Structure of matter

& Electric Charges

first lecture

Dr. Faten Monjed Hussein

Electrical physics for medical physics

الفيزياء الكهربائية للفيزياء الطبية

المستوى الأول

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Structure of matter

Atomic Model

In 1897, J. J. Thomson identified the electron as a charged particle and as a constituent of the atom. This led to the first atomic model that contained internal structure.

Atomic model was developed in1911 in which each atom is made up of electrons surrounding a central nucleus.(A nucleus of gold is shown in Figure 1.2)

This model leads, however, to a new question: Does The nucleus has structure? That is, is the nucleus a single particle or a collection of particles?

By the early 1930s, a model evolved that described two basic entities in the nucleus: protons and neutrons.

The proton carries a positive electric charge, and a specific chemical element is identified by the number of protons in its nucleus. This number is called the **atomic number** of the element.

For instance, the nucleus of hydrogen atom contains one proton (so the atomic number of hydrogen is 1).

The nucleus of a helium atom contains two protons (atomic number 2),
and the nucleus of a uranium atom contains 92 protons (atomic number 92).

In addition to atomic number, a second number—**mass number,** defined as the number of protons plus neutrons in a nucleus—characterizes atoms.

The atomic number of a specific element never varies (i.e., the number of protons does not vary), but the mass number can vary (i.e., the number of neutrons varies).

This model leads, however, to a new question:

Is that, where the process of breaking down stops?

Protons, neutrons, and a host of other exotic particles are now known to be composed of six different varieties of particles called **quarks,** which have been given the names of *up, down,*

Strange, charmed, bottom and *top.* The up, charmed, and top quarks have electric charges of +2/3 that of the proton, whereas the down, strange, and bottom quarks have charges of -1/3 that of the proton.

The proton consists of two up quarks and one down quark as shown at the bottom of Figure 1.2 and labeled u and d.

This structure predicts the correct charge for the proton. Likewise, the neutron consists of two down quarks and one up quark, giving a net charge of zero.

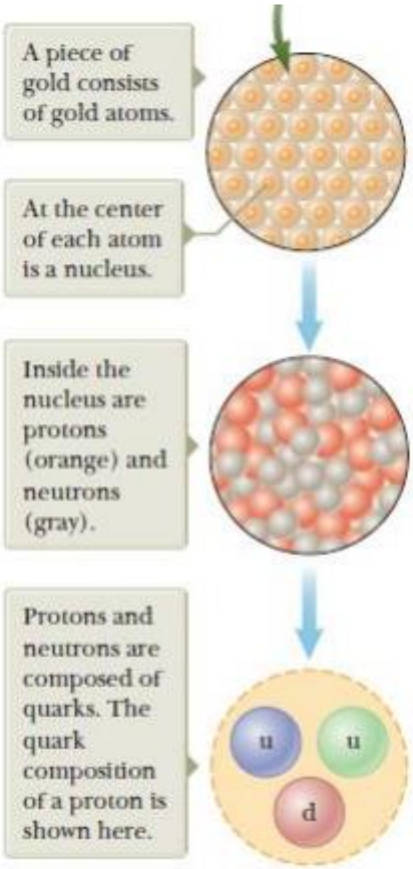


Figure 1.2 Levels of organization in matter.

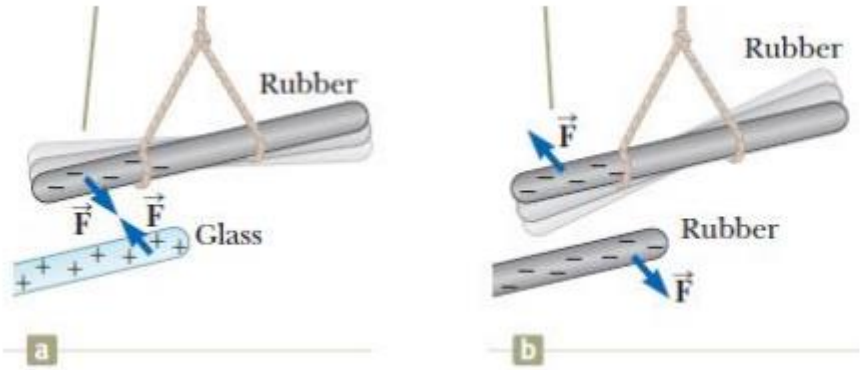
The Charge

In a series of simple experiments, it was found that there are two kinds of electric charges, which were given the names **positive** and **negative** by Benjamin Franklin (1706–1790).

Electrons are identified as having negative charge, and protons are positively charged.

To verify that there are two types of charge, suppose a hard rubber rod that has been rubbed on fur is suspended by a string

When a glass rod that has been rubbed on silk is brought near the rubber rod, the two attract each other (Fig. 23.1a). On the other hand, if two charged rubber rods (or two charged glass



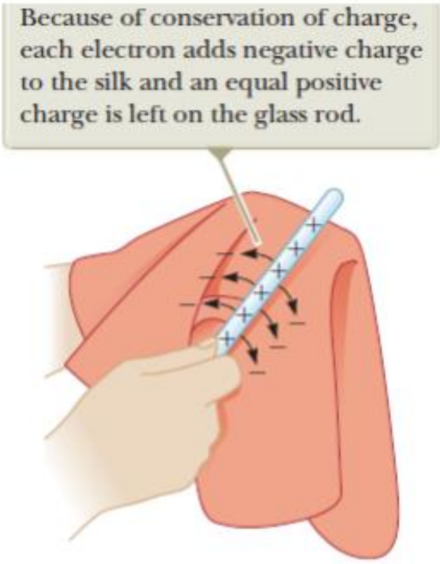


Figure 23.2 When a glass rod is rubbed with silk, electrons are transferred from the glass to the silk. Also, because the charges are transferred in discrete bundles, the charges on the two objects are $\pm e$, or $\pm 2e$, or $\pm 3e$, and so on.

rods) are brought near each other as shown in Figure 23.1b, the two repel each other.

This observation shows that the rubber and glass have two different types of charge on them. On the basis of these observations, we conclude that **charges of the same sign repel one another and charges with opposite signs attract one another.**

Electric charge is conserved

In 1909, Robert Millikan (1868–1953) discovered that electric charge always occurs as integral multiples of a fundamental amount of charge *e*.

The electric charge *q* is said to be **quantized,** where *q* is the standard. Symbol used for charge as a variable. That is, electric charge exists as discrete

- Electric charge is always conserved in an isolated system
  - For example, charge is not created in the process of rubbing two objects together
  - The electrification is due to a transfer of charge from one object to another
- The electric charge, *q*, is said to be quantized
  - *q* is the standard symbol used for charge as a variable
  - Electric charge exists as discrete packets
  - $q = \pm Ne$
    - *N* is an integer
    - *e* is the fundamental unit of charge
    - $|e| = 1.6 \times 10^{-19}$ C
    - Electron: $q = -e$
    - Proton: $q = +e$

Conductors:

- Electrical conductors are materials in which some of the electrons are free electrons
  - Free electrons are not bound to the atoms
  - These electrons can move relatively freely through the material
  - Examples of good conductors include copper, aluminum and silver
  - When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material

Semiconductors

- The electrical properties of semiconductors are somewhere between those of insulators and conductors
- Examples of semiconductor materials include silicon and germanium

Insulators

- Electrical insulators are materials in which all of the electrons are bound to atoms
  - These electrons cannot move relatively freely through the material
  - Examples of good insulators include glass, rubber and wood
  - When a good insulator is charged in a small region, the charge is unable to move to other regions of the material

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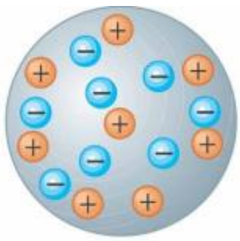
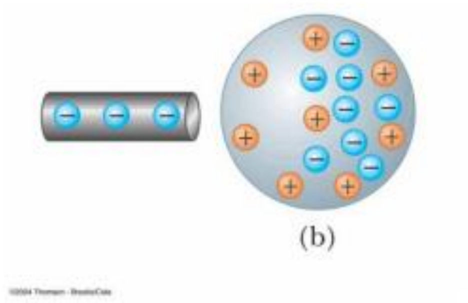
Charging by Induction

- Charging by induction requires no contact with the object inducing the charge
- Assume we start with a neutral metallic sphere
- The sphere has the same number of positive and negative charges



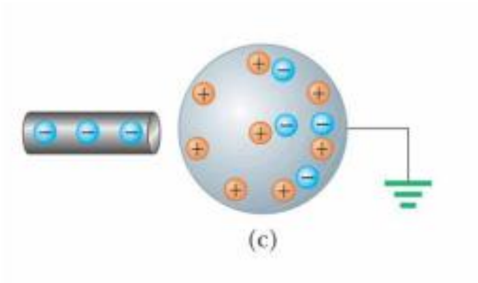
(a)

- A charged rubber rod is placed near the sphere
- It does **not** touch the sphere
- The electrons in the neutral sphere are redistributed



(b)

- The sphere is grounded
- Some electrons can leave the sphere through the ground wire



(c)

- The ground wire is removed
- There will now be more positive charges
- The charges are not uniformly distributed
- The positive charge has been ***induced*** in the sphere

- The rod is removed
- The electrons remaining on the sphere redistribute themselves
- There is still a net positive charge on the sphere
- The charge is now uniformly distributed

Coulomb's Law

Second lecture

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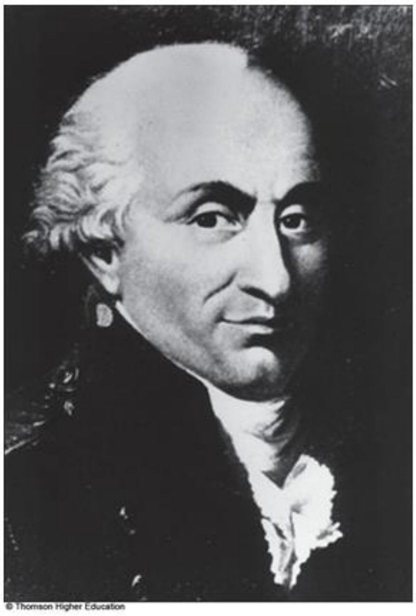
Coulomb’s Law

Charles Coulomb

- 1736 – 1806
- French physicist
- Major contributions were in areas of electrostatics and magnetism
- Also investigated in areas of
  - Strengths of materials
  - Structural mechanics
  - Ergonomics



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Point Charge

- The term **point charge** refers to a particle of zero size that carries an electric charge
- The electrical behavior of electrons and protons is well described by modeling them as point charges
- The electrical force between two stationary point charges is given by **Coulomb’s Law**

