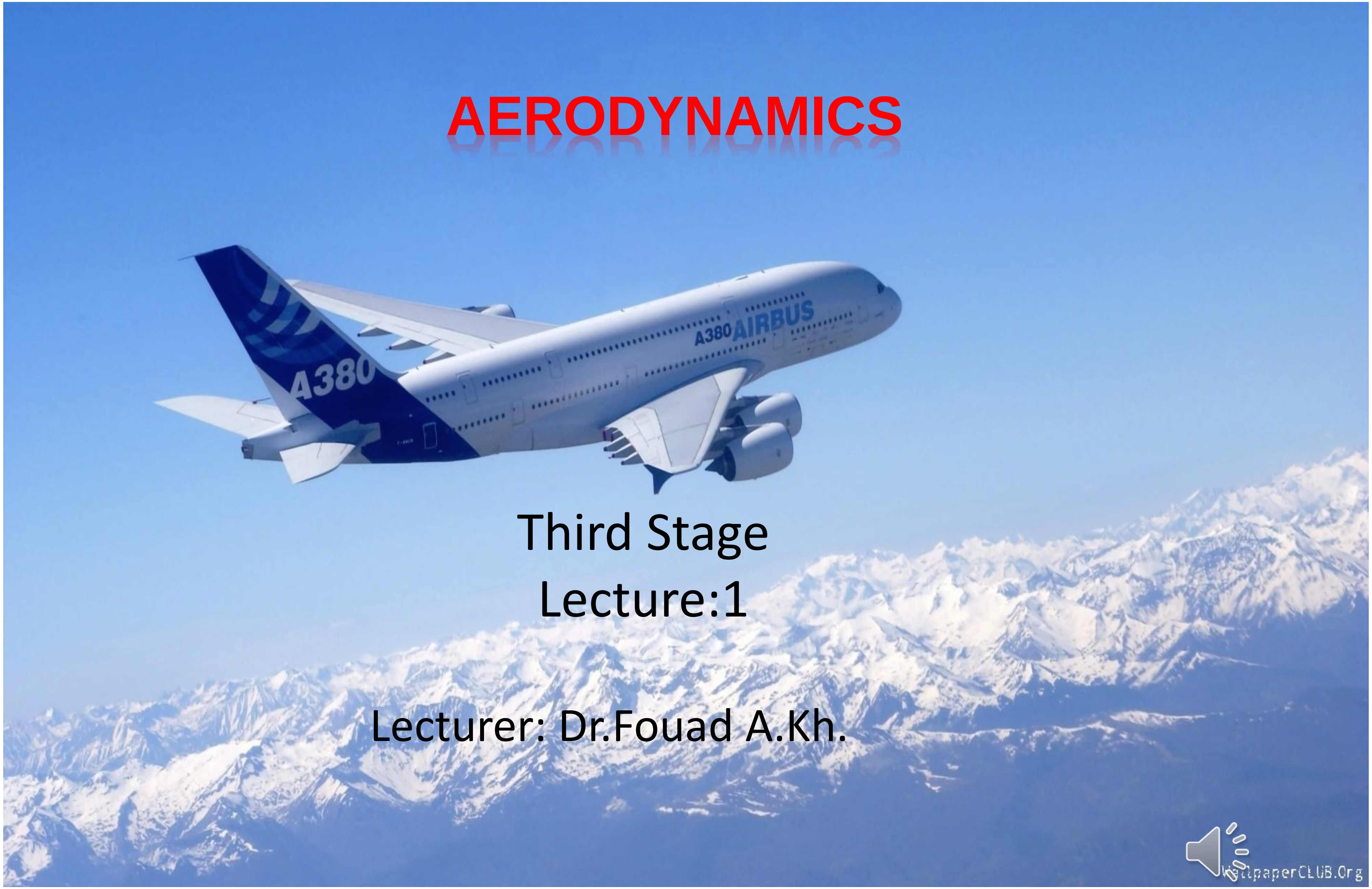
AERODYNAMICS



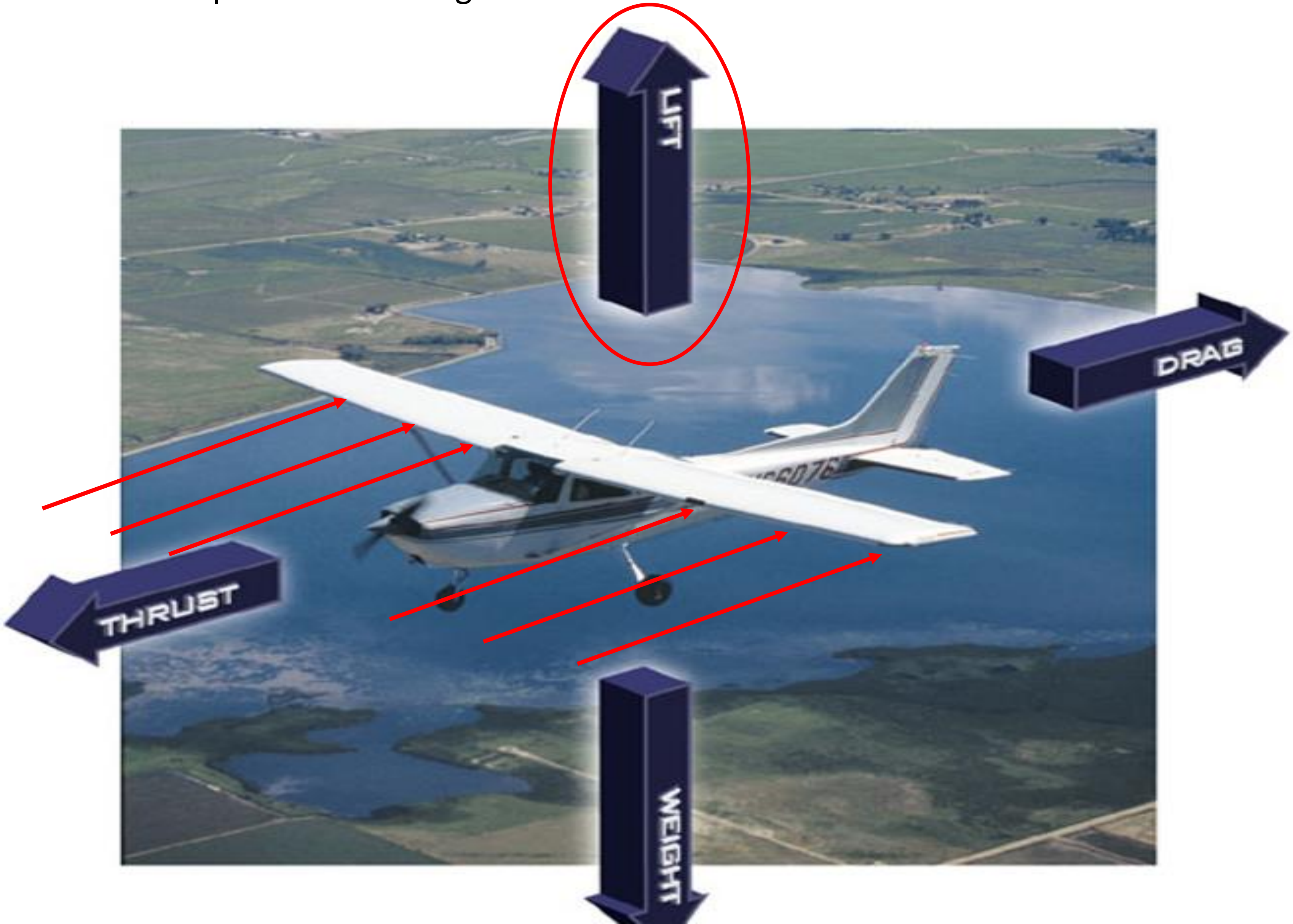
Third Stage

Lecture:1

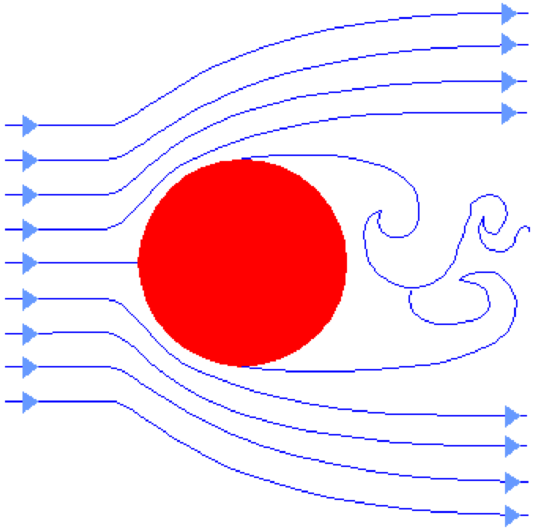
Lecturer: Dr.Fouad A.Kh.

