and outlet are the same, determine the following quantities at the mean radius:

(1) The blade loading coefficient and degree of reaction
(2) The gas angles
(3) The nozzle throat area

Solution

(1) $$\Psi = \frac{C_{pg}(T_{01} - T_{03})}{U^2} = \frac{(1147)(145)}{345^2} = 1.4$$

Using velocity diagram

$$U/Ca = \tan\beta_3 - \tan\alpha_3$$

or

$$\tan\beta_3 = \frac{1}{\Phi} + \tan\alpha_3$$

$$= \frac{1}{0.75} + \tan 12°$$

$$\beta_3 = 57.1°$$

From Equations (7.14) and (7.15), we have

$$\Psi = \Phi\left(\tan\beta_2 + \tan\beta_3\right)$$

and

$$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

From which

$$\tan\beta_3 = \frac{1}{2\Phi}(\Psi + 2\Lambda)$$

Therefore

$$\tan 57.1^\circ = \frac{1}{2\times 0.75}(1.4 + 2\Lambda)$$

Hence

$$\Lambda = 0.4595$$

(2) $$\tan\beta_2 = \frac{1}{2\Phi}(\Psi - 2\Lambda)$$

$$= \frac{1}{2\times 0.75}(1.4 - [2][0.459])$$

$$\beta_2 = 17.8^\circ$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\Phi}$$

$$= \tan 17.8^\circ + \frac{1}{0.75} = 0.321 + 1.33 = 1.654$$

$$\alpha_2 = 58.8^\circ$$

(3) $$Ca_1 = U\Phi$$

$$= (345)(0.75) = 258.75 \text{ m/s}$$

$$C_2 = \frac{Ca_1}{\cos\alpha_2} = \frac{258.75}{\cos 58.8^\circ} = 499.49 \text{ m/s}$$

$$T_{02} - T_2 = \frac{C_2^2}{2C_p} = \frac{499.49^2}{(2)(1147)} = 108.76\,\text{K}$$

$$T_2 - T_{2s} = \frac{(T_N)(499.49^2)}{(2)(1147)} = \frac{(0.05)(499.49^2)}{(2)(1147)} = 5.438\,\text{K}$$

$$T_{2s} = T_2 - 5.438$$

$$T_2 = 1100 - 108.76 = 991.24\,\text{K}$$

$$T_{2s} = 991.24 - 5.438 = 985.8\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_{2s}}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = 4 \times \left(\frac{985.8}{1100}\right)^4 = 2.58$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(2.58)(100)}{(0.287)(991.24)} = 0.911\,\text{kg/m}^3$$

(4) Nozzle throat area $= \dfrac{m}{\rho_1 C_1} = \dfrac{24}{(0.907)(499.49)} = 0.053\,\text{m}^2$

$$A_1 = \frac{m}{\rho_1 Ca_1} = \frac{24}{(0.907)(258.75)} = 0.102\,\text{m}^2$$

Design Example 7.8 A single-stage axial flow gas turbine with equal stage inlet and outlet velocities has the following design data based on the mean diameter:

Mass flow	20 kg/s
Inlet temperature, T_{01}	1150K
Inlet pressure	4 bar
Axial flow velocity constant through the stage	255 m/s
Blade speed, U	345 m/s
Nozzle efflux angle, α_2	60°
Gas-stage exit angle	12°

Calculate (1) the rotor-blade gas angles, (2) the degree of reaction, blade-loading coefficient, and power output and (3) the total nozzle throat area if the throat is situated at the nozzle outlet and the nozzle loss coefficient is 0.05.

Solution

(1) From the velocity triangles

$$C_{w2} = Ca \tan \alpha_2$$

$$= 255 \tan 60° = 441.67\ \text{m/s}$$

$$C_{w3} = Ca \tan \alpha_3 = 255 \tan 12° = 55.2\ \text{m/s}$$

$$V_{w2} = C_{w2} - U = 441.67 - 345 = 96.67\ \text{m/s}$$

$$\beta_2 = \tan^{-1}\frac{V_{w2}}{Ca} = \tan^{-1}\frac{96.67}{255} = 20.8°$$

Also $V_{w3} = C_{w3} + U = 345 + 55.2 = 400.2\ \text{m/s}$

$$\therefore\ \beta_3 = \tan^{-1}\frac{V_{w3}}{Ca} = \tan^{-1}\frac{400.2}{255} = 57.5°$$

(2) $$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

$$= \frac{255}{2\times 345}(\tan 57.5° - \tan 20.8°) = 0.44$$

$$\Psi = \frac{Ca}{U}\left(\tan\beta_2 + \tan\beta_3\right)$$

$$= \frac{255}{345}(\tan 20.8° + \tan 57.5°) = 1.44$$

Power $W = mU(C_{w2} + C_{w3})$

$$= (20)(345)(441.67 + 54.2) = 3421.5\,\text{kW}$$

(3) $$\lambda_N = \frac{C_p\left(T_2 - T_2'\right)}{\frac{1}{2}C_2^2},\ C_2 = Ca\ \sec\alpha_2 = 255\sec 60° = 510\ \text{m/s}$$

or $$T_2 - T_2' = \frac{(0.05)(0.5)(510^2)}{1147} = 5.67$$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1150 - \frac{510^2}{(2)(1147)} = 1036.6\,\text{K}$$

$$T_2' = 1036.6 - 5.67 = 1030.93\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_2}\right)^{\gamma/(\gamma-1)} = \left(\frac{1150}{1030.93}\right)^4 = 1.548$$

$$p_2 = \frac{4}{1.548} = 2.584\,\text{bar}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.584\times 100}{0.287\times 1036.6} = 0.869\,\text{kg/m}^3$$

$$m = \rho_2 A_2 C_2$$

$$A_2 = \frac{20}{0.869\times 510} = 0.045\,\text{m}^2$$

Illustrative Example 7.9 A single-stage axial flow gas turbine has the following data

Mean blade speed	340 m/s
Nozzle exit angle	15°
Axial velocity (constant)	105 m/s
Turbine inlet temperature	900°C
Turbine outlet temperature	670°C
Degree of reaction	50%

Calculate the enthalpy drop per stage and number of stages required.

Solution

At 50%,

$$\alpha_2 = \beta_3$$

$$\alpha_3 = \beta_2$$

$$C_2 = \frac{U}{\cos 15°} = \frac{340}{\cos 15°} = 351.99 \text{ m/s}$$

$$\text{Heat drop in blade moving row} = \frac{C_2^2 - C_3^2}{2C_p} = \frac{(351.99)^2 - (105)^2}{(2)(1147)}$$

$$= \frac{123896.96 - 11025}{(2)(1147)}$$

$$= 49.2\,\text{K}$$

$$\text{Therefore heat drop in a stage} = (2)(49.2) = 98.41\,\text{K}$$

$$\text{Number of stages} = \frac{1173 - 943}{98.41} = \frac{230}{98.4} = 2$$

Design Example 7.10 The following particulars relate to a single-stage turbine of free vortex design:

Inlet temperature, T_{01}	1100K
Inlet pressure, p_{01}	4 bar
Mass flow	20 kg/s
Axial velocity at nozzle exit	250 m/s
Blade speed at mean diameter	300 m/s
Nozzle angle at mean diameter	25°
Ratio of tip to root radius	1.4

The gas leaves the stage in an axial direction, find:

(1) The total throat area of the nozzle.
(2) The nozzle efflux angle at root and tip.
(3) The work done on the turbine blades.

