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المرحلة الثانية

الرياضيات

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Sequences

Definition

A sequence is a list of numbers $a_1, a_2, a_3, \ldots, a_n$ in a given order. Each of a_1, a_2, a_3 and so on represents a number. These are the terms of the sequence.

For example, the sequence $2, 4, 6, 8, \ldots, 2n$ has first term $a_1 = 2$, second term $a_2 = 4$ and nth term $a_n = 2n$. The integer n is called the index of a_n,

Note

Order is important. The sequence $2, 4, 6, 8, \ldots$ is not the same as the sequence $4, 2, 6, 8, \ldots$

Infinite sequence

Definition

The infinite sequence of numbers is

a function whose domain is the set of positive integers. The general behavior of the following sequence are

① $a_n = 2n = 2, 4, 6, 8, 10, 12, \dots$

② $a_n = \frac{1}{n} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$

③ $a_n = \frac{n+1}{n} = 2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \dots$

So, $\frac{n+1}{n}$ is also called the general term of the sequence.

Example

write the five terms of the following sequence

① $\{n-10\} = \{-9, -8, -7, -6, -5\}$

② $\{\frac{1}{2n}\} = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\}$ ✗

③ $\{\frac{2n-1}{3n+2}\} = \{\frac{1}{5}, \frac{3}{8}, \frac{5}{11}, \frac{7}{14}, \frac{9}{17}\}$

④ $\{2(-1)^n\} = \{-2, 2, -2, 2, -2\}$

⑤ $\{\frac{1+(-1)^n}{n}\} = \{0, 1, 0, \frac{1}{2}, 0\}$

* The limit of infinite sequences

Convergence and Divergence

Sometimes the numbers in a sequence approach a single value as the index n increases

$\lim\limits_{n\to\infty} a_n =$ L أي عدد → convergence (موجود)

±∞ → Divergence (غير موجود)

for example

① $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots, \frac{1}{n}, \dots\}$

the terms approach zero as n gets large.

② $\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots, 1-\frac{1}{n}, \dots\}$

the terms approach 1

In this case, it can be said that the sequence converges to this specific number

Definition

If the sequence $\{a_n\}$ converges to the number L, we write $\lim\limits_{n\to\infty} a_n = L$, or simply $a_n \xrightarrow{\text{converge to}} L$ and call L the limit of the sequence. On the other hand, if the sequence never converging to a single value or whose terms get larger than any number as n increases, then it can be said that the sequence diverges to infinity or negative infinity.

and write

① $\lim_{n\to\infty} a_n = \infty$ or $a_n \to \infty$ diverges to infinity

② $\lim_{n\to\infty} a_n = -\infty$ or $a_n \to -\infty$ diverges to negative infinity

For example

① $\{1, -1, 1, -1, 1, -1, \ldots, (-1)^{n+1}, \ldots\}$

② $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \ldots, \sqrt{n}, \ldots\}$

Calulating limits of Sequences

Theorem ①

Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers, and let A and B be real numbers. The following rules hold if $\lim_{n\to\infty} a_n = A$ and $\lim_{n\to\infty} b_n = B$

① Sum Rule : $\lim_{n\to\infty} (a_n + b_n) = A + B$ مهم

② Difference Rule : $\lim_{n\to\infty} (a_n - b_n) = A - B$

③ Constant Multiple Rule : $\lim_{n\to\infty} (K \cdot b_n) = K \cdot B$ (K is any number)

④ Product Rule : $\lim_{n\to\infty} (a_n \cdot b_n) = AB$

⑤ Quotient Rule : $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$ if $B \neq 0$

Notes

① $\frac{\text{عدد}}{\infty} = 0$ ② $\frac{\infty}{\text{عدد}} = \infty$

Examples

Find the value of limit

① $\lim_{n\to\infty} \left(\frac{-1}{n}\right) = -1 \lim_{n\to\infty} \frac{1}{n} = -1 \cdot \frac{1}{\infty} = -1 \cdot 0 = 0$ conv