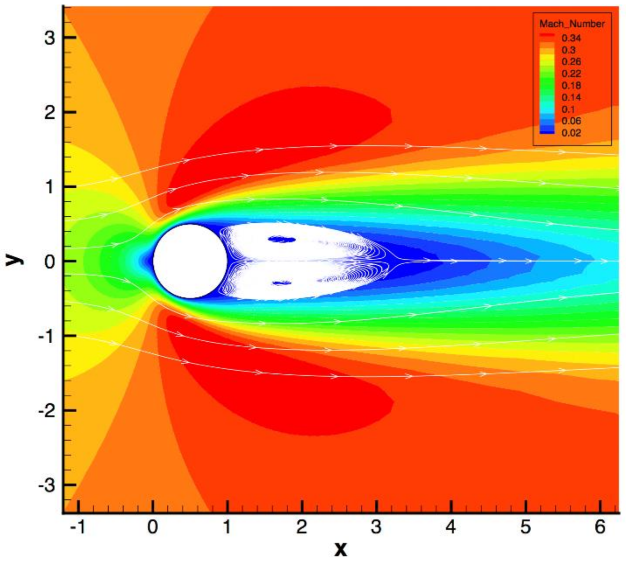
Definition

Aerodynamics is the science that study of objects in motion through the air and the forces that produce or change such motion.