Take

$$C_{pg} = 1.147\,\text{kJ/kg K},\ \gamma = 1.33$$

Solution

For no loss up to throat

$$\frac{p^*}{p_{01}} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = \left(\frac{2}{2.33}\right)^4 = 0.543$$

$$p^* = 4 \times 0.543 = 2.172\,\text{bar}$$

Also $T^* = 1100\left(\frac{2}{2.33}\right)^4 = 944\,\text{K}$

$$T_{01} = T^* + \frac{C^2}{2C_{pg}}$$

$$C^* = \sqrt{2C_{pg}\left(T_{01} - T^*\right)}$$

$$= \sqrt{(2)(1147)(1100-944)} = 598\ \text{m/s}$$

$$\rho^* = \frac{p^*}{RT^*} = \frac{(2.172)(100)}{(0.287)(944)} = 0.802\,\text{kg/m}^3$$

(1) Throat area

$$A = \frac{m}{\rho C^*} = \frac{20}{(0.802)(598)} = 0.042\,\text{m}^2$$

(2) Angle α_1, at any radius r and α_{1m} at the design radius r_m are related by the equation

$$\tan\alpha_1 = \frac{r_m}{r_1}\tan\alpha_{1m}$$

Given

$$\frac{\text{Tip radius}}{\text{Root radius}} = \frac{r_t}{r_r} = 1.4$$

$$\therefore \quad \frac{\text{Mean radius}}{\text{Root radius}} = 1.2$$

$$\alpha_{1m} = 25°$$

$$\tan\alpha_{1r} = \frac{r_{mean}}{r_{root}} \times \tan\alpha_{1m}$$

$$= 1.2 \times \tan 25° = 0.5596$$

$$\therefore \quad \alpha_{1r} = 29.23°$$

$$\tan\alpha_{1t} = \frac{r_r}{r_t} \times \tan\alpha_{1r} = \left(\frac{1}{1.4}\right)(0.5596) = 0.3997$$

$$\therefore \quad \alpha_{1t} = 21.79°$$

(3) $$C_{w2} = \frac{r_m}{r_r} x C_{w2m} = \frac{r_m}{r_r}\frac{Ca_2}{\tan\alpha_{2m}} = 1.2x\frac{250}{\tan 25°} = 643 \text{ m/s}$$

$$W = mUC_{w2} = \frac{(20)(300)(643)}{1000} = 3858\,\text{kW}$$

7.8 RADIAL FLOW TURBINE

In Sec. 7.1 "Introduction to Axial Flow Turbines", it was pointed out that in axial flow turbines the fluid moves essentially in the axial direction through the rotor. In the radial type the fluid motion is mostly radial. The mixed flow machine is characterized by a combination of axial and radial motion of the fluid relative to the rotor. The choice of turbine depends on the application, though it is not always clear that any one type is superior. For small mass flows, the radial machine can be made more efficient than the axial one. The radial turbine is capable of a high-pressure ratio per stage than the axial one. However, multistaging is very much easier to arrange with the axial turbine, so that large overall pressure ratios are not difficult to obtain with axial turbines. The radial flow turbines are used in turbochargers for commercial (diesel) engines and fire pumps. They are very compact, the maximum diameter being about 0.2 m, and run at very high speeds. In inward flow radial turbine, gas enters in the radial direction and leaves axially at outlet. The rotor, which is usually manufactured of

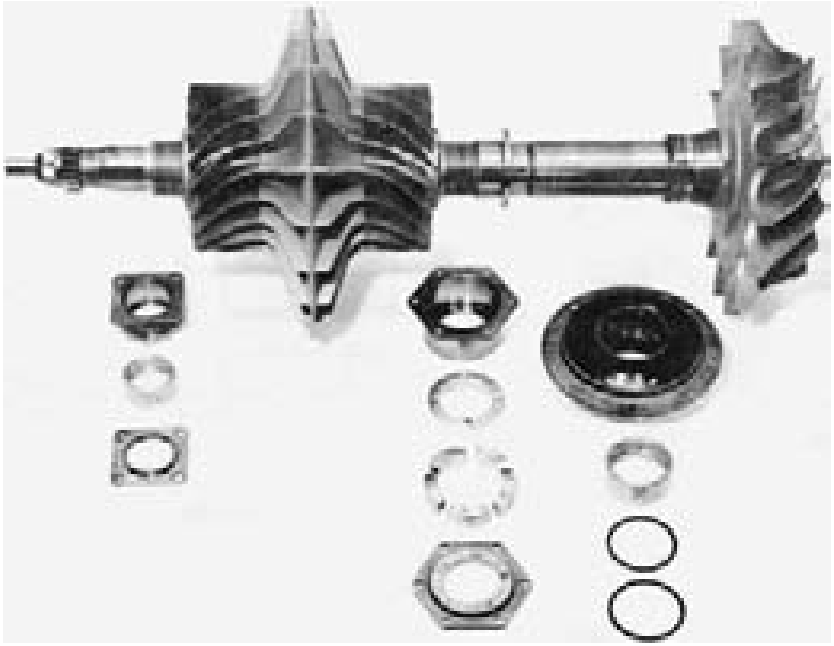


Figure 7.8 Radial turbine photograph of the rotor on the right.

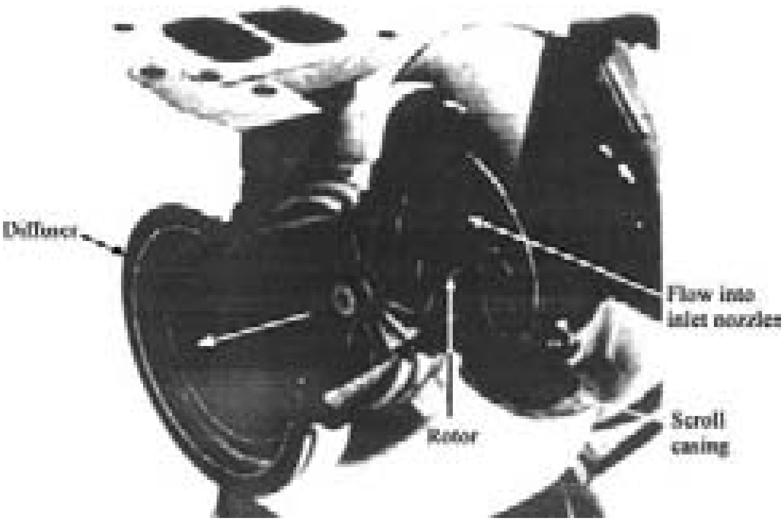


Figure 7.9 Elements of a 90° inward flow radial gas turbine with inlet nozzle ring.

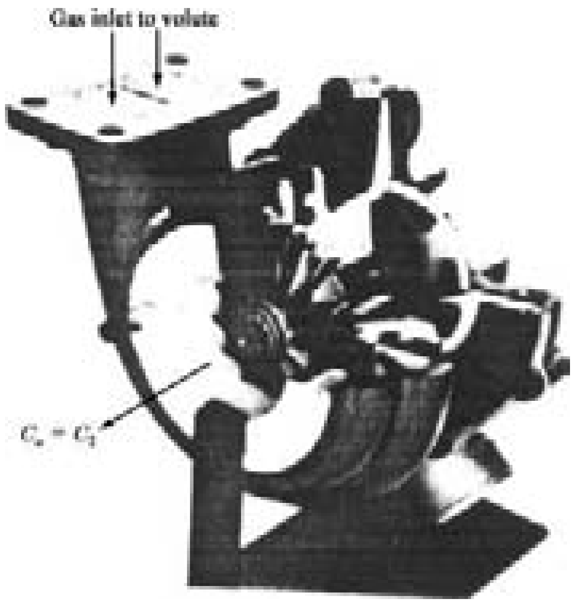


Figure 7.10 A 90° inward flow radial gas turbine without nozzle ring.

cast nickel alloy, has blades that are curved to change the flow from the radial to the axial direction. Note that this turbine is like a single-faced centrifugal compressor with reverse flow. Figures 7.8–7.10 show photographs of the radial turbine and its essential parts.

7.9 VELOCITY DIAGRAMS AND THERMODYNAMIC ANALYSIS

Figure 7.11 shows the velocity triangles for this turbine. The same nomenclature that we used for axial flow turbines, will be used here. Figure 7.12 shows the Mollier diagram for a 90° flow radial turbine and diffuser.