(constant multiple Rule)

② $\lim_{n\to\infty} \left(\frac{n-1}{n}\right) = \lim_{n\to\infty} \left(\frac{n}{n} - \frac{1}{n}\right) = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)$

$= \lim_{n\to\infty} 1 - \lim_{n\to\infty} \frac{1}{n} = 1 - 0 = 1$ conv

(difference Rule)

③ $\lim_{n\to\infty} \frac{5}{n^2} = 5 \lim_{n\to\infty} \frac{1}{n} \cdot \lim_{n\to\infty} \frac{1}{n} = 5 \cdot 0 \cdot 0 = 0$ conv

(product Rule)

④ $\lim_{n\to\infty} \frac{4 - 7n^6}{n^6 + 3} = \lim \frac{\left(\frac{4}{n^6}\right) - 7}{1 + \left(\frac{3}{n^6}\right)} = \frac{0 - 7}{1 + 0} = -7$

$\frac{\frac{4}{n^6} - \frac{7n^6}{n^6}}{\frac{n^6}{n^6} + \frac{3}{n^6}} = \frac{0 - 7}{1 + 0} = -7$

(Sum and Quotient Rule

Theorem (2)

The following six sequences converge قوانين
to the limits listed below

(1) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

(2) $\lim_{n\to\infty} \sqrt[n]{n} = 1$

(3) $\lim_{n\to\infty} x^{\frac{1}{n}} = 1 \quad (x>0)$

(4) $\lim_{n\to\infty} x^{n} = 0 \quad (|x|<1)$

$= \infty \quad (|x| \geq 1)$

(5) $\lim_{n\to\infty} (1+\frac{x}{n})^{n} = e^{x} \quad (\text{any } x)$

(6) $\lim \frac{x^n}{n!} = 0 \quad (\text{any } x)$

Note

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$ (L'-Hopitals Rule)

Examples:

Find the value of limit and determine
converges or diverges

(1) $\frac{\ln(n^2)}{n}$

$\lim_{n\to\infty} \frac{\ln(n^2)}{n} = \lim_{n\to\infty} \frac{2\ln(n)}{n} = 2\lim_{n\to\infty} \frac{\ln(n)}{n} = 2 \cdot 0 = 0$

formula (1) (Converges)

Note

اذا كانت a_n كسرية

① اذا كانت درجة البسط = درجة المقام فان

$$\lim_{n \to \infty} a_n = \frac{\text{معامل اكبر اس بالبسط}}{\text{معامل اكبر اس بالمقام}}$$

② اذا كانت درجة البسط اكبر من درجة المقام فان

$$\lim_{n \to \infty} a_n = \infty$$

③ اذا كانت درجة البسط اقل من درجة المقام فان

$$\lim_{n \to \infty} a_n = 0$$

حيث يتم ايجاد قيمة $\lim_{n \to \infty} a_n$ اما بطريقة القسمة على اكبر اس أو بقاعدة (L'-Hopitals)

Examples

Find the limit of each sequence, which of the sequences converges, and which diverges

① $\dfrac{3n^2 - 5n}{5n^2 + 2n - 6}$

$$\lim_{n \to \infty} \frac{3n^2 - 5n}{5n^2 + 2n - 6} = \lim_{n \to \infty} \frac{3 - \frac{5}{n}}{5 + \frac{2}{n} - \frac{6}{n^2}} = \frac{3 - 0}{5 + 0 - 0} = \frac{3}{5}$$

(converges)

Or (by using L'-Hopitals Rule)

$$\lim \frac{3n^2 - 5n}{5n^2 + 2n - 6} = \lim_{n \to \infty} \frac{6n - 5}{10n + 2} = \lim_{n \to \infty} \frac{6}{10} = \frac{3}{5}$$

(converges)