- The force is inversely proportional to the square of the separation *r* between the charges and directed along the line joining them
- The force is proportional to the product of the charges, q_1 and q_2, on the two particles
- The force is attractive if the charges are of opposite sign
- The force is repulsive if the charges are of like sign
- The force is a conservative force
- Mathematically:

$$F = k_e \frac{|q_1||q_2|}{r^2}$$

The SI unit of charge is the **coulomb** (C)

k_e is called the **Coulomb constant**

$k_e = 8.9876 \times 10^9 \text{ N·m}^2/\text{C}^2 = 1/(4\pi\varepsilon_o)$

ε_o is the **permittivity of free space**

$\varepsilon_o = 8.8542 \times 10^{-12} \text{ C}^2 / \text{N·m}^2$

Remember the charges need to be in coulombs, *e* is the smallest unit of charge **(except quarks)**

- $e = 1.6 \times 10^{-19}$ C
- So 1 C needs 6.24×10^{18} electrons or protons

- Typical charges can be in the μC range
- Remember that force is a *vector* quantity

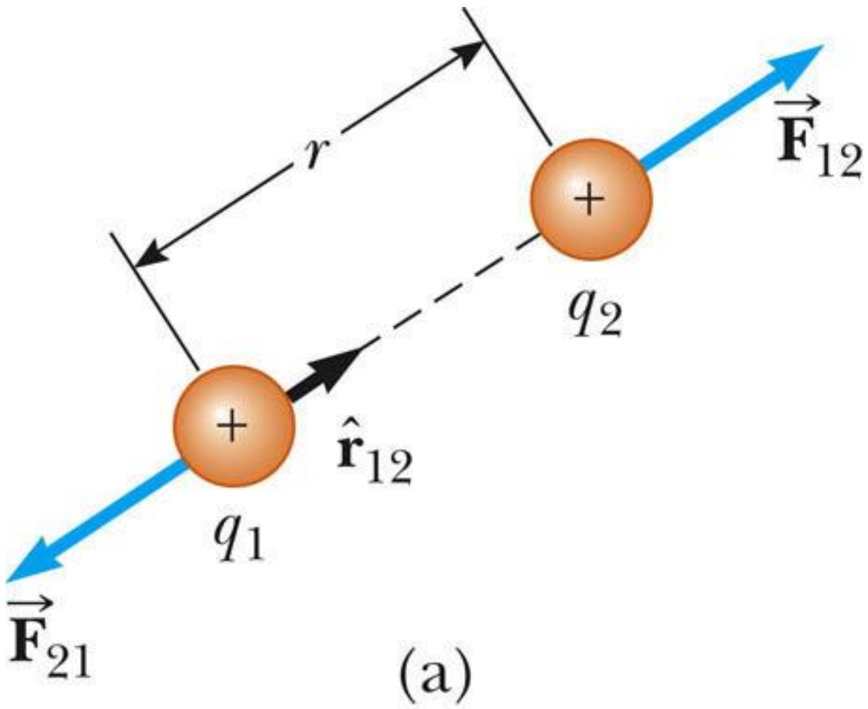
TABLE 23.1

Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

Vector Nature of Electric Forces:

- In vector form, is a unit vector directed from q_1 to q_2
  - The like charges produce a repulsive force between them



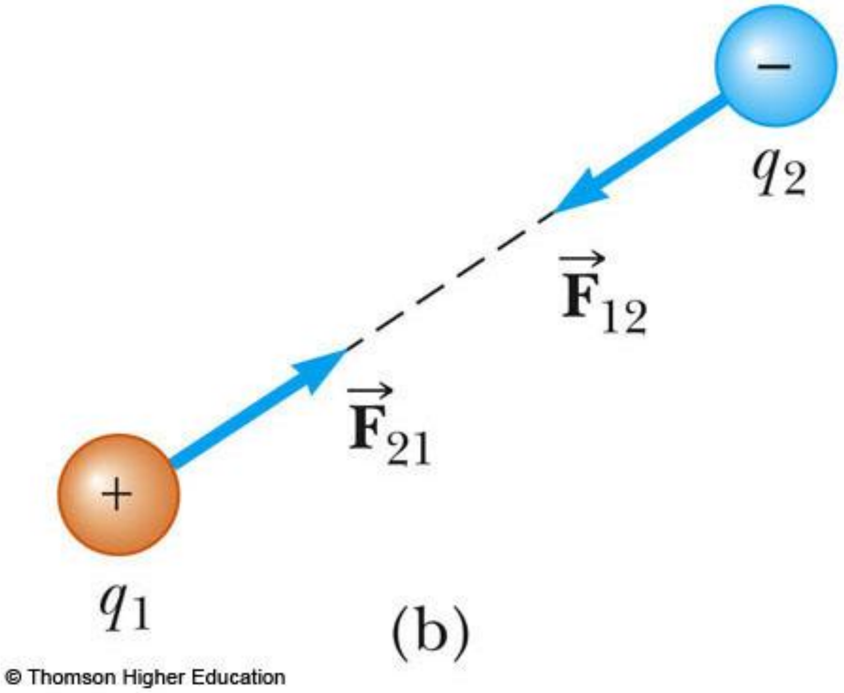
(a)

- Electrical forces obey Newton's Third Law
- The force on q_1 is equal in magnitude and opposite in direction to the force on q_2
- With like signs for the charges, the product q_1q_2 is positive and the force is repulsive

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$$

-
- Two point charges are separated by a distance r

- The unlike charges produce an attractive force between them
- With unlike signs for the charges, the product q1q2 is negative and the force is attractive
- Use the active figure to investigate the force for different positions



The Superposition Principle

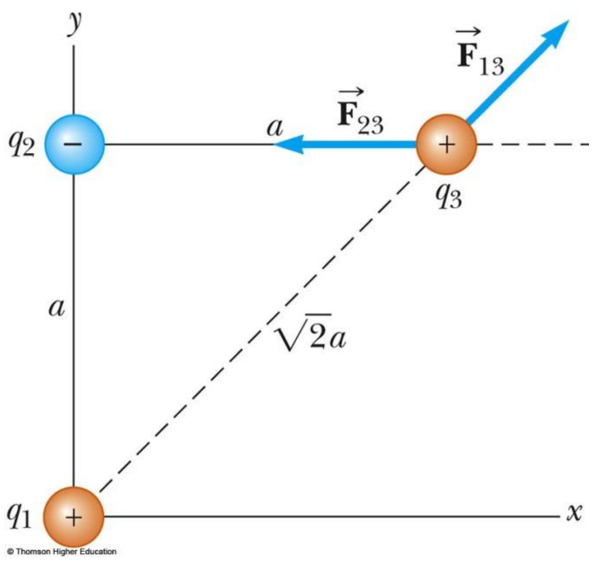
- The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present
- Remember to add the forces as vectors

- The resultant force on q_1 is the vector sum of all the forces exerted on it by other charges:

$$\overrightarrow{F_1} = \overrightarrow{F_{21}} + \overrightarrow{F_{31}} + \overrightarrow{F_{41}}$$

EX1:

Consider three point charges located at the corners of a right triangle as shown in Figure below, where $q_1=q_3=5.0\ \mu C$, $q_2=-2.0\ \mu C$, and $a=0.10$ m. Find the resultant force exerted on q_3.



Solution:

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= 8.99\times10^9 N.m^2/C^2 \frac{(2.0\times10^{-6}C)(5.0\times10^{-6}C)}{(0.10m)^2} = 9.0N$$

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= 8.99\times10^9 N.m^2/C^2 \frac{(5.0\times10^{-6}C)(5.0\times10^{-6}C)}{2(0.10m)^2} = 11N$$

$$F_{13x} = F_{13}\cos 45^\circ = 7.9N$$

$$F_{13y} = F_{13}\sin 45^\circ = 7.9N$$

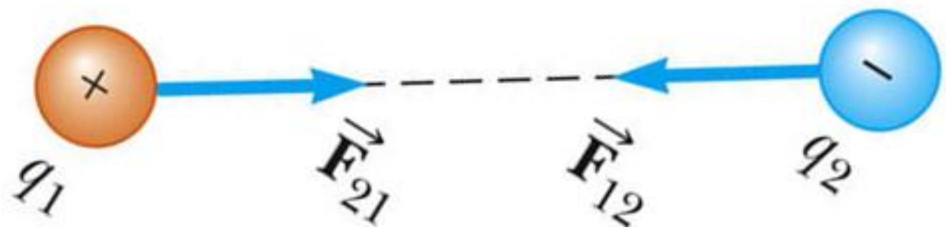
$$F_{3x} = F_{13x} + F_{23x} = 7.9N + (-9.0N) = -1.1N$$

$$F_{3y} = F_{13y} + F_{23y} = 7.9N + 0 = 7.9N$$

$$\vec{F}_3 = (-1.1\hat{i} + 7.9\hat{j})N$$

EX.2

Two point charges, $q_1 = +25nC$ and $q_2 = -75nC$, are separated by a distance of 3cm. Find the magnitude and direction of the electric force that q_1 exerts on q_2;



Solution:

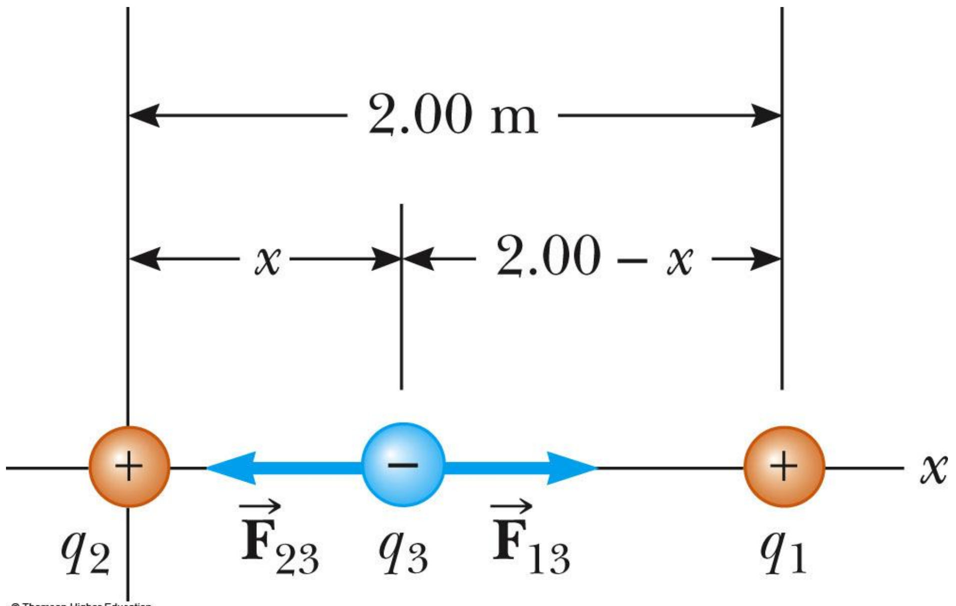
$$F_{12} = K_e \frac{|q_1||q_2|}{r^2}$$

$$F_{12} = 9 \times 10^9 \frac{|25 \times 10^{-9}||-75 \times 10^{-9}|}{(3 \times 10^{-2})^2}$$

$$F_{12} = 1.875 \times 10^{-5} N$$

Ex.4:

Three point charges lie along the x axis as shown in Figure. The positive charge q_1=15μc is at x = 2.00 m, the positive charge q_2=6μc is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3?