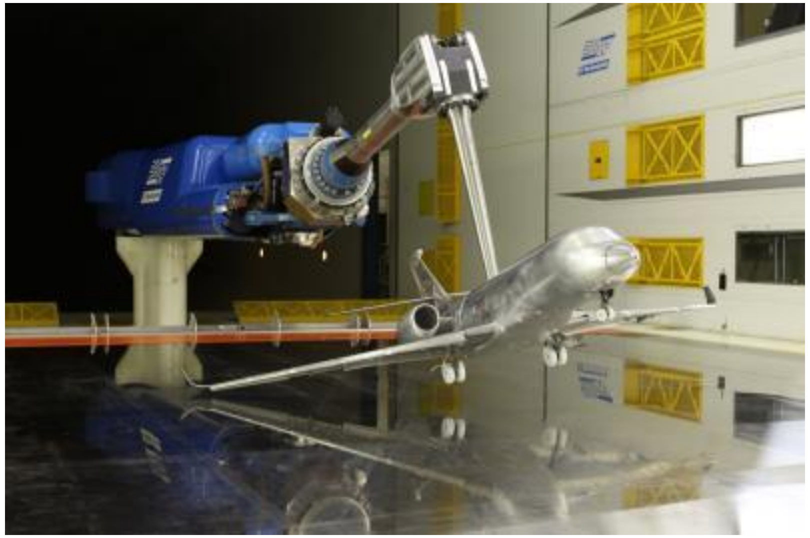
Flow around cylinder





Flow around aero foil

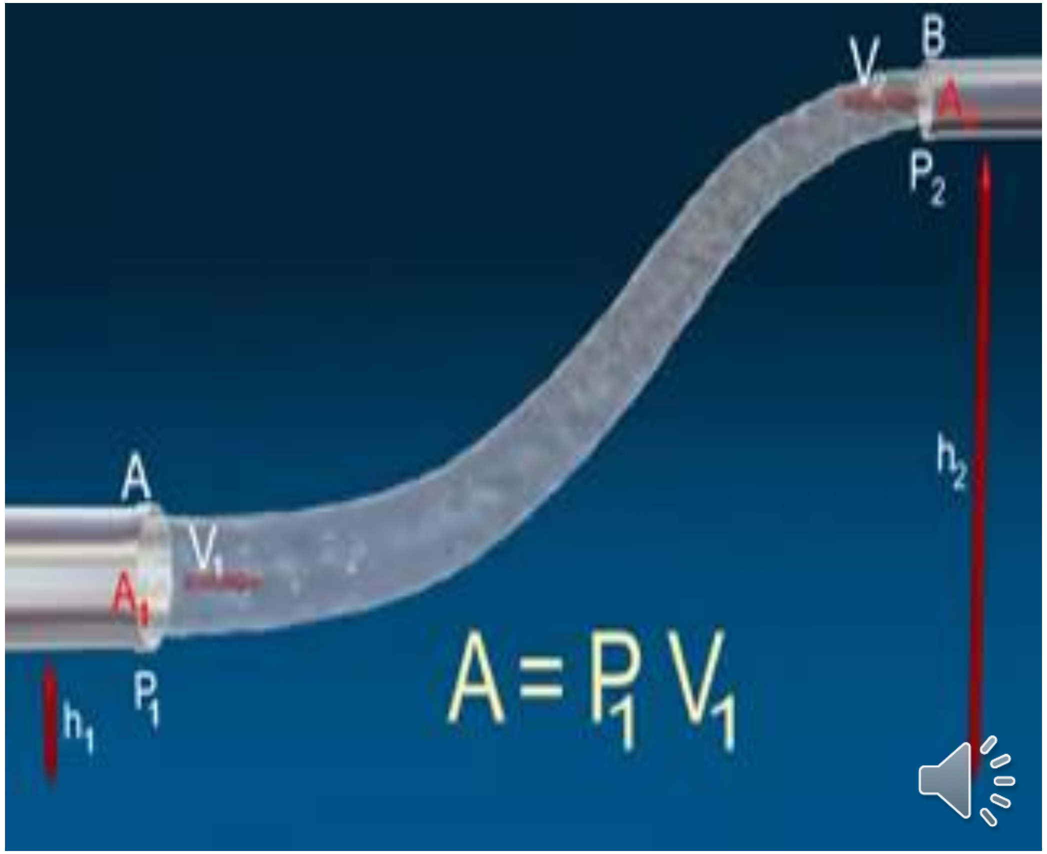
Flow around aircraft



Navier-Stokes Equations

-An important equation which give us mathematical description for the fluid flow (internal or external flow)

-fundamental partial differentials equations that describe the flow of fluids. Using the rate of stress and rate of strain



The Navier-Stokes Equations

2-Momentum Equation:

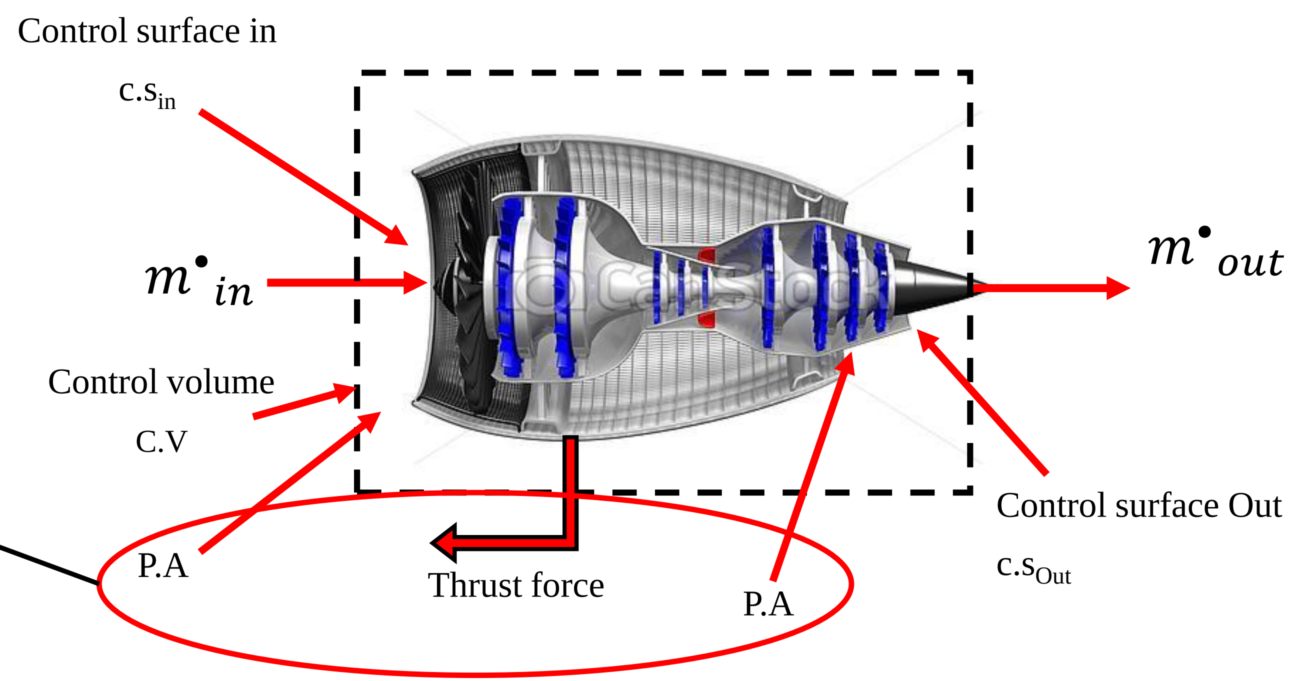
Newton's second law of motion: the resultant of force applied to particle which may be at rest or in motion is equal to rate of change of momentum of the particle in the direction of the resultant force