As no work is done in the nozzle, we have $h_{01} = h_{02}$. The stagnation pressure drops from p_{01} to p_1 due to irreversibilities. The work done per unit mass flow is given by Euler's turbine equation

$$W_t = (U_2 C_{w2} - U_3 C_{w3}) \tag{7.36}$$

If the whirl velocity is zero at exit then

$$W_t = U_2 C_{w2} \tag{7.37}$$

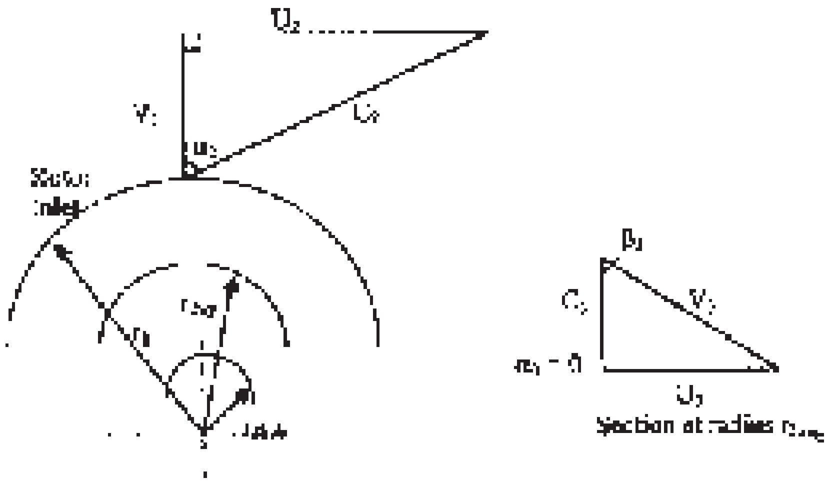


Figure 7.11 Velocity triangles for the 90° inward flow radial gas turbine.

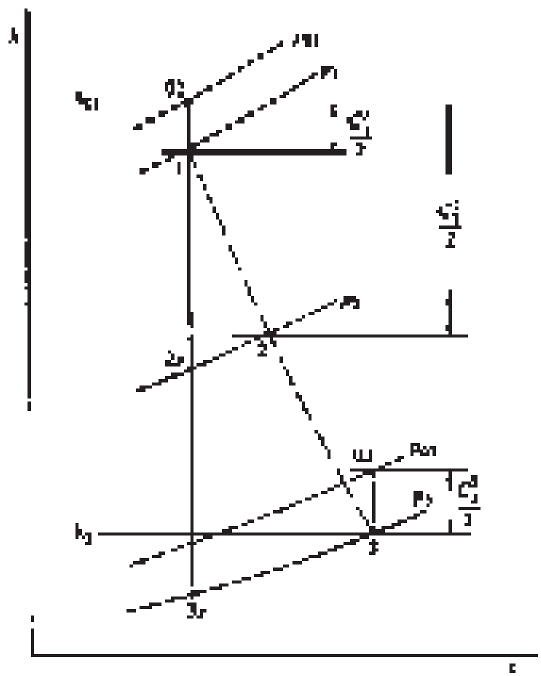


Figure 7.12 Mollier chart for expansion in a 90° inward flow radial gas turbine.

For radial relative velocity at inlet

$$W_t = U_2^2 \qquad (7.38)$$

In terms of enthalpy drop

$$h_{02} - h_{03} = U_2 C_{w2} - U_3 C_{w3}$$

Using total-to-total efficiency

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03\,ss}},$$

efficiency being in the region of 80–90%

7.10 SPOUTING VELOCITY

It is that velocity, which has an associated kinetic energy equal to the isentropic enthalpy drop from turbine inlet stagnation pressure p_{01} to the final exhaust pressure. Spouting velocities may be defined depending upon whether total or static conditions are used in the related efficiency definition and upon whether or not a diffuser is included with the turbine. Thus, when no diffuser is used, using subscript 0 for spouting velocity.

$$\frac{1}{2} C_0^2 = h_{01} - h_{03\,ss} \qquad (7.39)$$

or

$$\frac{1}{2} C_0^2 = h_{01} - h_{3\,ss} \qquad (7.40)$$

for the total and static cases, respectively.

Now for isentropic flow throughout work done per unit mass flow

$$W = U_2^2 = C_0^2/2 \qquad (7.41)$$

or

$$U_2/C_0 = 0.707 \qquad (7.42)$$

In practice, U_2/C_0 lies in the range $0.68 < \frac{U_2}{C_0} < 0.71$.

7.11 TURBINE EFFICIENCY

Referring to Fig. 7.12, the total-to-static efficiency, without diffuser, is defined as

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3\,ss}}$$

$$= \frac{W}{W + \frac{1}{2} C_3^2 + (h_3 - h_{3ss}) + (h_{3s} - h_{3\,ss})} \qquad (7.43)$$

Nozzle loss coefficient, ξ_n, is defined as

$$\xi_n = \frac{\text{Enthalpy loss in nozzle}}{\text{Kinetic energy at nozzle exit}}$$

$$= \frac{h_{3s} - h_{3\,ss}}{0.5C_2^2(T_3/T_2)} \tag{7.44}$$

Rotor loss coefficient, ξ_r, is defined as

$$\xi_r = \frac{h_3 - h_{3s}}{0.5V_3^2} \tag{7.45}$$

But for constant pressure process,

$$T\,ds = dh,$$

and, therefore

$$h_{3s} - h_{3ss} = (h - h_{2s})(T_3/T_2)$$

Substituting in Eq. (7.43)

$$\eta_{ts} = \left[1 + \frac{1}{2}\left(C_3^2 + V_3^2\xi_r + C_2\xi_n T_3/T_2\right)W\right]^{-1} \tag{7.46}$$

Using velocity triangles

$$C_2 = U_2 \operatorname{cosec}\alpha_2, V_3 = U_3 \operatorname{cosec}\beta_3, C_3 = U_3 \cot\beta_3, W = U_2^2$$

Substituting all those values in Eq. (7.44) and noting that $U_3 = U_2\, r_3/r_2$, then

$$\eta_{ts} = \left[1 + \frac{1}{2}\left\{\xi_n \frac{T_3}{T_2}\operatorname{cosec}^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2\left(\xi_r \operatorname{cosec}^2\beta_3 + \cot^2\beta_3\right)\right\}\right]^{-1} \tag{7.47}$$

Taking mean radius, that is,

$$r_3 = \frac{1}{2}(r_{3t} + r_{3h})$$

Using thermodynamic relation for T_3/T_2, we get

$$\frac{T_3}{T_2} = 1 - \frac{1}{2}(\gamma - 1)\left(\frac{U_2}{a_2}\right)^2\left[1 - \cot^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2\cot^2\beta_3\right]$$

But the above value of T_3/T_2 is very small, and therefore usually neglected. Thus

$$\eta_{ts} = \left[1 + \frac{1}{2}\left\{\xi_n \operatorname{cosec}^2\alpha_2 + \left(\frac{r_{3\,av}}{r_2}\right)^2\left(\xi_r \operatorname{cosec}^2\beta_{3\,av} + \cot^2\beta_{3\,av}\right)\right\}\right]^{-1} \tag{7.48}$$

Equation (7.46) is normally used to determine total-to-static efficiency. The η_{ts} can also be found by rewriting Eq. (7.43) as

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3\,ss}} = \frac{(h_{01} - h_{3\,ss}) - (h_{03} - h_3) - (h_3 - h_{3s}) - (h_{3s} - h_{3\,ss})}{(h_{01} - h_{3ss})}$$

$$= 1 - \left(C_3^2 + \xi_n C_2^2 + \xi_r V_3^2\right)/C_0^2 \quad (7.49)$$

where spouting velocity C_0 is given by

$$h_{01} - h_{3\,ss} = \frac{1}{2}C_0^2 = C_p T_{01}\left[1 - \left(p_3/p_{01}\right)^{\gamma-1/\gamma}\right] \quad (7.50)$$