Analyze Write an expression for the net force on charge q_3 when it is in equilibrium:	$\vec{F}_3 = \vec{F}_{23} + \vec{F}_{13} = -k_e \frac{\lvert q_2\rvert \lvert q_3\rvert}{x^2}\hat{i} + k_e \frac{\lvert q_1\rvert \lvert q_3\rvert}{(2.00 - x)^2}\hat{i} = 0$
Move the second term to the right side of the equation and set the coefficients of the unit vector $\hat{i}$ equal:	$k_e \frac{\lvert q_2\rvert \lvert q_3\rvert}{x^2} = k_e \frac{\lvert q_1\rvert \lvert q_3\rvert}{(2.00 - x)^2}$
Eliminate k_e and $\lvert q_3\rvert$ and rearrange the equation:	$(2.00 - x)^2 \lvert q_2\rvert = x^2 \lvert q_1\rvert$ $(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \mathrm{C}) = x^2(15.0 \times 10^{-6}\ \mathrm{C})$
Reduce the quadratic equation to a simpler form:	$3.00x^2 + 8.00x - 8.00 = 0$
Solve the quadratic equation for the positive root:	$x = \boxed{0.775\ \mathrm{m}}$

Electric Field

third lecture

Dr. Faten Monjed Hussein

physics for medical physics

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Electric Field

Introduction:

- The electric force is a field force, Field forces can act through space
- The effect is produced even with no physical contact between objects
- Faraday developed the concept of a field in terms of electric fields

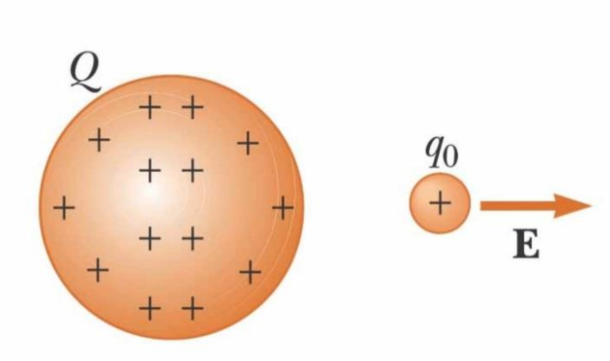
Electric Field – Definition

- An electric field is said to exist in the region of space around a charged object this charged object is the source charge
- When another charged object, the test charge, enters this electric field, an electric force acts on it
- The electric field is defined as the electric force on the test charge per unit charge
- The electric field vector, $\vec{\boldsymbol{E}}$, at a point in space is defined as the electric force $\vec{\boldsymbol{F}}$

acting on a positive test charge, q_o placed at that point divided by the test charge:

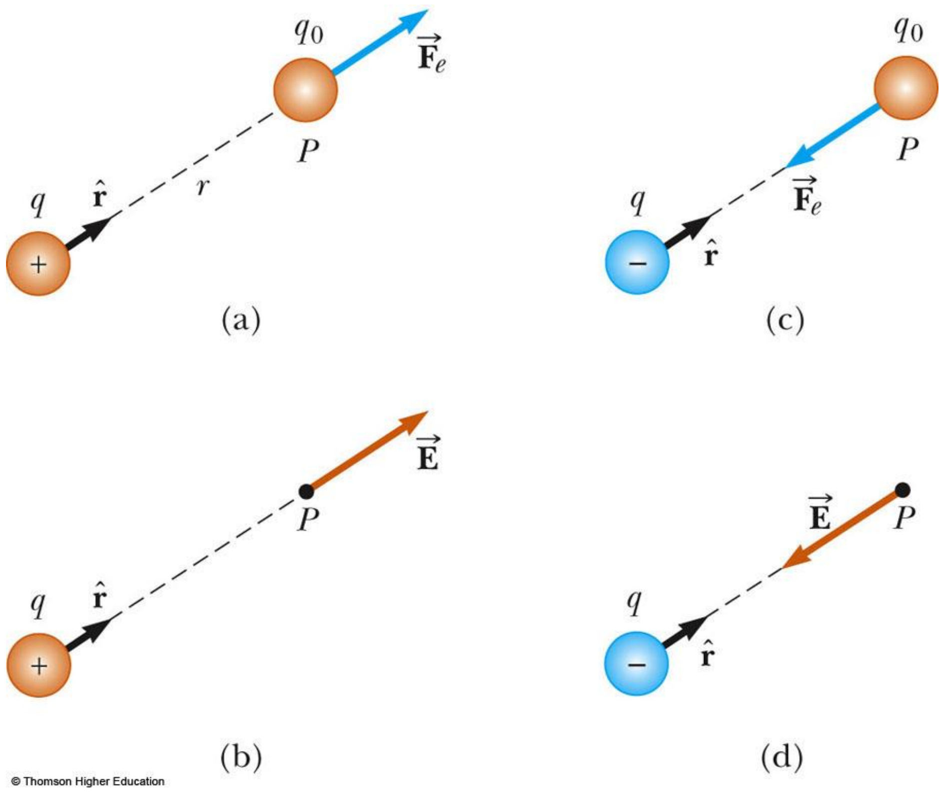
$$\vec{E} \equiv \frac{\vec{F}}{q_0}$$

- The field produced by some charge or charge distribution, separate from the test charge
- The existence of an electric field is a property of the source charge.
- The presence of the test charge is not necessary for the field to exist
- The test charge serves as a detector of the field



- The direction of $\vec{\boldsymbol{E}}$ is that of the force on a positive test charge
- The SI units of $\vec{\boldsymbol{E}}$ are **N/C**
- $\vec{F} = q_o\vec{E}$. This is valid for a point charge only (of zero size)

- For larger objects, the field may vary over the size of the object
- If q is positive, the force and the field are in the same direction
- If q is negative, the force and the field are in opposite directions
- the electric field will be: $\vec{E} = \frac{\vec{F_e}}{q_o} = k_e \frac{|q|}{r^2} \hat{r}$



Example:

What is the magnitude of the electric field at apoint 2 m from a point charge q=4 nC?

Solution:

$$E = K_e \frac{|q|}{r^2}$$

$$E = 9 \times 10^9 \frac{|4 \times 10^{-9}|}{2^2} = 9 \frac{N}{C}$$

$$E = 9 \frac{N}{C} \vec{i}$$

....

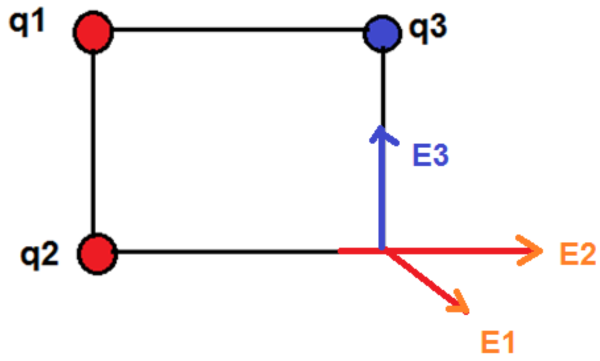
Superposition with Electric Fields

- At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges:

$$\vec{E} = K_e \sum_i \frac{q_i}{r_i^2} \hat{r}$$

Example:

Charges q_1=5μC , q_2=3 μC and q_3=-6 μC are located on the corner of square (its edge a=2 Cm), Find the components of the net electric field at the point P, shown in figure.



$$E = K_e \frac{|q|}{r^2}$$

$$E_1 = 9 \times 10^9 \frac{|5 \times 10^{-6}|}{\left(2\sqrt{2} \times 10^{-2}\right)^2}$$

$$= 5.63 \times 10^7 N/C$$

$$E_2 = 9 \times 10^9 \frac{|3 \times 10^{-6}|}{(2 \times 10^{-2})^2} = 6.75 \times 10^7 N/C$$

$$E_3 = 9 \times 10^9 \frac{|-6 \times 10^{-6}|}{(2 \times 10^{-2})^2} = 13.5 \times 10^7 \frac{N}{C}$$

$$\theta_1 = -45 \ , \theta_2 = 0, \quad \theta_3 = 90$$

$$\vec{E_1} = \left(5.63 \times 10^7 \cos(-45)\right)\vec{i} + \left(5.63 \times 10^7 \sin(-45)\right)\vec{j}$$

$$= 3.98 \times 10^7 \vec{i} - 3.98 \times 10^7 \vec{j}$$

$$\vec{E_2} = \left(6.75 \times 10^7 \cos(0)\right)\vec{i} + \left(6.75 \times 10^7 \sin(0)\right)\vec{j} = 6.75 \times 10^7 \vec{i}$$

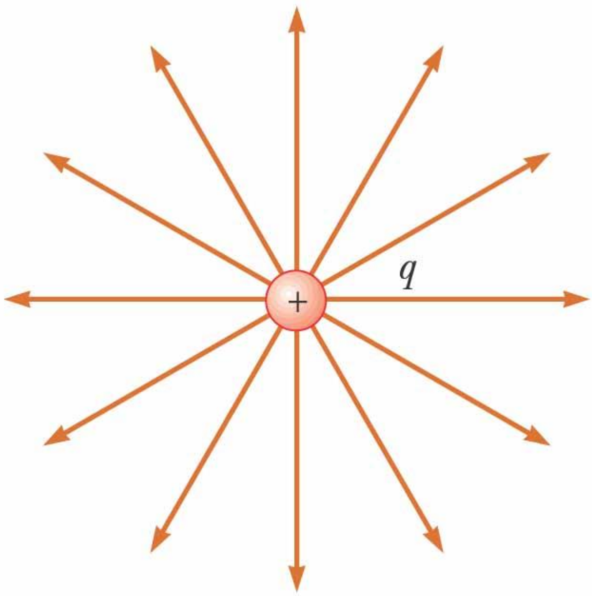
$$\vec{E_3} = \left(13.5 \times 10^7 \cos(90)\right)\vec{i} + \left(13.5 \times 10^7 \sin(90)\right)\vec{j}$$

$$= 13.5 \times 10^7 \vec{j}$$

$$\overrightarrow{E_P} = (25.55 \times 10^7)\vec{i} + (12.52 \times 10^7)\vec{j}$$