$$\sum F_x = \frac{d(mV_x)}{dt}$$

$$\sum F_x = m.a$$

$$a = \frac{dV}{dt}$$

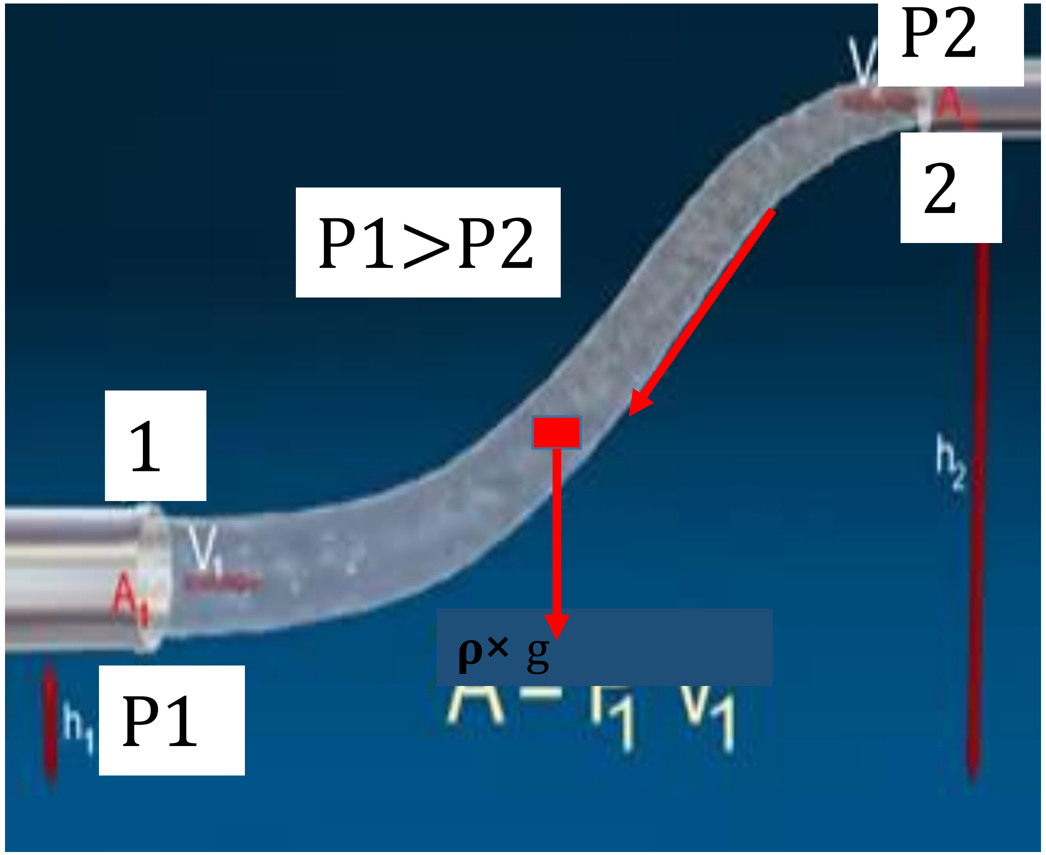
$$\sum F_x = \frac{d(mV_x)}{dt} = m.a$$



When the momentum equation is applied to an infinitesimal control volume (c.v.), it can be written in the form:

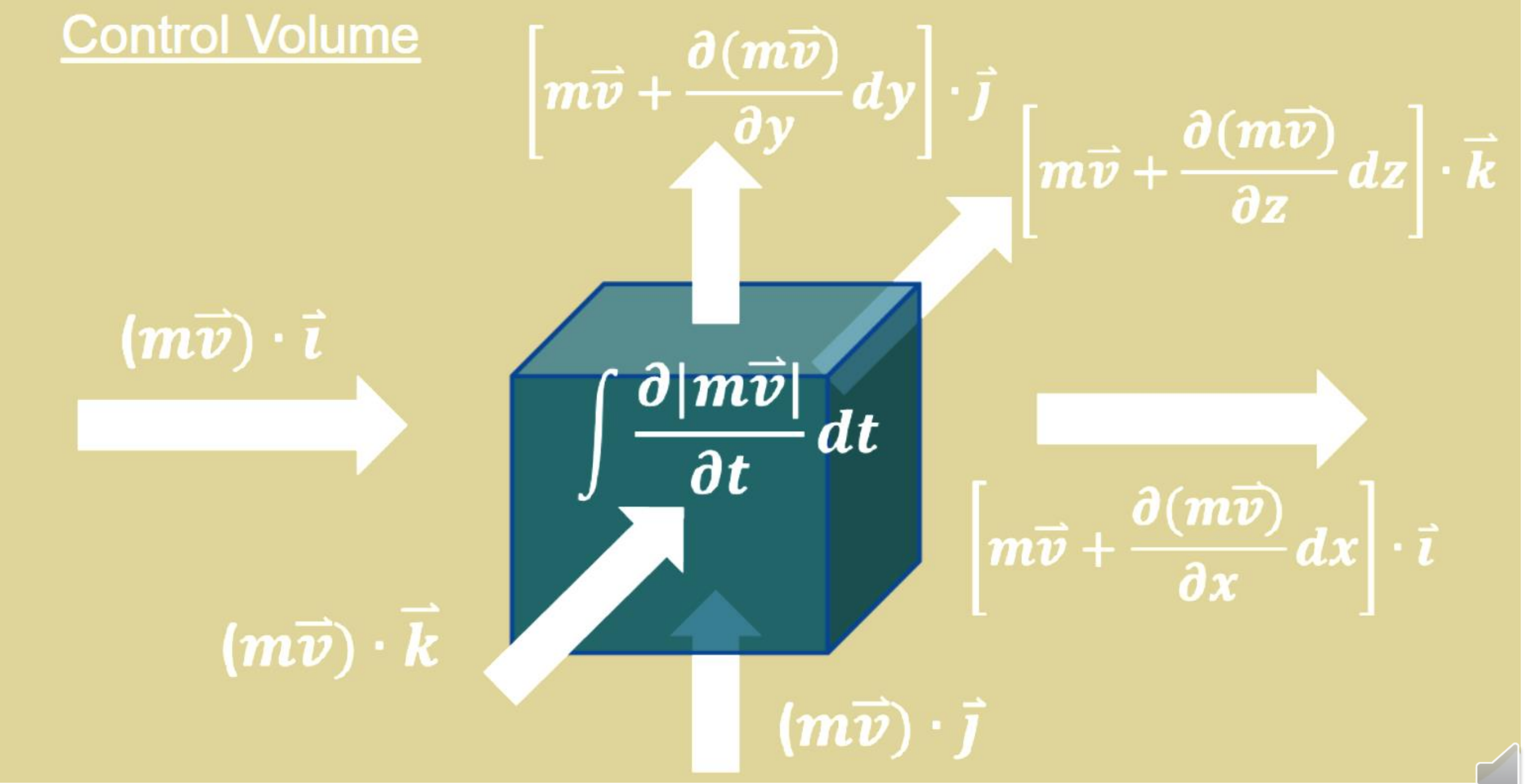
Rate of increase of momentum within the C.V. + Net rate at which momentum leaves the C.V.=

= Body force + pressure force + viscous force



- The second law is based on the conservation of momentum

$$\Sigma\vec{F} = \frac{D(m\vec{v})}{Dt} = \frac{\partial(m\vec{v})}{\partial t} + \frac{\partial(m\vec{v})}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial(m\vec{v})}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial(m\vec{v})}{\partial z}\frac{\partial z}{\partial t}$$



$$m \cdot \frac{D\ \vec{V}}{Dt} = \sum \vec{F} \qquad (Eq\ 1)$$

We express the total force as the sum of body forces and surface forces

$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface}$. Thus $(Eq\ 1)$ can be written as

$$m \cdot \frac{D\ \vec{V}}{Dt} = \sum \vec{F}_{body} + \sum \vec{F}_{surface} \qquad (Eq\ 2)$$

Body forces: Gravity force, Electromagnetic forse, Centrifugal force

Surface forces: Pressure forces, Viscous forces

We cosider the x-component of (*Eq* 2).