The relationship between η_{ts} and η_{tt} can be obtained as follows:

$$W = U_2^2 = \eta_{ts} W_{ts} = \eta_{ts}(h_{01} - h_{3ss}), \text{ then}$$

$$\eta_{tt} = \frac{W}{W_{ts} - \frac{1}{2}C_3^2} = \frac{1}{\frac{1}{\eta_{ts}} - \frac{C_3^2}{2W}}$$

$$\therefore \frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2W} = \frac{1}{\eta_{ts}} - \frac{1}{2}\left(\frac{r_{3\,av}}{r_2} - \cot\beta_{3\,av}\right)^2 \quad (7.51)$$

Loss coefficients usually lie in the following range for 90° inward flow turbines

$$\xi_n = 0.063\text{–}0.235$$

and

$$\xi_r = 0.384\text{–}0.777$$

7.12 APPLICATION OF SPECIFIC SPEED

We have already discussed the concept of specific speed N_s in Chapter 1 and some applications of it have been made already. The concept of specific speed was applied almost exclusively to incompressible flow machines as an important parameter in the selection of the optimum type and size of unit. The volume flow rate through hydraulic machines remains constant. But in radial flow gas turbine, volume flow rate changes significantly, and this change must be taken into account. According to Balje, one suggested value of volume flow rate is that at the outlet Q_3.

Using nondimensional form of specific speed

$$N_s = \frac{N Q_3^{1/2}}{(\Delta h_0')^{3/4}} \quad (7.52)$$

where N is in rev/s, Q_3 is in m^3/s and isentropic total-to-total enthalpy drop (from turbine inlet to outlet) is in J/kg. For the 90° inward flow radial turbine,

$U_2 = \pi N D_2$ and $\Delta h_{0s} = \frac{1}{2} C_0^2$, factorizing the Eq. (7.52)

$$N_s = \frac{Q_3^{1/2}}{\left(\frac{1}{2} C_0^2\right)^{3/4}} \left(\frac{U_2}{\pi D_2}\right) \left(\frac{U_2}{\pi N D_2}\right)^{1/2}$$

$$= \left(\frac{\sqrt{2}}{\pi}\right)^{3/2} \left(\frac{U_2}{C_0}\right)^{3/2} \left(\frac{Q_3}{N D_2^3}\right)^{1/2} \quad (7.53)$$

For 90° inward flow radial turbine, $U_2/C_0 = \frac{1}{\sqrt{2}} = 0.707$, substituting this value in Eq. (7.53),

$$N_s = 0.18 \left(\frac{Q_3}{N D_2^3}\right)^{1/2}, \quad \text{rev} \quad (7.54)$$

Equation (7.54) shows that specific speed is directly proportional to the square root of the volumetric flow coefficient. Assuming a uniform axial velocity at rotor exit C_3, so that $Q_3 = A_3 C_3$, rotor disc area $A_d = \pi D_2^2/4$, then

$$N = U_2/(\pi D_2) = \frac{C_0 \sqrt{2}}{2 \pi D_2}$$

$$\frac{Q_3}{N D_2^3} = \frac{A_3 C_3 2 \pi D_2}{\sqrt{2} C_0 D_2^2} = \frac{A_3}{A_d} \frac{C_3}{C_0} \frac{\pi^2}{2\sqrt{2}}$$

Therefore,

$$N_s = 0.336 \left(\frac{C_3}{C_0}\right)^{\frac{1}{2}} \left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \quad \text{rev} \quad (7.55)$$

$$= 2.11 \left(\frac{C_3}{C_0}\right)^{\frac{1}{2}} \left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \quad \text{rad} \quad (7.56)$$

Suggested values for C_3/C_o and A_3/A_d are as follows:

$$0.04 < C_3/C_0 < 0.3$$

$$0.1 < A_3/A_d < 0.5$$

Then $\quad 0.3 < N_s < 1.1, \quad \text{rad}$

Thus the N_s range is very small and Fig. 7.13 shows the variation of efficiency with N_s, where it is seen to match the axial flow gas turbine over the limited range of N_s.

Design Example 7.11 A small inward radial flow gas turbine operates at its design point with a total-to-total efficiency of 0.90. The stagnation pressure and temperature of the gas at nozzle inlet are 310 kPa and 1145K respectively. The flow leaving the turbine is diffused to a pressure of 100 kPa and the velocity of

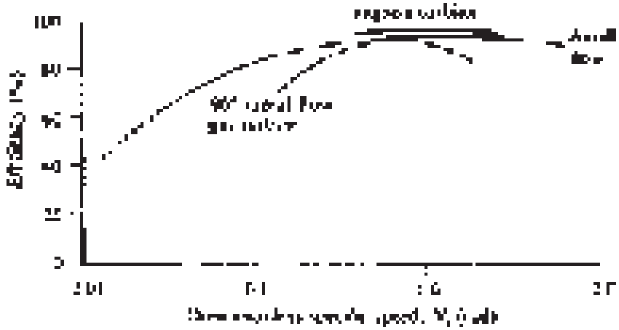


Figure 7.13 Variation of efficiency with dimensionless specific speed.

flow is negligible at that point. Given that the Mach number at exit from the nozzles is 0.9, find the impeller tip speed and the flow angle at the nozzle exit. Assume that the gas enters the impeller radially and there is no whirl at the impeller exit. Take

$$C_p = 1.147 \quad \text{kJ/kg K}, \quad \gamma = 1.333.$$

Solution

The overall efficiency of turbine from nozzle inlet to diffuser outlet is given by

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03\,ss}}$$

Turbine work per unit mass flow

$$W = U_2^2 = C_p(T_{01} - T_{03}), \quad (C_{w3} = 0)$$

Now using isentropic p–T relation

$$T_{01}\left(1 - \frac{T_{03ss}}{T_{01}}\right) = T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\gamma-1/\gamma}\right]$$

Therefore

$$U_2^2 = \eta_{tt} C_p T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\gamma-1/\gamma}\right]$$

$$= 0.9 \times 1147 \times 1145\left[1 - \left(\frac{100}{310}\right)^{0.2498}\right]$$

$\therefore$ Impeller tip speed, $U_2 = 539.45$ m/s

The Mach number of the absolute flow velocity at nozzle exit is given by

$$M = \frac{C_1}{a_1} = \frac{U_1}{a_1 \sin \alpha_1}$$

Since the flow is adiabatic across the nozzle, we have

$$T_{01} = T_{02} = T_2 + \frac{C_2^2}{2C_p} = T_2 + \frac{U_2^2}{2C_p \sin^2 \alpha_2}$$

or $$\frac{T_2}{T_{01}} = 1 - \frac{U_2^2}{2C_p T_{01} \sin^2 \alpha_2}, \text{ but } C_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \quad \frac{T_2}{T_{01}} = 1 - \frac{U_2^2(\gamma - 1)}{2\gamma R T_{01} \sin^2 \alpha_2} = 1 - \frac{U_2^2(\gamma - 1)}{2a_{01}^2 \sin^2 \alpha_2}$$

But $$\left(\frac{T_2}{T_{01}}\right)^2 = \frac{a_2}{a_{01}} = \frac{a_2}{a_{02}} \qquad \text{since } T_{01} = T_{02}$$

and $$\frac{a_2}{a_{02}} = \frac{U_2}{M_2 a_{02} \sin \alpha_2}$$

$$\therefore \quad \left(\frac{U_2}{M_2 a_{02} \sin \alpha_2}\right)^2 = 1 - \frac{U_2^2(\gamma - 1)}{2a_{02}^2 \sin^2 \alpha_2}$$

and $$1 = \left(\frac{U_2}{a_{02} \sin \alpha_2}\right)^2 \left(\frac{(\gamma - 1)}{2} + \frac{1}{M_2^2}\right)$$

or $$\sin^2 \alpha_2 = \left(\frac{U_2}{a_{02}}\right)^2 \left(\frac{(\gamma - 1)}{2} + \frac{1}{M_2^2}\right)$$

But $$a_{02}^2 = \gamma R T_{02} = (1.333)(287)(1145) = 438043 \text{ m}^2/\text{s}^2$$

$$\therefore \quad \sin^2 \alpha_2 = \frac{539.45^2}{438043} \left(\frac{0.333}{2} + \frac{1}{0.9^2}\right) = 0.9311$$

Therefore nozzle angle $\alpha_2 = 75°$

Illustrative Example 7.12 The following particulars relate to a small inward flow radial gas turbine.