Electric Field Lines, Positive Point Charge

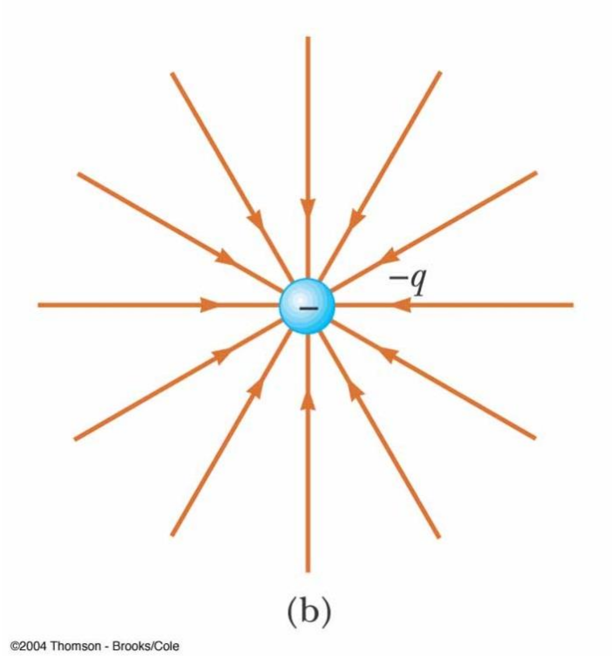
- The field lines radiate outward in all directions
- In three dimensions, the distribution is spherical
- The lines are directed away from the source charge



(a)

Electric Field Lines, Negative Point Charge

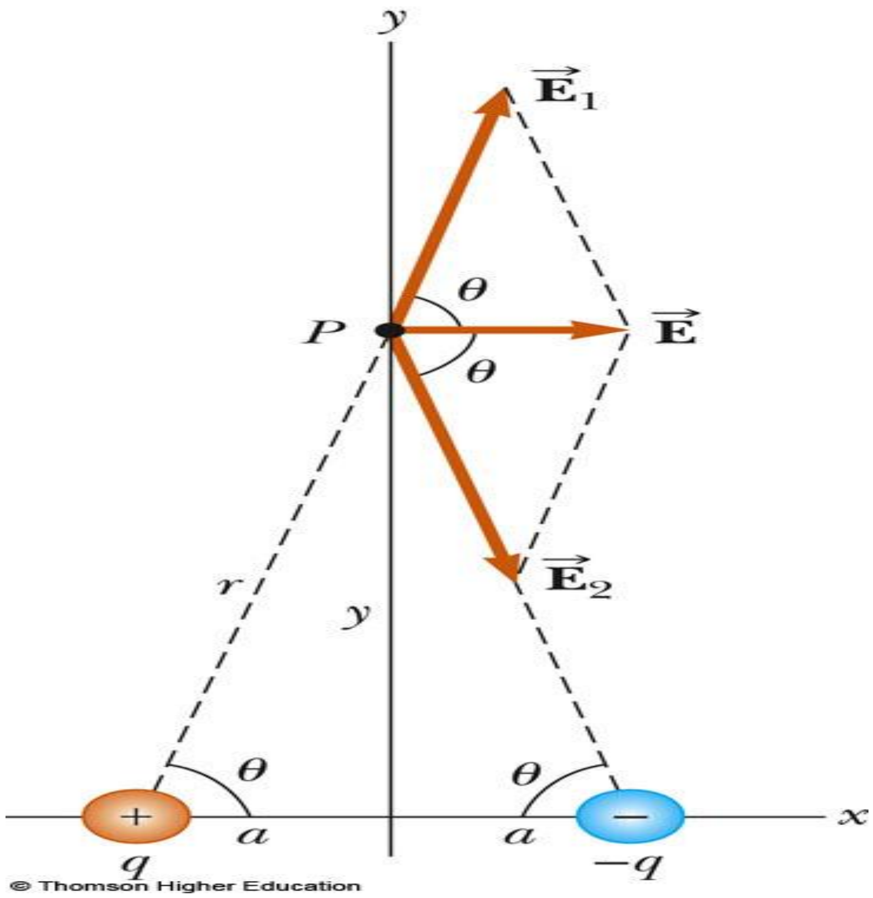
- The field lines radiate inward in all directions
- The lines are directed toward the source charge



EX.

Charges |q|=2μC, and are located as in figure. (a=2 Cm), $\theta = 60$, r=4 cm, y=3.46 cm

Find the components of the net electric field at the point P, shown in figure.



$$E_1 = E_2 = 9 \times 10^9 \times \frac{|2 \times 10^{-6}|}{(4 \times 10^{-2})^2} = 1.125 \times 10^7 N/C$$

$$\theta_1 = \theta_2 = \theta = 60$$

$$\vec{E_1} = \left(1.125 \times 10^7 \cos(60)\right)\vec{i} + \left(1.125 \times 10^7 \sin(60)\right)\vec{j}$$
$$= 0.563 \times 10^7 \vec{i} + 0.974 \times 10^7 \vec{j}$$

$$\vec{E_2} = \left(1.125 \times 10^7 \cos(-60)\right)\vec{i} + \left(1.125 \times 10^7 \sin(-60)\right)\vec{j}$$
$$= 0.563 \times 10^7 \vec{i} - 0.974 \times 10^7 \vec{j}$$

$$\overrightarrow{E_{tot}} = \left(1.125 \times 10^7\right)\vec{i} + (0)\vec{j} = 1.125 \times 10^7 \vec{i}$$

$$E_{tot} = \sqrt{(1.125 \times 10^7)^2} = 1.125 \times 10^7 N/C$$

Electric Flux and Gauss's Law

physics for medical physics

forth lecture

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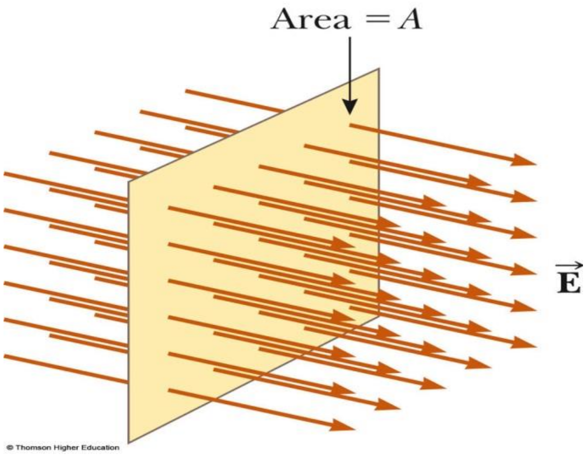
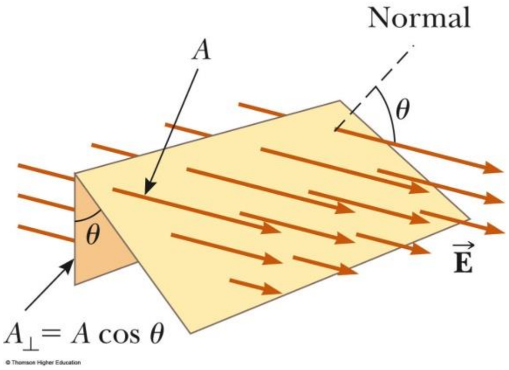
Electric Flux

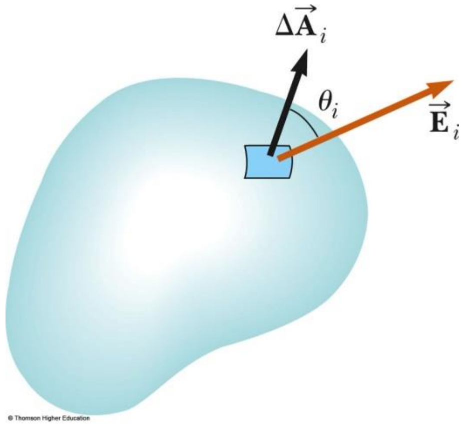
- ***Electric flux*** is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field
  - $\phi_E = \vec{E}.\vec{A}$





- The electric flux is proportional to the number of electric field lines penetrating some surface
- The field lines may make some angle θ with the perpendicular to the surface
- $\phi_E = EAcos\theta$
- The flux is a maximum when the surface is perpendicular to the field
- The flux is zero when the surface is parallel to the field

- **In the more general case, look at a small area element**



$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

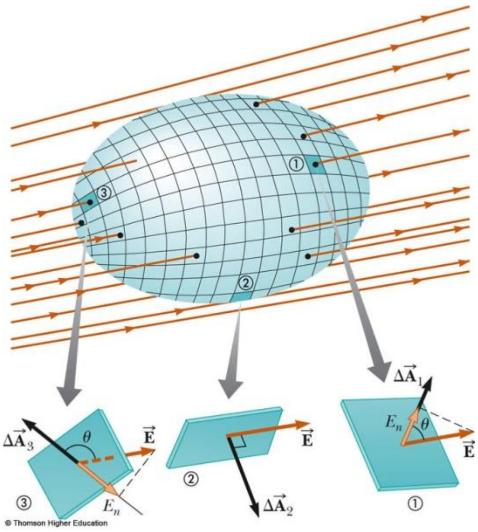
- **In general, this becomes:**

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum E_i \cdot \Delta A_i$$

$$\Phi_E = \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

- **In general, the value of the flux will depend both on the field pattern and on the surface**
- **The units of electric flux will be N·m^2/C**

Electric Flux, Closed Surface



- **Assume a closed surface**
- **The vectors $\overrightarrow{\Delta A_i}$ point in different directions**
- **At each point, they are perpendicular to the surface**
- **By convention, they point outward**

- **At (1), the field lines are crossing the surface from the inside to the outside; $\theta < 90^o$, Φ is positive**

- **At (2), the field lines graze surface; $\theta = 90^o$, $\Phi = 0$**

- **At (3), the field lines are crossing the surface from the outside to the inside; $180^o > \theta > 90^o$, Φ is negative**

- **The *net* flux through the surface is proportional to the net number of lines leaving the surface**

- **This net number of lines is the number of lines leaving the surface minus the number entering the surface**

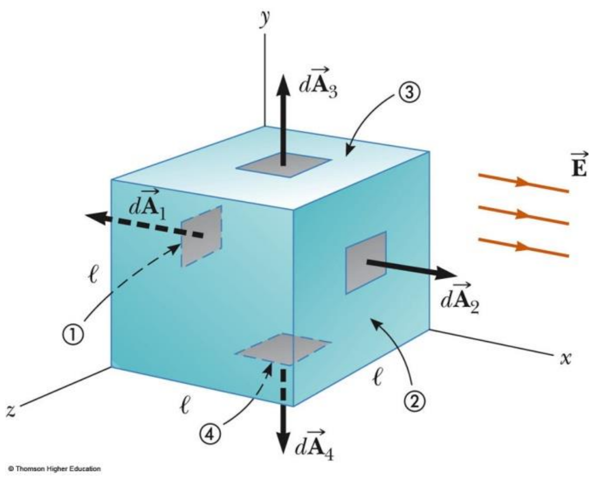
- **If E_n is the component of E perpendicular to the surface, then: $\phi_E = \oint \vec{E}.\overrightarrow{dA} = \oint E_n \, dA$**

Example:

Consider uniform electric field $\vec{E}$ directed in the empty space as shown in figure, and the cube has edge length (L), and is placed in the field. Find The Net electric Flux through the surfaces of the cube.