Since $m = \rho dxdydz$ and $\vec{V} = (u, v, w)$ we have

$$\rho dxdydz \cdot \frac{Du}{Dt} = \sum \vec{F}_{x,body} + \sum \vec{F}_{x,surface} \qquad (Eq\ 3)$$

x component:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

Viscous Term

Convective Term
Rate of change
Momentum Term

Pressure Term

Body force Term
Gravity

y- *component:*

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

z component:

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

[The vector form for these equations: $\rho\frac{D\vec{V}}{Dt}=-\nabla P+\rho\vec{g}+\mu\nabla^2\vec{V}$ **]**

AERODYNAMICS

LECTURER: DR. FOUAD

Steady Laminar Flow Between Parallel Plates

Lecture 2

Solutions of Viscous-Flow Equations

The equations of viscous flow derived in Lecture 1 are a system of nonlinear partial differential equations. No general analytical method yet exists for attacking this system for an arbitrary viscous-flow problem.

Over past 150 years, a considerable number of exact but particular solutions have been found which satisfy the complete equations for some special geometry, many of which are very illuminating about viscous flow phenomena.

Almost all the known particular solutions are for the case of incompressible Newtonian flow with constant transport properties.

x component:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

Viscous Term

Convective Term
Rate of change
Momentum Term

Pressure Term

Body force Term
Gravity

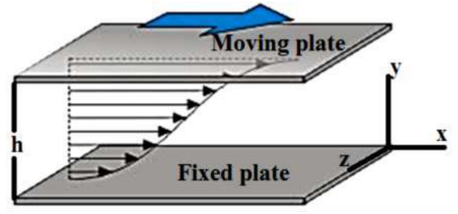
Basically, there are two types of exact solutions of the momentum equation:

1. ***Linear solutions***, where the convective term $\boldsymbol{V}\cdot\nabla$ vanishes

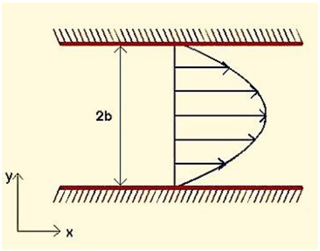
2. ***Nonlinear solutions***, where $\boldsymbol{V}\cdot\nabla$ exist

It is also possible to classify solutions by the type or geometry of flow involved:

√ 1. Couette (wall-driven) steady flows



√ 2. Poiseuille (pressure-driven) steady duct flows



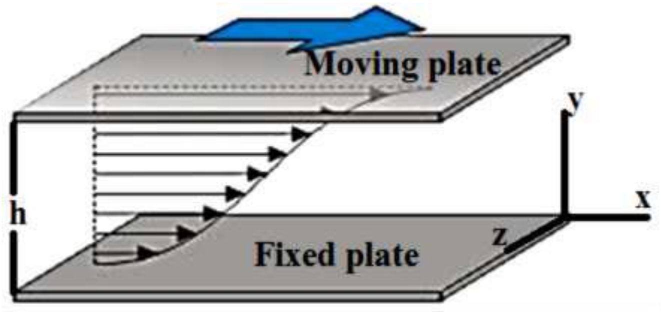
√ 3. Unsteady duct flows

4. Unsteady flows with moving boundaries

5. Duct flows with suction or injection

Couette flow

✓In fluid dynamics, Couette flow is the laminar flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to the other.



✓The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates.
✓This kind of flow has application in hydro-static lubrication, viscosity pumps and turbine.

Navier-Stokes Equation: *Cartesian Coordinates*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Continuity equation for 3-D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Continuity equation for study incompressible flow

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

X-momentum

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{\mu}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Y-momentum

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \frac{\mu}{3}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

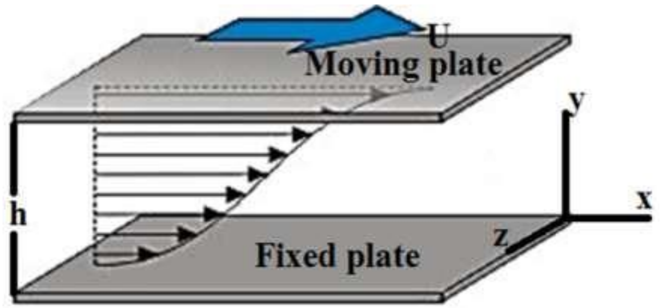
Z-momentum

Analytical solution oF Couette flow

We choose x to be the direction along which all fluid particles travel, and assume the plates are infinitely large in z-direction, so the z-dependence is not there.

- $u \neq 0, v = w = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} = 0$$



- means $u = u(y, z)$
- Now Steady Navier-Stroke equation can be reduce to

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

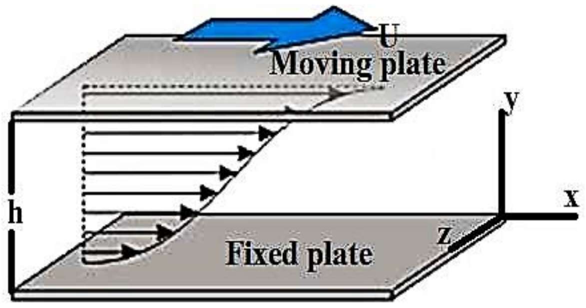
X-momentum

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0$ means $p = p(x)$ *only*

- The governing equation is:

$$\frac{\partial p}{\partial x} = \mu \left(\frac{\partial^2 u}{\partial y^2}\right)$$ By integrating we get → $$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$$



The boundary conditions are:

1- at $y = 0, \rightarrow u = 0$ $u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$ → $C_2 = 0 \; and$

2- $y = h, \rightarrow u = U$ → $C_1 = \frac{U}{h} - \frac{1}{2\mu}\frac{\partial p}{\partial x} \cdot h$

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$$

$$u = \frac{y}{h} U - \frac{h^2}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{y}{h}\left(1 - \frac{y}{h}\right)$$

The velocity profile in non-dimensional form

$$u = \frac{y}{h}U - \frac{h^2}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot \frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$\frac{u}{U} = \frac{y}{h} - \frac{h^2}{2\mu U} \cdot \frac{\partial p}{\partial x} \cdot \frac{y}{h}\left(1 - \frac{y}{h}\right)$$

Let $\quad P = -\frac{h^2}{2\mu} \cdot \left(\frac{\partial p}{\partial x}\right)$

Where P is non-dimensional Pressure gradient.