Rotor inlet tip diameter	92 mm
Rotor outlet tip diameter	64 mm
Rotor outlet hub diameter	26 mm
Ratio C_3/C_0	0.447

Ratio U_2/C_0 (ideal)	0.707
Blade rotational speed	30, 500 rpm
Density at impeller exit	1.75 kg/m^3

Determine

(1) The dimensionless specific speed of the turbine.
(2) The volume flow rate at impeller outlet.
(3) The power developed by the turbine.

Solution

(1) Dimensionless specific speed is

$$N_s = 0.336\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \text{ rev}$$

Now

$$A_3 = \frac{\pi\left(D_{3t}^2 - D_{3h}^2\right)}{4}$$

$$= \frac{\pi\left(0.064^2 - 0.026^2\right)}{4} = (2.73)(10^{-3}) \text{ m}^2$$

$$A_d = \frac{\pi D_2^2}{4} = \left(\frac{\pi}{4}\right)(0.092^2) = (6.65)(10^{-3}) \text{ m}^2$$

Dimensionless specific speed

$$N_s = 0.336\left(\frac{[0.447][2.73]}{6.65}\right)^{\frac{1}{2}}$$

$$= 0.144 \text{ rev}$$

$$= 0.904 \text{ rad}$$

(2) The flow rate at outlet for the ideal turbine is given by Eq. (7.54).

$$N_s = 0.18\left(\frac{Q_3}{ND_2^3}\right)^{1/2}$$

$$0.144 = 0.18\left(\frac{[Q_3][60]}{[30, 500][0.092^3]}\right)^{1/2}$$

Hence

$$Q_3 = 0.253 \text{ m}^3/\text{s}$$

(3) The power developed by the turbine is given by

$$W_t = \dot{m}U_3^2$$
$$= \rho_3 Q_3 U_3^2$$
$$= 1.75 \times 0.253 \times \left(\frac{\pi N D_2}{60}\right)^2$$
$$= 1.75 \times 0.253 \times \left(\frac{[\pi][30,500][0.092]}{60}\right)^2$$
$$= 9.565 \text{ kW}$$

PROBLEMS

7.1 A single-stage axial flow gas turbine has the following data:

Inlet stagnation temperature	1100K
The ratio of static pressure at the nozzle exit to the stagnation pressure at the nozzle inlet	0.53
Nozzle efficiency	0.93
Nozzle angle	20°
Mean blade velocity	454 m/s
Rotor efficiency	0.90
Degree of reaction	50%

$$C_{pg} = 1.147 \text{ kJ/kgK}, \quad \gamma = 1.33$$

Find (1) the work output per kg/s of air flow, (2) the ratio of the static pressure at the rotor exit to the stagnation pressure at the nozzle inlet, and (3) the total-to-total stage efficiency.

(282 kW, 0.214, 83.78%)

7.2 Derive an equation for the degree of reaction for a single-stage axial flow turbine and show that for 50% reaction blading $\alpha_2 = \beta_3$ and $\alpha_3 = \beta_2$.

7.3 For a free-vortex turbine blade with an impulse hub show that degree of reaction

$$\Lambda = 1 - \left(\frac{r_h}{r}\right)^2$$

where r_h is the hub radius and r is any radius.

7.4 A 50% reaction axial flow gas turbine has a total enthalpy drop of 288 kJ/kg. The nozzle exit angle is 70°. The inlet angle to the rotating blade row is inclined at 20° with the axial direction. The axial velocity is constant through the stage. Calculate the enthalpy drop per row of moving blades and the number of stages required when mean blade speed is 310 m/s. Take $C_{pg} = 1.147$ kJ/kgK, $\gamma = 1.33$.

(5 stages)

7.5 Show that for zero degree of reaction, blade-loading coefficient, $\Psi = 2$.

7.6 The inlet stagnation temperature and pressure for an axial flow gas turbine are 1000K and 8 bar, respectively. The exhaust gas pressure is 1.2 bar and isentropic efficiency of turbine is 85%. Assume gas is air, find the exhaust stagnation temperature and entropy change of the gas.

(644K, −0.044 kJ/kgK)

7.7 The performance date from inward radial flow exhaust gas turbine are as follows:

Stagnation pressure at inlet to nozzles, p_{01}	705 kPa
Stagnation temperature at inlet to nozzles, T_{01}	1080K
Static pressure at exit from nozzles, p_2	515 kPa
Static temperature at exit from nozzles, T_2	1000K
Static pressure at exit from rotor, p_3	360 kPa
Static temperature at exit from rotor, T_3	923K
Stagnation temperature at exit from rotor, T_{03}	925K
Ratio $\frac{r_{2\,\mathrm{av}}}{r_2}$	0.5
Rotational speed, N	25, 500 rpm

The flow into the rotor is radial and at exit the flow is axial at all radii. Calculate (1) the total-to-static efficiency of the turbine, (2) the impeller tip diameter, (3) the enthalpy loss coefficient for the nozzle and rotor rows, (4) the blade outlet angle at the mean diameter, and (5) the total-to-total efficiency of the turbine.

[(1) 93%, (2) 0.32 m, (3) 0.019, 0.399, (4) 72.2°, (5) 94%]

NOTATION

A	area
C	absolute velocity

C_0	spouting velocity
h	enthalpy, blade height
N	rotation speed
N_s	specific speed
P	pressure
r_m	mean radius
T	temperature
U	rotor speed
V	relative velocity
Y_N	nozzle loss coefficient in terms of pressure
α	angle with absolute velocity
β	angle with relative velocity
ΔT_{0s}	stagnation temperature drop in the stage
ΔT_s	static temperature drop in the stage
ε_n	nozzle loss coefficient in radial flow turbine
ε_r	rotor loss coefficient in radial flow turbine
ϕ	flow coefficient
η_s	isentropic efficiency of stage
Λ	degree of reaction

CHAPTER FIVE : AIR INTAKE

Introduction: Depending on the type of A/C air intakes are divided into two types a subsonic & a supersonic. The subsonic type is relatively simple while the supersonic type may be extremely complex & its design may be critical to the successful operation of the A/C. For an A/C cruising at M2.0, the intake may generate upto 7:1 pressure ratio & responsible for some 60% of the thrust.

Requirements:

① Deliver air to the engine at the required mass flow & velocity with minimum loss of available energy due to friction (i.e. minimum total pressure loss), and minimum [illegible] uniform stable airflow at the compressor face [illegible] velocity profile.

② Be insensitive to A/C manoeuvre.