- **For side 1, $\Phi_E = -El^2$**

- For side 2, $\Phi_E = El^2$
- For the other sides,
- $\Phi_E = 0$
- Therefore, $\Phi_{Etotal} = 0$



Karl Friedrich Gauss

- 1777 – 1855

Made contributions in

- Electromagnetism
- Number theory
- Statistics
- Non-Euclidean geometry
- Cometary orbital mechanics
- A founder of the German Magnetic Union
- Studies the Earth's magnetic field

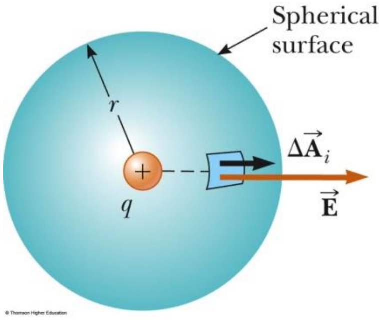
Gauss's Law

Introduction:

- **Gauss's law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface**

- **The closed surface is often called a *Gaussian surface***

- **Gauss's law is of fundamental importance in the study of electric fields**



- $\Phi_e = \oint \vec{E}.\overrightarrow{dA} = E_n \oint dA = \frac{q_{in}}{\varepsilon_o}$

  - q_{in} **is the net charge inside the surface,**
  - E_n **the normal component of $\vec{E}$ on surface vector**

- $\vec{E}$ **represents the electric field at any point on the surface**

- **Although Gauss's law can, in theory, be solved to find $\vec{E}$ for any charge configuration, in practice it is limited to symmetric situations.**

- **the net flux through any closed surface surrounding a point charge q is given by $\frac{q_{in}}{\varepsilon_0}$ and is independent of the shape of the surface**

- **the electric flux through a closed surface that surrounds no charge is zero**

EX.1

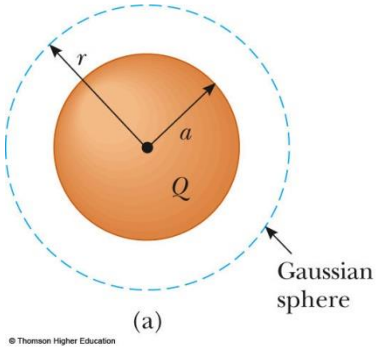
Find electric field due to spherically symmetric charge distribution (Q) (charge volume density is uniform), on insulator sphere which has radius (a), at point p where

;1) r>a, and at 2) r <a

Solution:

1)

$$\Phi_e = \oint \vec{E}.\vec{dA} = \oint E dA = \frac{Q_{in}}{\varepsilon_o}$$



(a)

$$\Phi_e = 4\pi r^2 E = \frac{Q_{in}}{\varepsilon_o}$$

$$E = \frac{1}{4\pi r^2}\frac{Q_{in}}{\varepsilon_o} = \frac{1}{4\pi\varepsilon_o}\frac{Q_{in}}{r^2} = k_e\frac{Q_{in}}{r^2}$$

2)

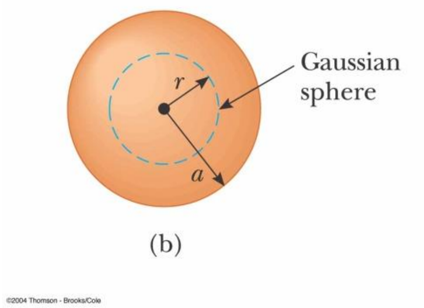
Select a sphere as the Gaussian surface, *r* < *a*

$q_{in} < Q$

$q_{in} = (Qr^3)/a^3$

$$\Phi_e = \oint \vec{E}.\vec{dA} = \oint E dA = \frac{Q_{in}}{\varepsilon_o}$$

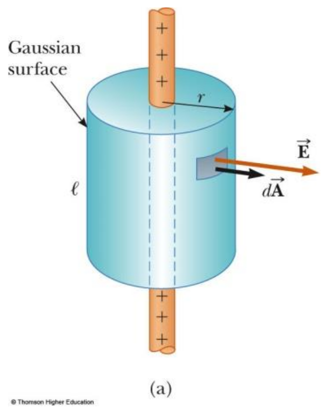
$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = k_e\frac{Q}{a^3}r$$



(b)

EX.2 Find electric Field at a Distance (r) from a Line of positive Charge (charges is uniformly distributed on the line). Use Gauss's law to find the field

$$\Phi_e = \oint \vec{E}.\vec{dA} = \oint E dA = \frac{Q_{in}}{\varepsilon_o}$$



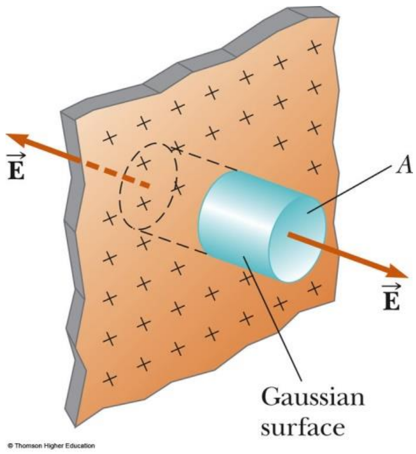
(a)

$$\Phi_e = E(2\pi r L) = \frac{Q_{in}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o} = 2k_o\frac{\lambda}{r}$$

EX.3

Find electric Field Due to a Plane of positive Charge at a Distance (r) from it. (charges is uniformly distributed on the surface)



- The flux through each end of the cylinder is *EA* and so the total flux is 2*EA*
- The total charge in the surface is σA
- Applying Gauss's law
- $\Phi_E = 2EA = \dfrac{\sigma A}{\varepsilon_o}$ and $E = \dfrac{\sigma}{2\varepsilon_o}$
- Note, this does not depend on *r*

Electric Potential

Dr. F. M. Alfeel

physics for medical physics

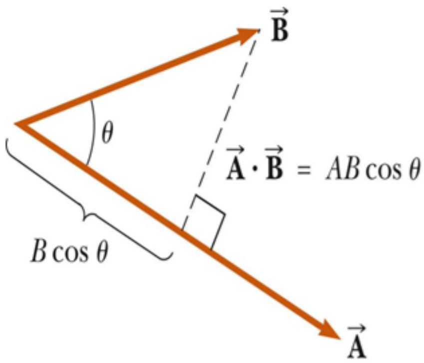
الفيزياء الكهربائية للفيزياء الطبية

forth lecture المحاضرة الرابعة

first year المستوى الأول

introduction

- Scalar Product of Two Vectors:
- The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$
  - It is also called the dot product
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\,cos\,\theta$
  - θ is the angle *between* A and B
- Applied to work, this means



$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

Conservative Forces and Potential Energy

- Define a potential energy function, U, such that the work done by a conservative force equals the decrease in the potential energy of the system
- The work done by such a force, F, is

$$W_C = \int_{x_i}^{x_f} F_x\,dx = -\Delta U$$

  - ΔU is negative when F and x are in the same direction

Work And Energy

- In the general case of a net force whose magnitude and direction may vary

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum \vec{\mathbf{F}}\right) d\vec{\mathbf{r}}$$

- The Work-Kinetic Energy Theorem states $\Sigma W = K_f - K_i = \Delta K$

$$K = \frac{1}{2} m v^2$$

Electrical Potential Energy

- When a test charge is placed in an electric field, it experiences a force $\vec{\mathbf{F}} = q_o \vec{\mathbf{E}}$
  - The force is conservative
- Because the force is conservative, the line integral does not depend on the path taken by the charge
- This is the change in potential energy of the system

- The work done by the electric field is

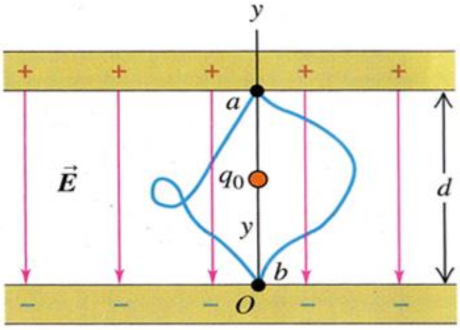
$$\vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- As this work is done by the field, the potential energy of the charge-field system is changed by
- $\Delta U = -q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$
- For a finite displacement of the charge from A to B,

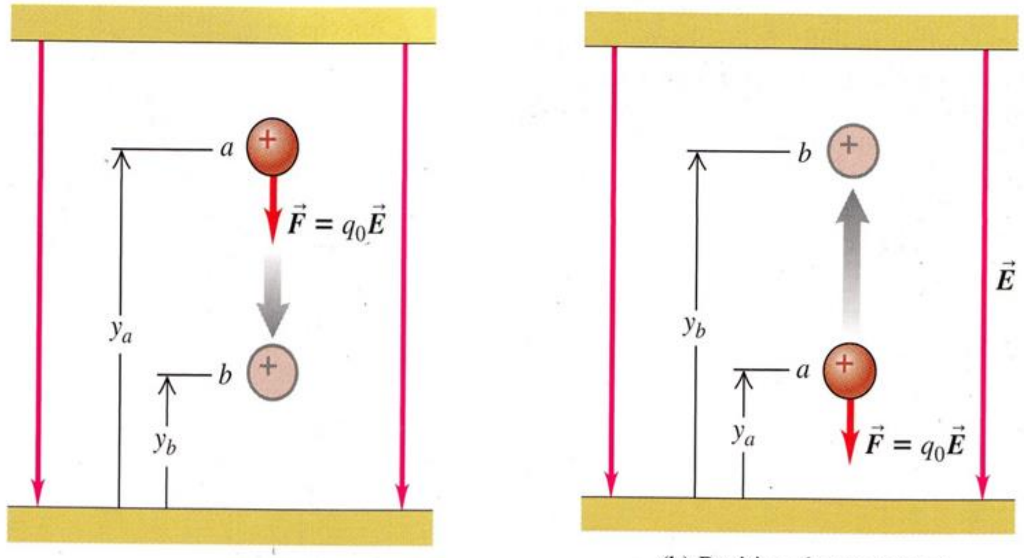
$$\Delta U = U_B - U_A = -q_o \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Electric Potential Energy in a Uniform Field:

- **In Fig. below a pair of charged parallel metal plates sets up a uniform, downward electric field with magnitude E.**



A test charge q_0 moving from a to b experiences a force of magnitude q_0E. The work done by this force is $W_{a\to b} = q_0Ed$ and is independent of the particle's path.