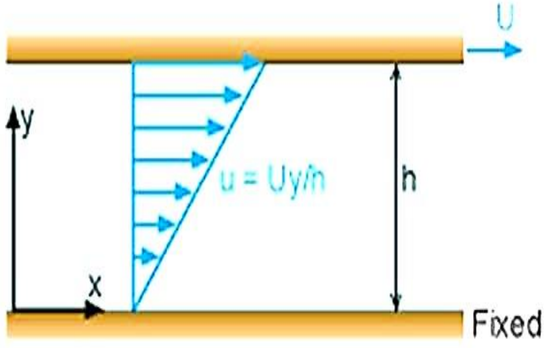


Fig. Simple couette flow

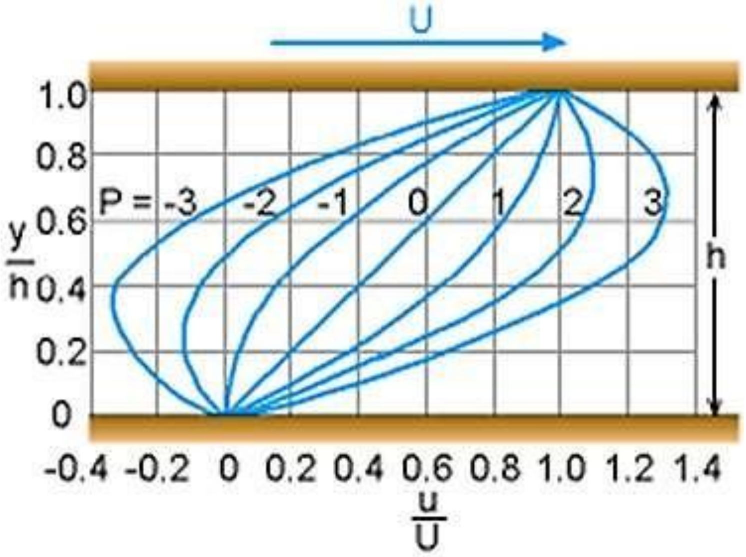
- For simple shear flow, there is no pressure gradient in the direction of the flow.
- when $P = 0$ the equation reduced to:

$$\frac{u}{U} = \frac{y}{h} \qquad \text{(simple couette flow)}$$

The velocity profiles for various P

- For P < o, the fluid motion created by the top plate is not strong enough to overcome the adverse pressure gradient, hence backflow (i.e., u/U is negative) occurs at the lower-half region.

- For P>o, the fluid motion created by top plate is enough strong to overcome the adverse pressure gradient, hence u/U is +ve over the whole gap.



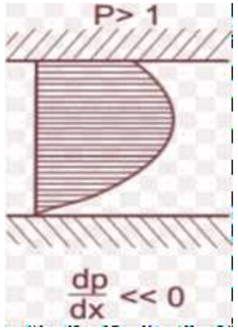
Velocity Profiles

Maximum and minimum velocity and it's location

- For maximum velocity : $\frac{\partial u}{\partial y} = 0$
- $\frac{\partial u}{\partial y} = \frac{U}{h} + \frac{PU}{h}\left(1 - 2\frac{y}{h}\right) = 0$ ⟹ $\frac{y}{h} = \frac{1}{2} + \frac{1}{2P}$
- It is interesting to note that maximum velocity for P=1 occurs at y/h =1 and equals to U. For P>1, the maximum velocity occurs at a location y/h<1.
- This means that with P>1, the fluid particles attain a velocity higher than that of the moving plate at a location somewhere below the moving plate.
- For P=-1 the minimum velocity occurs, at y/h=0. For P<-1, the minimum velocity occurs at allocation y/h>1, means occurrence of back flow near the fixed plate.

The Max. velocity : $u_{max} = \frac{U(1+P)^2}{4P}$ For $P \geq 1$

The Min. velocity : $u_{min} = \frac{U(1+P)^2}{4P}$ For $P \leq 1$



Volume flow rate and average velocity

- The volume flow rate per unit width is:

$$Q = \int_0^h u\,dy = U \int_0^h \left[\frac{y}{h} + P\frac{y}{h}\left(1 - \frac{y}{h}\right)\right] dy \longrightarrow Q = \left(\frac{1}{2} + \frac{P}{6}\right) U \cdot h$$

- The Average velocity:

$$u_{avg} = \frac{volume\ flow\ rate\ (Q)}{area\ per\ unit\ width\ (h \times 1)}$$

$$u_{avg} = \left(\frac{1}{2} + \frac{P}{6}\right) U$$

Shear stress distribution

- By invoking Newton's law of viscosity:

$$\tau = \mu \frac{du}{dy} = \mu \frac{d}{dy}\left[\mathrm{U}\left\{\frac{y}{h} + P\frac{y}{h}\left(1 - \frac{y}{h}\right)\right\}\right]$$

- In the dimensionless form, the shear stress distribution becomes

$$\frac{h\,\tau}{\mu\,U} = 1 + P\left(1 - \frac{2y}{h}\right)$$

- Shear stress varies linearly with the distance from the boundary.

- For P=0, Shear stress remains constant across the flow passage: $\tau = \frac{\mu U}{h}$

- At y=h/2, i.e., at the center of the flow passage, shear stress is independent of pressure gradient (P).

MCQ:

3. For fluid flows obeying conservation of mass, what is the value of k if v=4x+ky denotes the velocity at any point in the flow?

a) -4

b) 4

c) -2

d) 2

View Answer

Answer: a

Explanation: A flow obeys conservation of mass if $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}$=0. Comparing v=4x+ky with $v=v_x+v_y$, we get v_x=4x and v_y=ky.

Using the conservation of mass, we get $\frac{\partial (4x)}{\partial x} + \frac{\partial (ky)}{\partial y}$=0

4+k=0

K=-4.

4. The following equation represents the momentum equation for a fluid flow that is approximated by a two-dimensional model. What does k stand for?