Intake efficiency: The intake isentropic efficiency [illegible] defined as the ratio of total pressure after compression to the free stream total pressure.

$$\eta_i = \frac{P_{01}}{P_{0\infty}}$$

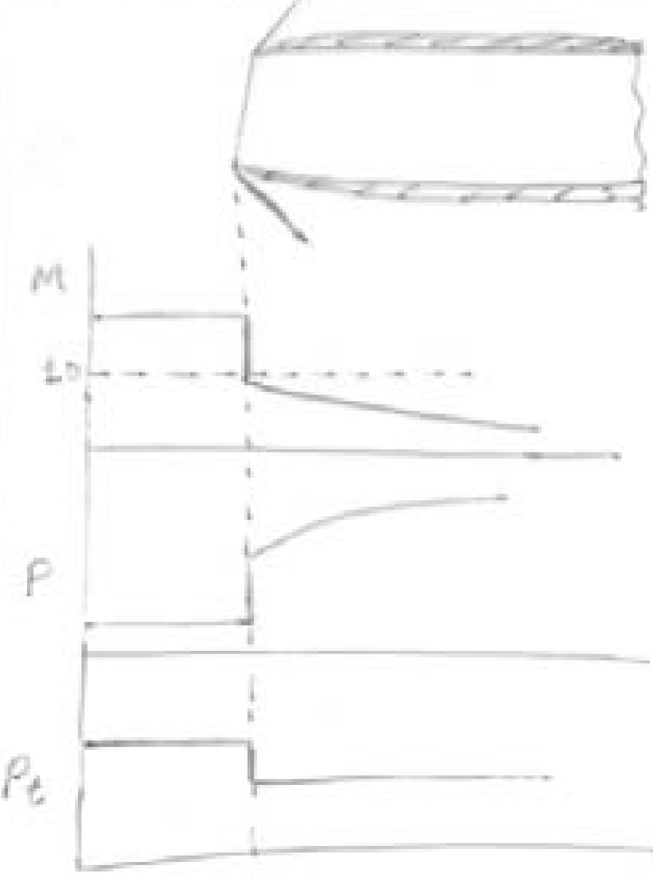
5.4 The Pitot Intake: In subsonic operation, the job of intake is simply to capture the required mass flow of air & to deliver it to the compressor at the required speed. This involves converting dynamic pressure to static pressure in a diffuser. This type is known as the pitot type & is generally used in subsonic A/Cs.

The subsonic pitot intake has thick well-rounded lips to prevent flow separation. If this type is well designed, it will operate with high η_i over wide operating range

(b) <u>The supersonic pitot intake</u>. If a pitot intake is driven at supersonic speed, a normal shock forms at the lip.

The M is instantaneously reduced below unity through the shock so, the intake still acts as a subsonic diffuser.

Because the N.S is very strong there is a sharp loss of total pressure across it & hence η_i is low. e.g at $M=2.0$ η_i is ~ 70% compared with 97% for more efficient types. At the same time, because of the drop in M, there is a rise in static pressure, so the intake acts as a compressor.



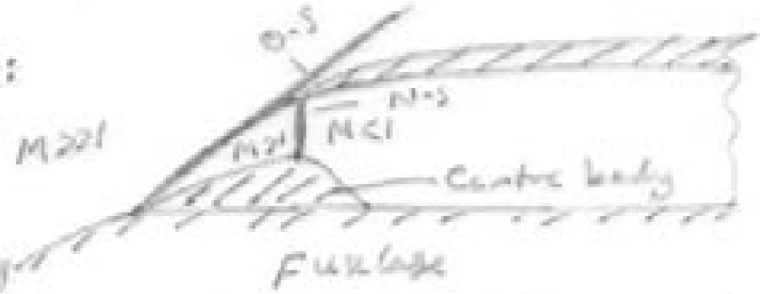
The pitot intake is used on some A/c; with a supersonic capability because at low supersonic M, the simplicity gained outweighs the slight loss of η_i.

One problem of the supersonic intake is that the lips must be sharp to prevent shock detachment, this can cause problems at low speeds.

6.4 The multi-shock intake

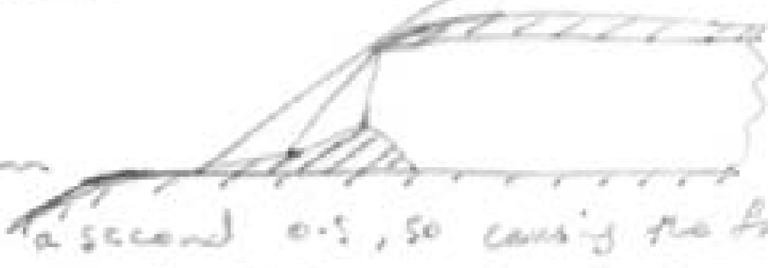
For A/C which operates at high M, η_i is unacceptably low because of the large loss across the strong N-S. This loss can be reduced by doing most of the compression through a series of oblique shocks, and using a final N-S to bring M below unity. This is because O-S are weak & involve only small losses, while the N-S now occurs at a lower M and is therefore much weaker than in a Pitot intake.

(a) The 2-Shock intake:



The 2-shock intake is the simplest type of multishock intakes, consisting of 1 O-S & followed by 1 N-S. The O-S is induced by introducing a centre body into the intake, which is so designed that the O-S is touching the cowl lip at the design M. Since the O-S shock angle = f(M), the shock will not touch the lip except at the design point.

(b) The 3-Shock intake



An even higher η_i can be obtained by introducing a second O-S, so causing the final N-S to occur at even lower M, such an intake is known as a 3-shock intake. The 2nd O-S is caused by designing a discontinuity into the centre body.

(c) The Isentropic intake

If comp. were achieved through a series of O-S, each is infinitesimally stronger than its predecessor, the total press. loss would be zero. In practice, this is not possible, but by using a suitably curved compression surface, nearly isentropic comp. is possible with a very high η_i (total pressure recovery).

5.5 (a) Capture Area Ratio: It is the ratio of the x-sectional area of the captured free-stream stream-tube to that of the intake entry plane. If the shock system lies ahead of the cowl lip, then stream lines will be deflected & $\frac{A_\infty}{A_i} < 1$.

(b) Critical operation

An intake is said to be in the critical condition when the shock system intersects the cowl lip & the capture area ratio is then unity. Critical operation should occur at the A/C design Mach No.

(c) Sub-critical operation

If the engine requires less air than the intake is delivering, then a back pressure will rise ahead of the comp. This back pressure will expel the N.S, which becomes slightly curved & the excess air will spill over the cowl, so that intake delivery matches engine demand. Sub-critical operation is undesirable because it gives rise to high drag forces.

$$\frac{A_\infty}{A_i} < 1$$

(d) Super-critical operation

This operation occurs when the engine demands more air than the intake can deliver. The suction created causes the N.S to occur inside the intake, and therefore at a higher M (since the diffuser is now acting as a supersonic duct & accelerating the air ahead of the shock), the N.S is stronger & increases stagnation press. drop. At the same time, η_i will be reduced, & no drag is generated.

Intake Drag: Three kinds of drag force are generated by an air intake: Cowl drag, Friction drag and pre-entry drag. These forces can become very large under certain conditions.

Cowl and Friction Drag: Cowl drag is the pressure drag exerted on the vertical projection of the cowl by the high pressure flow field behind the external shock system.

This drag is very small & reduces with capture area ratio. It may actually have a negative value during sub-critical operation, because the subsonic flow over the cowl creates a suction force. Friction drag is also quite small. It remains sensibly constant at a given Mach no.

Pre-entry Drag: The thrust generated by a power plant arises from the rate of change of momentum imposed on the free stream air by that power plant. During sub-critical operation, however, some pressure rise occurs in the free stream, this momentum change does not react on the ... & so gives no thrust. The pre-entry drag is very large when the capture area ratio is small, this is why sub-critical operation is undesirable.

Boundary layer Effects: If an intake is located close to ... structure (as in a wing-root or under-wing intake), so that all part of the Boundary layer is swallowed, there is a considerable reduction in pressure recovery. This is caused by complex shock - B.L interactions, so, it is desirable to bleed off the B.L. ahead of the intake. There are several ways of doing this. The Phantom, for example, use a perforated ramp & an inter ramp void.

Variable geometry intakes

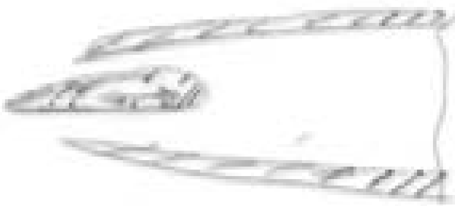
A fixed geometry intake has the great advantage of simplicity but will only operate (efficiently) at design M. at any off design operation, 2 - adverse phenomena will occur.

(a) The O-S. will not be touching the lip, as the S. angle depends on M ⇒ spillage below design M.