(a) Positive charge moves in direction of $\vec{E}$: field does positive work on charge, potential energy U decreases

(b) Positive charge moves in direction opposite $\vec{E}$: field does negative work on charge, potential energy U increases

Electric Potential

- The potential energy per unit charge, U/q_o, is the **electric potential** v: $V = \dfrac{U}{q_o}$

- The potential is a scalar quantity
- As a charged particle moves in an electric field, it will experience a change in potential $\Delta V = \dfrac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

Units

- J potential energy unit
- C charge unit
- 1 V = 1 J/C Electric Potential
- 1 N/C = 1 V/m electric field unit
- Electron-Volts is potential energy unit
- 1 eV = 1.60 x 10^{-19} J

Potential Difference in a Uniform Field

- The equations for electric potential can be simplified if the electric field is uniform

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B ds = -Ed$$

- Electric field lines always point in the direction of decreasing electric potential

Example: Electric force and electric potential

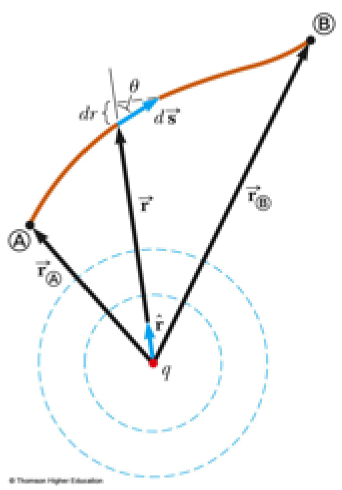
A proton (charge $e = 1.62 \times 10^{-19} C$) moves in straight line from point a to point b inside a linear accelerator, a total distance d=0.5 m. the electric field is$E = 1.5 \times 10^{7} N/C$ uniform along this line, with magnitude in the direction from a to b. Determine a) the force on the proton; b) the work done on it by the field; c) the potential difference $V_a - V_b$.

SOLUTION:

$$(a) F = qE$$
$$= (1.602 \times 10^{-19})(1.5 \times 10^{7})$$
$$F = 2.4 \times 10^{-12} N$$
$$(b) W_{a\to b} = Fd = (2.4 \times 10^{-12})(0.5)$$
$$W_{a\to b} = 1.2 \times 10^{-12} J$$
$$(c) V_a - V_b = \frac{W_{a\to b}}{q} = \frac{1.2 \times 10^{-12}}{1.602 \times 10^{-19}}$$
$$V_a - V_b = 7.5 \times 10^{6} J/C$$

The potential difference between points *A* and *B* will be

- Potential difference between points *A* and *B* will be $V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$



- The electric potential is independent of the path between points *A* and *B*

- It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$

- Then the potential at some point *r* is

$$V = k_e \frac{q}{r}$$

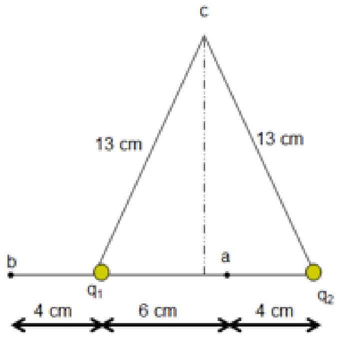
Electric Potential with Multiple Charges

- The electric potential due to several point charges is the sum of the potentials due to each individual charge
  - This is another example of the superposition principle
  - The sum is the algebraic sum $V = k_e \sum_i \frac{q_i}{r_i}$
    - $V = 0$ at $r = \infty$

Ex. 2 . Potential due to two point charges

- An electric dipole consists of two point charges, **12nC** and **-12nC** , placed **10 cm** apart (Fig. below).
- Compute the potentials at points
- a, b, and c by adding

the potentials due to either charges



solution

- A)

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

$$(a)V_a = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$V_a = (9\times10^9)\frac{12\times10^{-9}}{0.06} + (9\times10^9)\frac{-12\times10^{-9}}{0.04}$$

$$V_a = 1800 - 2700 = -900V$$

- B)

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

$$(b)V_b = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$V_b = (9\times10^9)\frac{12\times10^{-9}}{0.04} + (9\times10^9)\frac{-12\times10^{-9}}{0.14}$$

$$V_b = 2700 - 770 = 1930V$$

- C)

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

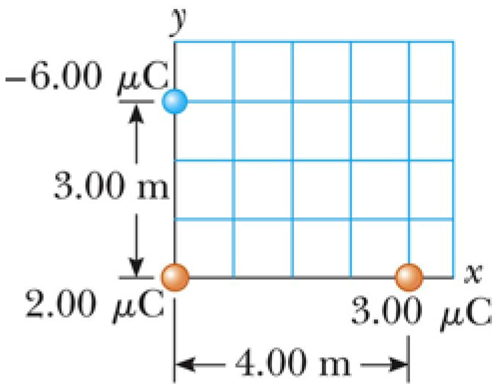
$$(c)V_c = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$V_c = (9\times10^9)\frac{12\times10^{-9}}{0.13} + (9\times10^9)\frac{-12\times10^{-9}}{0.13}$$

$$V_c = 830 - 830 = 0V$$

EX.3

- As shown in Figure below: a charge q1 = 2 µC is located at the origin and a charge q2 = -6 µ C is located at (0,3.00)m. (A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00,0) m.



(b)

- Potion at p

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99\times10^9)\left(\frac{2.00\times10^{-6}}{4.00} + \frac{-6.00\times10^{-6}}{5.00}\right)$$

$$V_P = -6.29\times10^3 V$$

Electric Potential, continue

- The potential is a scalar quantity
  - Since energy is a scalar
- As a charged particle moves in an electric field, it will experience a change in potential

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- The *difference* in potential is the meaningful quantity
- We often take the value of the potential to be zero at some convenient point in the field

- **Using previous equations, and then dividing it by to obtain**

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a)$$
$$= V_a - V_b$$

- **where** $V_a = U_a/q_0$ **is the potential energy per unit charge at point *a*, and similarly for :** V_b
- **We call** V_a **the potential at point *a* and** V_b **potential at point *b*, respectively.**

- **Potential difference can be measured using instrument called *voltmeter***

Potential Difference in a Uniform Field

- The equations for electric potential can be simplified if the electric field is uniform:

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B ds = -Ed$$

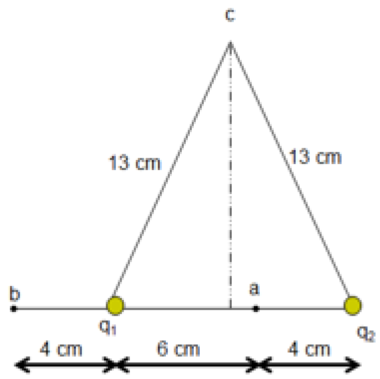
- The negative sign indicates that the electric potential at point B is lower than at point A

  - Electric field lines always point in the direction of decreasing electric potential

- A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field
  - An electric field does work on a positive charge when the charge moves in the direction of the electric field

- The charged particle gains kinetic energy equal to the potential energy lost by the charge-field system
  - Another example of Conservation of Energy

- **Compute the potential energy associated with a point charge of +4 nC if it is placed at points a, b, and c. from ex.2**
-



EXECUTE: At point *a*,

$$U_a = qV_a = (4.0 \times 10^{-9}\,\mathrm{C})(-900\,\mathrm{J/C}) = -3.6 \times 10^{-6}\,\mathrm{J}$$

At point *b*,

$$U_b = qV_b = (4.0 \times 10^{-9}\,\mathrm{C})(1930\,\mathrm{J/C}) = 7.7 \times 10^{-6}\,\mathrm{J}$$

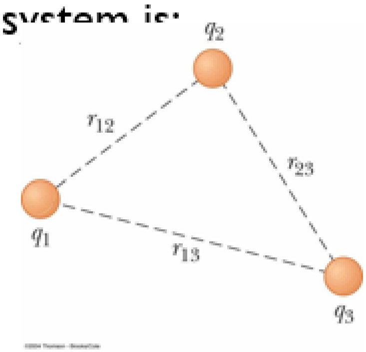
At point *c*,

$$U_c = qV_c = 0$$

All of these values correspond to *U* and *V* being zero at infinity.