$\rho \frac{\partial v_x}{\partial t} - \frac{\partial}{\partial x}(2k \frac{\partial v_x}{\partial x}) - \frac{\partial}{\partial y}[k(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x})] + \frac{\partial P}{\partial x} - f_x = 0$

a) Thermal conductivity

b) Fluid viscosity

c) Density

d) Pressure

View Answer

Answer: b

Explanation: By using constitutive relations the momentum equation is expressed as $\rho \frac{\partial v_x}{\partial t} - \frac{\partial}{\partial x}(2k \frac{\partial v_x}{\partial x}) - \frac{\partial}{\partial y}[k(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x})] + \frac{\partial P}{\partial x} - f_x = 0$, where v_x, v_y are the velocity components, P is the pressure, k is the viscosity, f_x is the component of the body force vector, and ρ is the density.

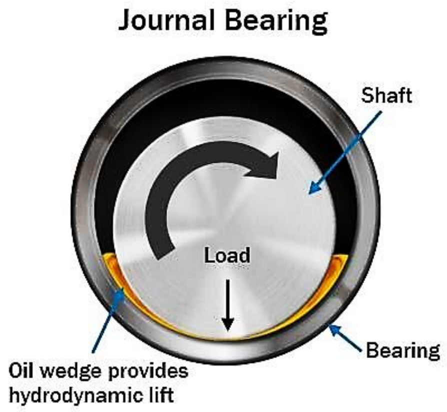
3-What is an assumption made while considering Couette flow?
a) Flow is unparallel
b) No slip condition between two plates
c) Flow is inviscid
d) Both the plates are stationary
View Answer
Answer: b
Explanation: In analyzing Coutte flow, we have two flat plates kept parallel to each other with viscous fluid contained between the two. One major assumption made is that there is a no slip condition thus resulting in no relative motion between the fluid and the plate.

8. What happens to the shear stress if the thickness between the two plates is increased in a Couette flow?
a) Increases
b) Decreases
c) Remains same
d) Becomes infinite

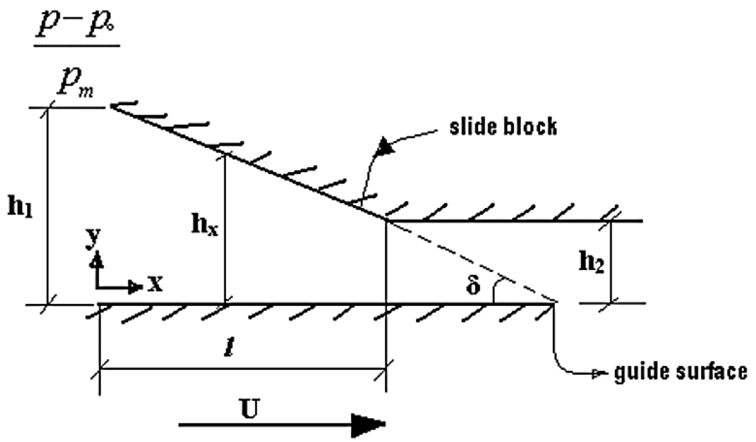
View Answer

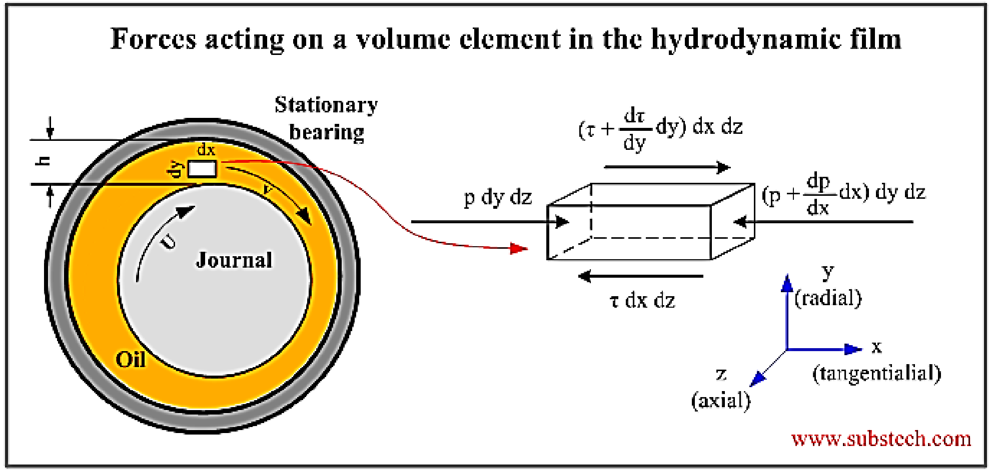
Answer: b
Explanation: The relation between the shear stress and the viscous shear layer is given by:
$\tau = \mu(\frac{u_e}{D})$
Where, u_e is the velocity at y = D that is at the upper plate.
τ is the shear stress
D is the thickness of the vicsous shear layer/distance between the two parallel plates
As peer the formula, then the thickness of the viscous shear layer increases, the shear stress decreases provided the other properties remain the same.

Hydrodynamic Lubrication (Sliding Bearing)



Large forces are developed in small clearance when the surfaces are slightly inclined and one is in motion so that fluid is wedged into the decreasing space. Usually the oils employed for lubrication are highly viscous and the flow is of laminar nature.





Assumptions:

The acceleration is zero.

The body force is small and can be neglected.

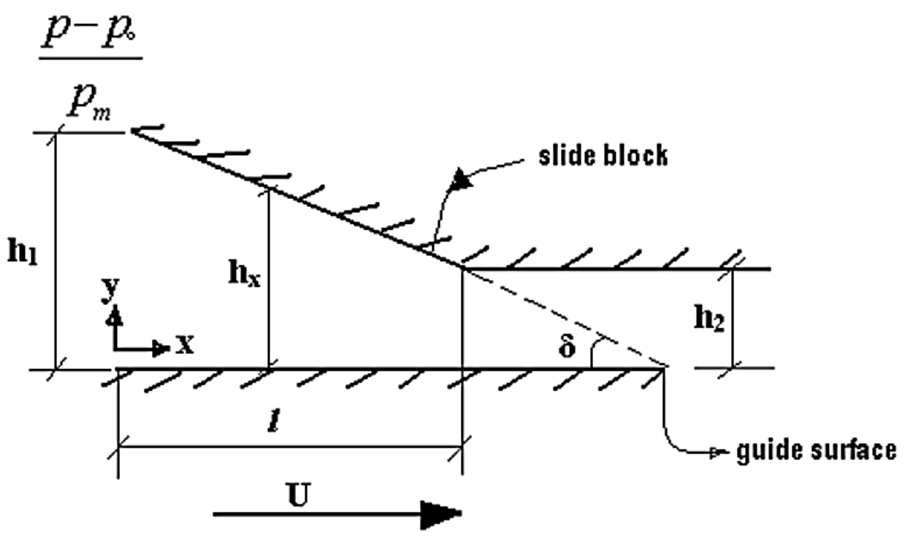
Also $\frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial z^2}$