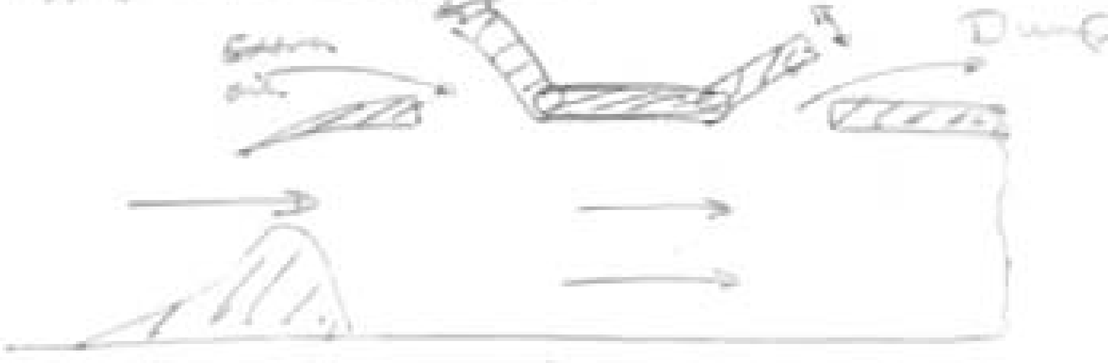
(b) The engine airflow demand will not be matched by the intake delivery, causing either sub or super critical operation.

For A/c which spend long periods at high M. the penalties caused by intake fixed geometry may be unacceptable. In such case, V.G.I. systems must be fitted. Any such system will have the following features.

(a) a moveable centre-body to position the O-S on the lip or movable ramps may be used.



(b) a system of auxiliary intakes & dump doors to take in extra air when engine demand exceeds delivery or dump air when intake supply exceeds demand



(c) some system of B.L. bleed

The Phantom VG intake system uses 3-S ramp, with movable ramps & auxiliary air doors.

6-5-..

x .. up stream.
y .. down "

The Total pressure ratio across a N.S can be calculated using:-

$$\frac{P_{ty}}{P_{tx}} = \left\{ \frac{\frac{\varkappa+1}{2} M_x^2}{1 + \frac{\varkappa-1}{2} M_x^2} \right\}^{\frac{\varkappa}{\varkappa-1}} \left\{ \frac{2\varkappa}{\varkappa+1} M_x^2 - \frac{\varkappa-1}{\varkappa+1} \right\}^{-\frac{1}{\varkappa-1}} \quad \text{- - - - (x)}$$

where M_x .. is the upstream Mach no. & M_y is calculated using:-

$$M_y = \left\{ \frac{M_x^2 + \frac{2}{\varkappa-1}}{\frac{2\varkappa}{\varkappa-1} M_x^2 - 1} \right\}^{1/2} \quad \text{- - - (xx)}$$

For oblique S.W, the total press ratio can be calculated using eq -(x) but with $M_x = M_1 \sin\beta$ & M_2 is calculated by eq -(xx) but with $M_y = M_2 \sin(\beta - \theta)$

and

$$\tan(\beta - \theta) = \frac{\frac{2}{\varkappa+1} + \frac{\varkappa-1}{\varkappa+1}(M_1 \sin\beta)^2}{M_1^2 \sin\beta \cos\beta}$$

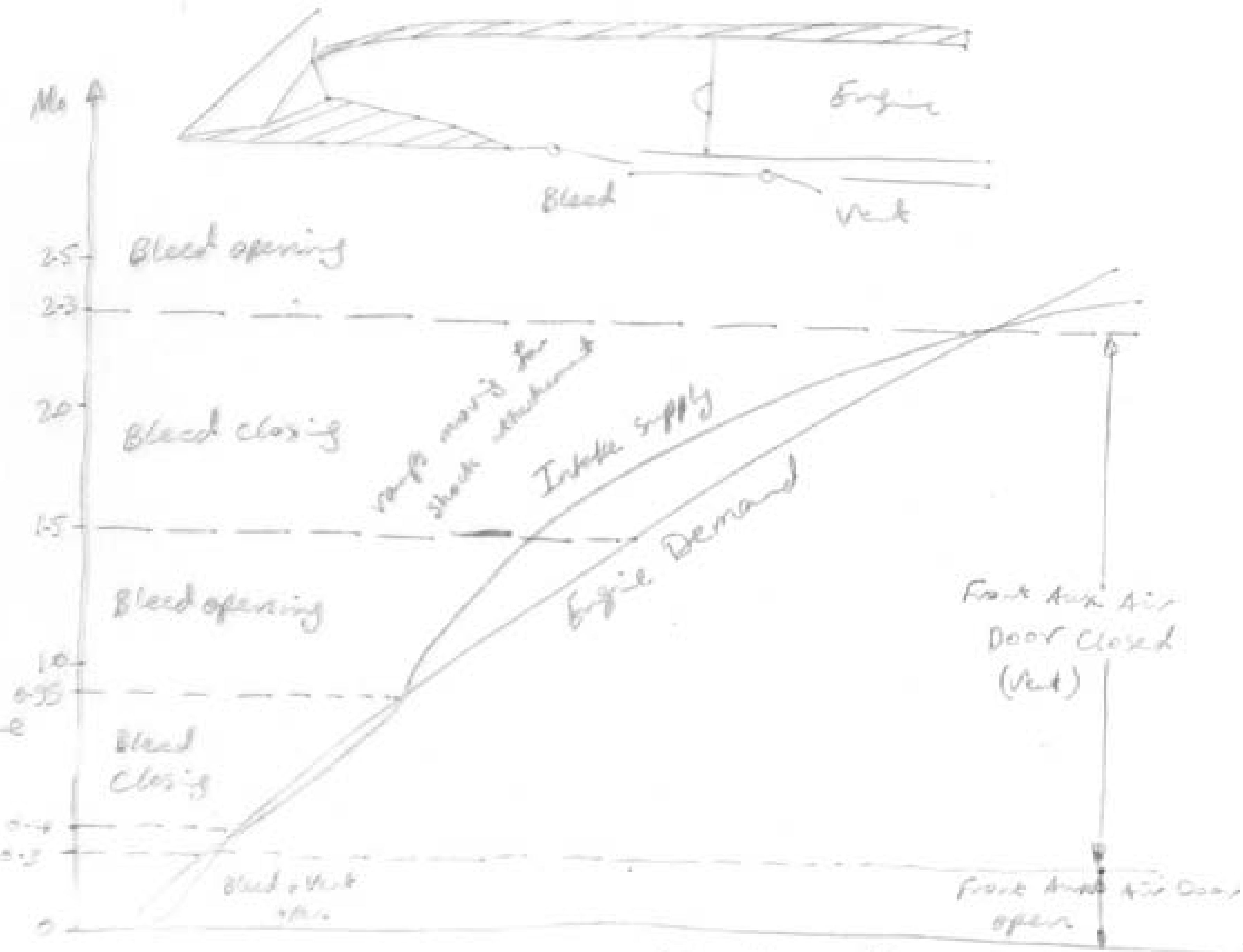
$$\Rightarrow \tan\theta = \frac{2\cot\beta\,(M_1^2 \sin^2\beta - 1)}{2 + M_1^2(\varkappa + 1 - 2\sin^2\beta)}$$

and

$$(\sin^2\beta)_{\theta_{max}} = \frac{(\varkappa+1)M_1^2 - 4 + \{(\varkappa+1)[(\varkappa+1)M_1^4 + 8(\varkappa-1)M_1^2 + 16]\}^{1/2}}{4\varkappa M_1^2}$$

$$(\sin^2\beta)_{M_2=1} = \frac{(\varkappa+1)M_1^2 - (3-\varkappa) + \{(\varkappa+1)[(\varkappa+1)M_1^4 - 2(3-\varkappa)M_1^2 + \varkappa + 9]\}^{1/2}}{4\varkappa M_1^2}$$

Intake-Engine Matching



Air Mass Flow

CHAPTER ~~Seven~~ Six: Exhaust Nozzle

7.1 Purpose: The purpose of the Exhaust Nozzle is to increase the velocity of the exhaust gas before discharge from the nozzle & to collect & straighten the gas flow. For large values of thrust, the kinetic energy must be high which implies a high exhaust velocity. The max. thrust for a given engine is obtained when the exit Press equals the ambient.