Potential Energy of Multiple Charges

- Consider two charged particles
- The potential energy of the system is:

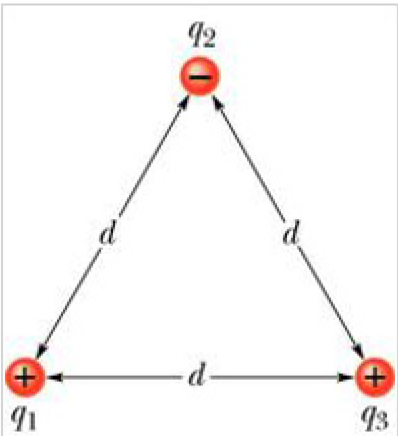


$$U = k_e \frac{q_1 q_2}{r_{12}}$$

- If there are more than two charges, then find *U* for each pair of charges and add them
- For three charges: $U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$
  - The result is independent of the order of the charges

Example4:

- Figure below shows three point charges held in fixed positions by forces that are not shown .what is the electric potential energy U of the system of charges? Assume that d=12cm and
- $q_1 = +q,\ q_2 = -4q, q_3 = +2q$, q=150nC



Solution:

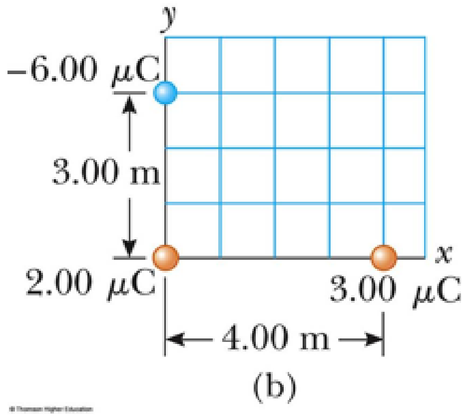
$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$U = 9\times10^9\times\left(\frac{(150\times10^{-9})\times(-600\times10^{-9})}{(12\times10^{-2})} + \frac{(150\times10^{-9})\times(300\times10^{-9})}{(12\times10^{-2})} + \frac{(300\times10^{-9})\times(-600\times10^{-9})}{(12\times10^{-2})}\right)$$
$$= (625 + 312.5 - 1250)\times10^{-5}J = -312.5\times10^{-5}J$$

Example

- Find the change in potential energy of the system of two charges plus a third charge

q3 = 3.00 mC as the latter charge moves from infinity to point P (Figure below)



Solution:

- Assign U_i=0 for the system to the configuration in which the charge q_3 is at infinity. Use equation 25.2 to evaluate the potential energy for the configuration in which the charge is at P: $U_f = q_3 V_P$
- Substitute numerical values to evaluate D U.

$$\Delta U = U_f - U_i = q_3V_P - 0$$
$$\Delta U = (3.00\times10^{-6})(-6.29\times10^{3}) = -1.89\times10^{-2}J$$

Capacitance and Dielectrics

Dr. Faten Monjed Hussein

physics for medical physics

الفيزياء الكهربائية للفيزياء الطبية

الملزمة الرابعة المستوى الأول

Capacitors

- Capacitors are devices that store electric charge
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

Definition of Capacitance

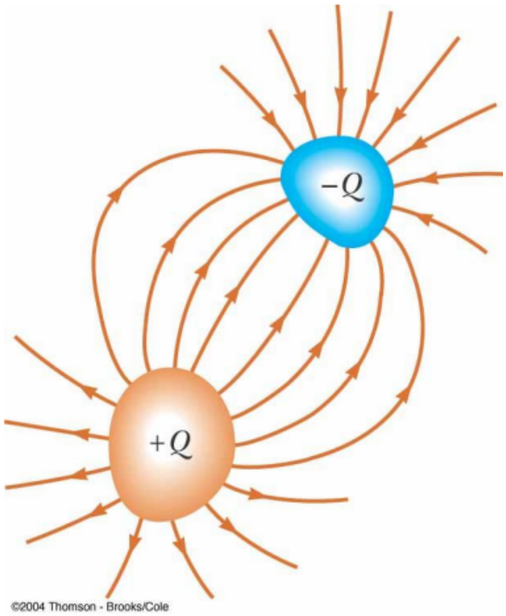
- The **capacitance**, C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors

$$C \equiv \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F)

Makeup of a Capacitor

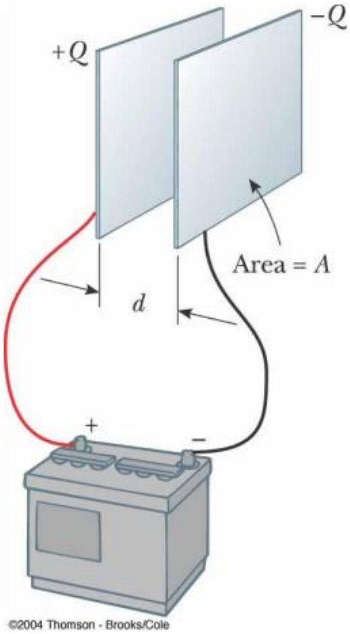
- A capacitor consists of two conductors
  - These conductors are called *plates*
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions
- A potential difference exists between the plates due to the charge



A capacitor Shapes

Parallel Plate Capacitor

- Each plate is connected to a terminal of the battery
  - The battery is a source of potential difference
- If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires



Parallel Plate Capacitor, cont

- This field applies a force on electrons in the wire just outside of the plates
- The force causes the electrons to move onto the negative plate
- This continues until equilibrium is achieved
  - The plate, the wire and the terminal are all at the same potential
- At this point, there is no field present in the wire and the movement of the electrons ceases

Parallel Plate Capacitor, final

- The plate is now negatively charged
- A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged
- In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery

Capacitance – Parallel Plates

- The charge density on the plates is $\sigma = Q/A$
  - A is the area of each plate, which are equal
  - Q is the charge on each plate, equal with opposite signs
- The electric field is uniform between the plates and zero elsewhere

Capacitance – Parallel Plates, cont.

- The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_o A} = \frac{\varepsilon_o A}{d}$$

External Example: **Size of a 1-F capacitor**

A parallel-plate capacitor has a capacitance of 1 F. If the plates are 1 mm apart, what is the area of the plates?

$$A = \frac{Cd}{\epsilon_0} = \frac{(1.0\,\mathrm{F})(1.0 \times 10^{-3}\,\mathrm{m})}{8.85 \times 10^{-12}\,\mathrm{F/m}}$$
$$= 1.1 \times 10^{8}\,\mathrm{m}^2$$

Example **properties of a parallel-plate capacitor**

The plates of a parallel-plate capacitor in vacuum are 5 mm apart and 2 m^2 in area. A potential difference of 10,000 V is applied across the capacitor. Compute a) the capacitance; b) the charge on each plate; and c) the magnitude of the electric field in the space between them.

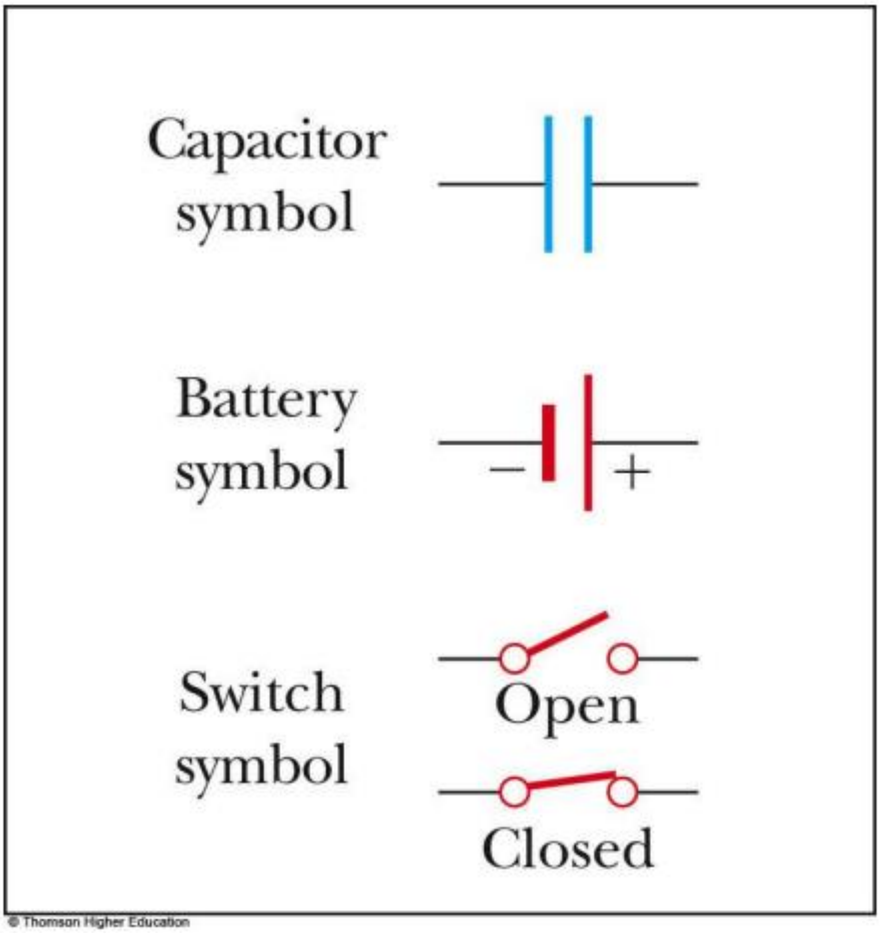
$$C = \epsilon_0 \frac{A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{F/m})(2.00\ \mathrm{m}^2)}{5.00 \times 10^{-3}\ \mathrm{m}}$$

$$= 3.54 \times 10^{-9}\ \mathrm{F} = 0.00354\ \mu\mathrm{F}$$

$$Q = CV_{ab} = (3.54 \times 10^{-9}\ \mathrm{C/V})(1.00 \times 10^{4}\ \mathrm{V})$$

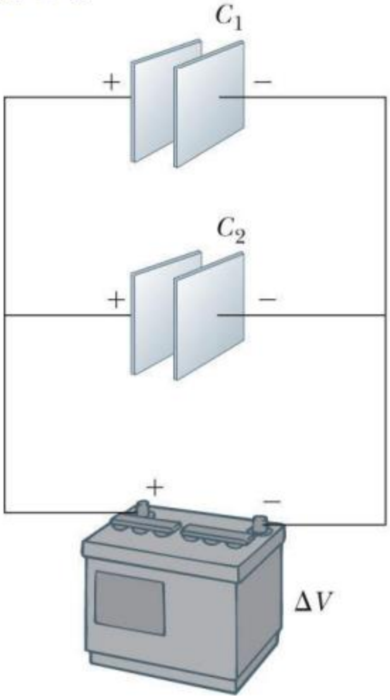
$$= 3.54 \times 10^{-5}\ \mathrm{C} = 35.4\ \mu\mathrm{C}$$

$$E = \frac{V_{ab}}{d} = \frac{1.00 \times 10^{4}\ \mathrm{V}}{5.00 \times 10^{-3}\ \mathrm{m}} = 2.00 \times 10^{6}\ \mathrm{V/m}$$



Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged



Capacitors in Parallel, 2

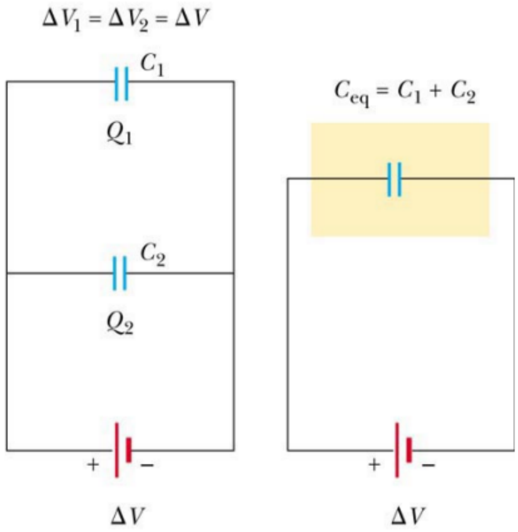
- The flow of charges ceases when the voltage across the capacitors equals that of the battery
- The potential difference across the capacitors is the same
  - And each is equal to the voltage of the battery
  - $\Delta V_1 = \Delta V_2 = \Delta V$
    - ΔV is the battery terminal voltage
- The capacitors reach their maximum charge when the flow of charge ceases
- The total charge is equal to the sum of the charges on the capacitors