7.2 Functions:

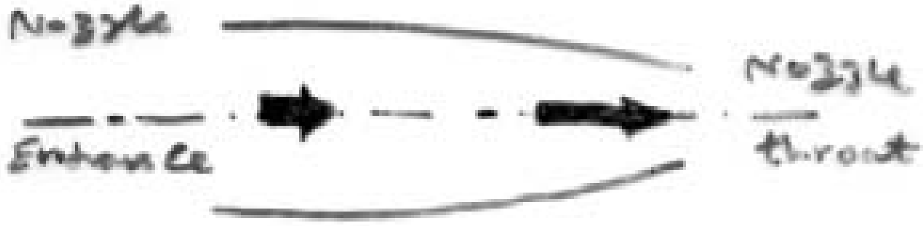
The function of the exhaust nozzle can be summarized in the following.

× Accelerate the flow to a high velocity with minimum total press. loss.
× Match exit & atmospheric press. as closely as possible.
× Permit AB operation without affecting engine operation.
× Allow cooling of walls if necessary.
× Mix core & bypass streams of turbofan if necessary.
× Allow for thrust reversing if desired.

* suppress jet noise & IR radiation, if desired.
* perform all the above with minimal cost, weight, drag, while meeting life & reliability goals.

7.3 Types:-

7.3.1 <u>Convergent nozzle</u>:- This is a simple convergent duct used when the pressure ratio (P_{te}/P_0) is low (< about 4). This type has generally been used in engines for subsonic A/cs.



7.3.2 <u>C-D Nozzle</u>

This nozzle is a convergent duct followed by a divergent duct. When the x-sectional area of the duct is at minimum, the nozzle is said to have a throat. The C-D nozzle is used if the nozzle press. ratio is high (greater than about >6). If the engine incorporates an A.B the nozzle throat is scheduled to leave the operating conditions of the engine upstream of the A.B unchanged (in other words; vary the exit nozzle area so that, the engine does not know that AB is working, also the exit area must be varied to match the different flow conditions

So that the engine does not know that the A.B is in operation). Also the exit area must be varied to match the different flow conditions & to produce the max. available thrust.

A_8 .. primary nozzle throat area , θ .. primary nozzle half angle
A_9 .. secondary " exit area, α secondary nozzle half angle.
L_s .. secondary nozzle length.

7.4 Nozzle coefficients

Gross thrust coefficient: C_{fg} is defined as the ratio of the actual gross thrust ($F_{g act}$) to the ideal gross thrust ($F_{g ideal}$)

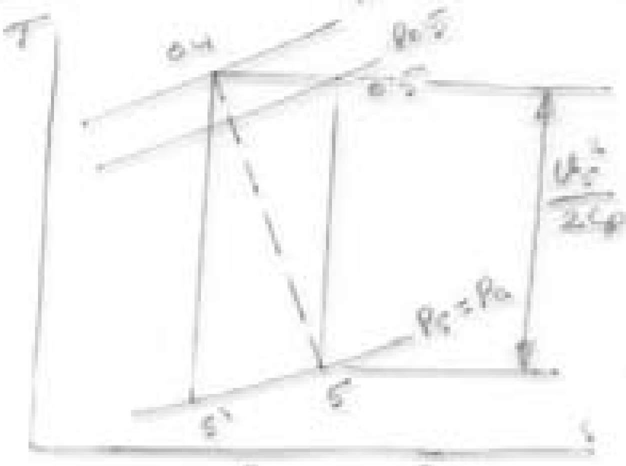
$$C_{fg} = F_{g act} / F_{g ideal} \quad --- \quad 7.1$$

Empirically derived coefficients are applied to eq. 7.1 to account for the losses of the actual nozzle flow. Each engine organization uses somewhat different coefficients, but each of the following losses are accounted for:-

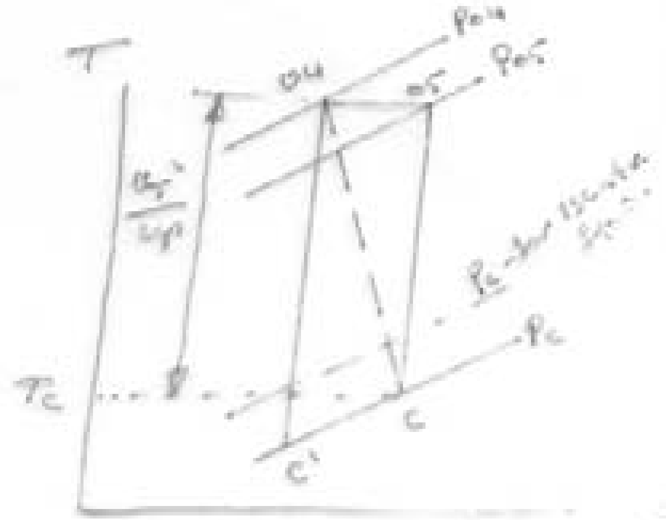
- Thrust loss due to the exhaust velocity vector angularity.
- due to reduction in velocity magnitude caused by friction in the boundary layer.
- due to loss of mass flow between stations 7 & 8 ...

If we assume isentropic flow, the critical pressure ratio

$$\frac{P_{04}}{P_c} = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} = 1.853 \quad \text{when } \gamma = 1.333.$$



when $\frac{P_{04}}{P_a} < \frac{P_{04}}{P_c}$



when $\frac{P_{04}}{P_a} > \frac{P_{04}}{P_c}$

$v_5 = \sqrt{\gamma R T_c}$

$$\eta_j = \frac{T_{04} - T_5}{T_{04} - T_5'}$$

when P_{04}, T_{04} are given, η_j & T_5 are assumed

$$T_{04} - T_5 = \eta_j T_{04}\left[1 - \left(\frac{1}{(P_{04}/P_5)}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$= \frac{v_5^2}{2c_p} \quad \text{as } T_{04} = T_{05}$$

∴ pressure ratios up to the critical value $P_5 = P_a$ & the specific pressure thrust $A_5(P_5 - P_a)/m = \text{zero}$

above the critical pressure ratio the nozzle is choked, $P_5 = P_c$ & $v_5 = \sqrt{\gamma R T_5}$,

$$\frac{T_{04}}{T_5} = \frac{T_{05}}{T_5} = 1 + \frac{v_5^2}{2c_p T_5} = 1 + \frac{\gamma-1}{2} M_5^2, \quad M_5 = 1 \Rightarrow$$

$$\boxed{\frac{T_{04}}{T_c} = \frac{\gamma+1}{2}}$$

$$T_c' = T_{04} - \frac{1}{\eta_j}(T_{04} - T_c)$$

$$P_c = P_{o4}\left(\frac{T_c}{T_{o4}}\right)^{\frac{\gamma}{\gamma-1}} = P_{o4}\left[1-\frac{1}{\eta_j}\left(1-\frac{T_c}{T_{o4}}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{o4}}{P_c} = \frac{1}{\left[1-\frac{1}{\eta_j}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/\gamma-1}}$$

Nozzle area:

$$A_5 = \frac{m}{\rho_c V_c}$$

$$\rho_c = \frac{P_c}{RT_c} \quad , V_c = \sqrt{\gamma R T_c} \text{ or } = \left[2c_p(T_{o4}-T_c)\right]^{1/2}$$

Specific thrust

$$F_s = (V_5 - V_a) + \frac{A_5}{m}(P_5 - P_a)$$

where V_a -- A/C forward speed.

station (4) -- turbine exit

" (5) -- Nozzle exit.

(c) -- critical