- The total charge is equal to the sum of the charges on the capacitors

  - $Q_{total} = Q_1 + Q_2$

Capacitors in Parallel, 3

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}
  - The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors

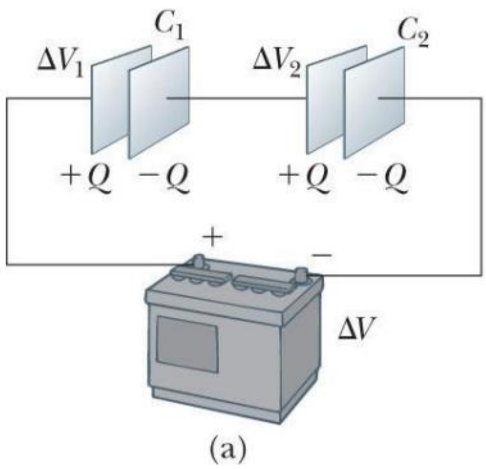


Capacitors in Parallel, final

- $C_{eq} = C_1 + C_2 + C_3 + \ldots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors
  - Essentially, the areas are combined
- Use the active figure to vary the battery potential and the various capacitors and observe the resulting charges and voltages on the capacitors

Capacitors in Series

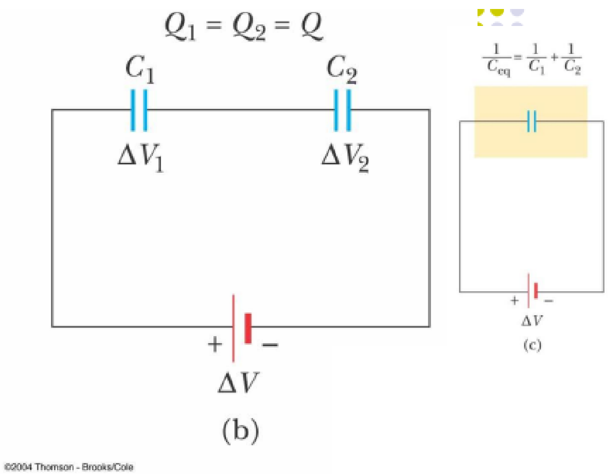
- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery



- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge
- All of the right plates gain charges of $-Q$ and all the left plates have charges of $+Q$

- An equivalent capacitor can be found that performs the same function as the series combination
- The charges are all the same

$Q_1 = Q_2 = Q$

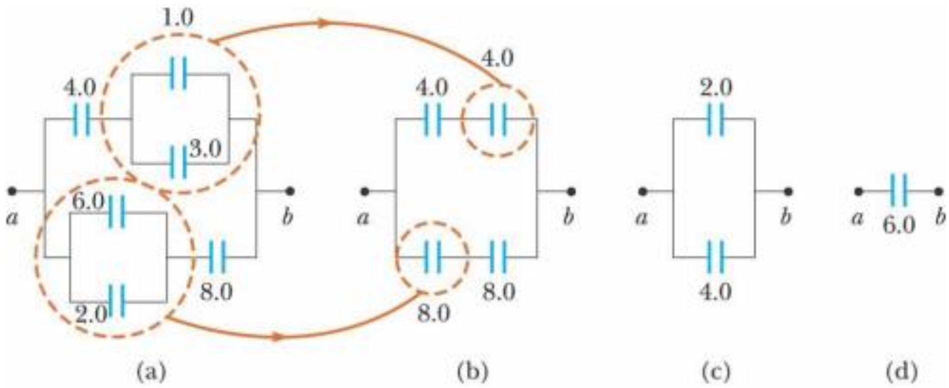


Capacitors in Series, final

- The potential differences add up to the battery voltage
  $\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \dots$
- The equivalent capacitance is

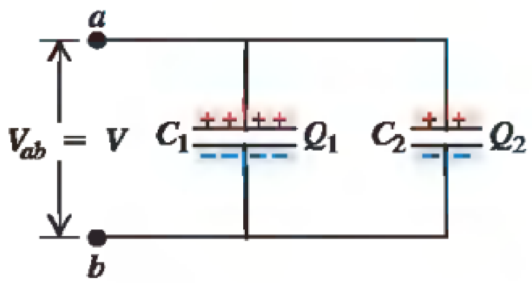
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

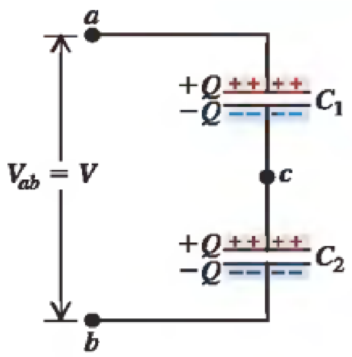
- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination



Example:

Two parallel plate capacitors with capacitance $C_1 = 6\ \mu F$ and $C_2 = 3\ \mu F$ respectivilty, and the potential across both of them is $V_{ab} = 18\ V$ (see figures below). Find the equivalent capacitance, and find the charge and potential difference for each capacitor when the two capacitors are connected a) in series and b) in parallel.





The series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6.0\ \mu F} + \frac{1}{3.0\ \mu F} \qquad C_{eq} = 2.0\ \mu F$$

$$Q = C_{eq}V = (2.0\ \mu F)(18\ V) = 36\ \mu C$$

$$V_{ac} = V_1 = \frac{Q}{C_1} = \frac{36\ \mu C}{6.0\ \mu F} = 6.0\ V$$

$$V_{cb} = V_2 = \frac{Q}{C_2} = \frac{36\ \mu C}{3.0\ \mu F} = 12.0\ V$$

The parallel combination

$$C_{eq} = C_1 + C_2 = 6.0\ \mu F + 3.0\ \mu F = 9.0\ \mu F$$

The potential difference across each of the two capacitors in parallel is the same as that across the equivalent capacitor, 18 V. The charges Q_1 and Q_2 are directly proportional to the capacitances C_1 and C_2, respectively:

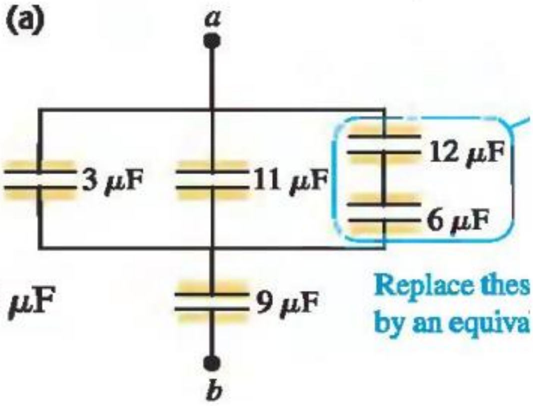
$$Q_1 = C_1V = (6.0\ \mu F)(18\ V) = 108\ \mu C$$
$$Q_2 = C_2V = (3.0\ \mu F)(18\ V) = 54\ \mu C$$

Example:

Find the equivalent capacitance of the combination shown in Figure below.

$$\frac{1}{C'} = \frac{1}{12\,\mu\mathrm{F}} + \frac{1}{6\,\mu\mathrm{F}} \qquad C' = 4\,\mu\mathrm{F}$$

(a)



$$C'' = 3\,\mu\mathrm{F} + 11\,\mu\mathrm{F} + 4\,\mu\mathrm{F} = 18\,\mu\mathrm{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{18\,\mu\mathrm{F}} + \frac{1}{9\,\mu\mathrm{F}} \qquad C_{eq} = 6\,\mu\mathrm{F}$$

Energy in a Capacitor – Overview, cont

- The electric potential energy is related to the separation of the positive and negative charges on the plates
- A capacitor can be described as a device that stores energy as well as charge

Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge q on it
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

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- The work done in charging the capacitor appears as electric potential energy U:

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

- This applies to a capacitor of any geometry
- The energy stored increases as the charge increases and as the potential difference increases
- In practice, there is a maximum voltage before discharge occurs between the plates

Capacitors with Dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, glass, and waxed paper
- With a dielectric, the capacitance becomes $C = \kappa C_o$
  - The capacitance increases by the factor κ when the dielectric completely fills the region between the plates
  - κ is the **dielectric constant** of the material

- For a parallel-plate capacitor, $C = \kappa\varepsilon_o(A/d)$
- In theory, d could be made very small to create a very large capacitance
- In practice, there is a limit to d
  - d is limited by the electric discharge that could occur though the dielectric medium separating the plates
- For a given d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material

- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching
    - This decreases d and increases C

Example

Suppose the parallel plates each have an area of 2000 cm^2 and are 1 cm apart. The capacitor is connected to a power supply and charged to a potential difference V_0=3000 V. It is then disconnected from the power supply, and a sheet of insulating plastic material is inserted between the plates, completely filling the space between them. We find that the potential difference decreases to 1000 V while the charge on each capacitor plate remains constant. Compute

a) the original capacitance C_0;
b) the magnitude of charge Q on each plate;
c) the capacitance C after the dielectric is inserted;
d) the dielectric constant K of the dielectric;

$$C_0 = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \mathrm{F/m}) \frac{2.00 \times 10^{-1}\ \mathrm{m}^2}{1.00 \times 10^{-2}\ \mathrm{m}}$$
$$= 1.77 \times 10^{-10}\ \mathrm{F} = 177\ \mathrm{pF}$$

$$Q = C_0 V_0 = (1.77 \times 10^{-10}\ \mathrm{F})(3.00 \times 10^{3}\ \mathrm{V})$$
$$= 5.31 \times 10^{-7}\ \mathrm{C} = 0.531\ \mu\mathrm{C}$$

$$C = \frac{Q}{V} = \frac{5.31 \times 10^{-7}\ \mathrm{C}}{1.00 \times 10^{3}\ \mathrm{V}} = 5.31 \times 10^{-10}\ \mathrm{F} = 531\ \mathrm{pF}$$

$$K = \frac{C}{C_0} = \frac{5.31 \times 10^{-10}\ \mathrm{F}}{1.77 \times 10^{-10}\ \mathrm{F}} = \frac{531\ \mathrm{pF}}{177\ \mathrm{pF}} = 3.00$$