The Navier-Stokes equation in the x-direction (eq. 1) reduces to:

$$\frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

Integration:

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + Ay + B$$



B.C

$$y = 0 \qquad u = U \quad \Rightarrow \quad B = U$$

$$y = h_x \qquad u = 0 \quad \Rightarrow \quad A = \frac{-h_x}{2\mu}\frac{dp}{dx} - \frac{U}{h_x}$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h_x y\right) + U\left(1 - \frac{y}{h_x}\right)$$

The volume flow rate in every section will be constant.

$$Q = w\int_0^{h_x} u.dy \qquad \text{assume w} = 1$$

$$\therefore\ Q = \frac{Uh_x}{2} - \frac{h_x^3}{12\mu}\frac{dp}{dx} \quad \text{--------(*)}$$

** For a constant taper bearing:

$$\delta = \frac{h_1 - h_2}{l}$$

$$\therefore\ h_x = (h_1 - \delta x)$$

Sub in eq.(*) and solving for $\frac{dp}{dx}$ produces:

$$\frac{dp}{dx} = \frac{6\mu U}{(h_1 - \delta x)^2} - \frac{12\mu Q}{(h_1 - \delta x)^3}$$

B.C

$x = 0 \qquad p = p_o = 0$

$x = l \qquad p = p_o = 0$

$$\Rightarrow\ Q = \frac{Uh_1 h_2}{h_1 + h_2} \quad \text{and} \quad C = \frac{-6\mu U}{\delta(h_1 + h_2)}$$

With these values inserted in eq.(**) we obtain the pressure distribution inside the bearing.

$$p(x) = \frac{6\mu Ux(h_x - h_2)}{h_x^2(h_1 + h_2)}$$

The load that the bearing will support per unit width is:

$$F = \int_0^l p(x).dx$$

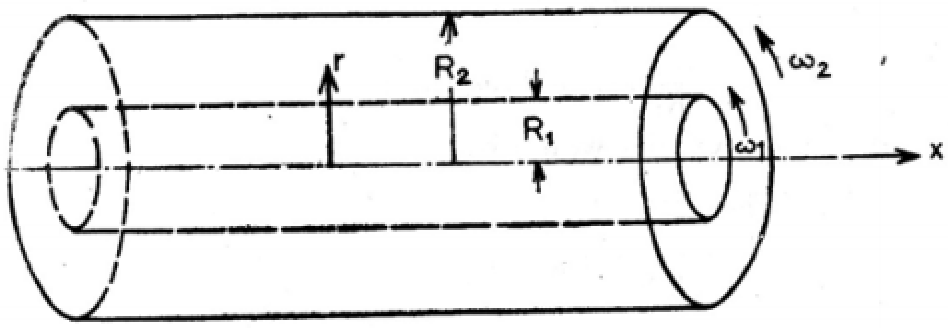
$$F = \frac{6\mu Ul^2}{(h_1 - h_2)^2}\left[\ln k - \frac{2(k-1)}{k+1}\right]$$

where

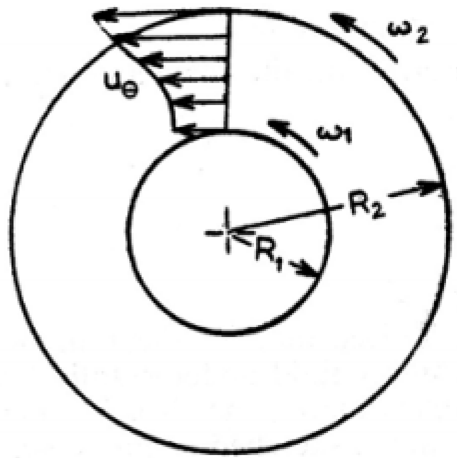
$$k = \frac{h_1}{h_2}$$

4- Laminar flow between concentric rotating cylinders:

Consider the purely circulatory flow of a fluid contained between two long concentric rotating cylinders of radius R_1 and R_2 at angular velocities ω_1 and ω_2.



(a) PICTORIAL REPRESENTATION



(b) VELOCITY PROFILE

In this case the Navier-Stokes equations in cylindrical coordinates are used.

r- direction:

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+w\frac{\partial u_r}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+g_r$$

θ- direction:

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+w\frac{\partial u_\theta}{\partial z}=\frac{-1}{\rho r}\frac{\partial p}{\partial \theta}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+g_\theta$$

In the above equations:

$u_r = 0$

$w = 0$

$$\frac{\partial}{\partial t}=0 \quad , \quad \frac{\partial u_\theta}{\partial \theta}=0 \quad , \quad \frac{\partial p}{\partial \theta}=0$$

body force = 0

The equation in θ- direction reduces to:

$$\frac{d^2 u_\theta}{dr^2}+\frac{d}{dr}\left(\frac{u_\theta}{r}\right)=0$$

Integration:

$$\frac{1}{r}\frac{d}{dr}(ru_\theta) = A$$

$$u_\theta = Ar + \frac{B}{r} \quad \text{---------(i)}$$

<u>B.C</u>

$$r = R_1 \qquad u_\theta = R_1\omega_1$$

$$r = R_2 \qquad u_\theta = R_2\omega_2$$

$$\Rightarrow \quad A = \omega_1 + \frac{R_2^2}{R_2^2 - R_1^2}(\omega_2 - \omega_1)$$

$$B = -\frac{R_1^2 R_2^2}{R_2^2 - R_1^2}(\omega_2 - \omega_1)$$

Sub. in eq.(i) yields:

$$u_\theta = \frac{1}{R_2^2 - R_1^2}\left[\left(\omega_2 R_2^2 - \omega_1 R_1^2\right)r - \frac{R_1^2 R_2^2}{r}(\omega_2 - \omega_1)\right] \quad \text{-------(ii)}$$

The shear stress may be evaluated by the equation:

$$\tau = \mu\left[r\frac{d}{dr}\left(\frac{u_\theta}{r}\right)\right]$$

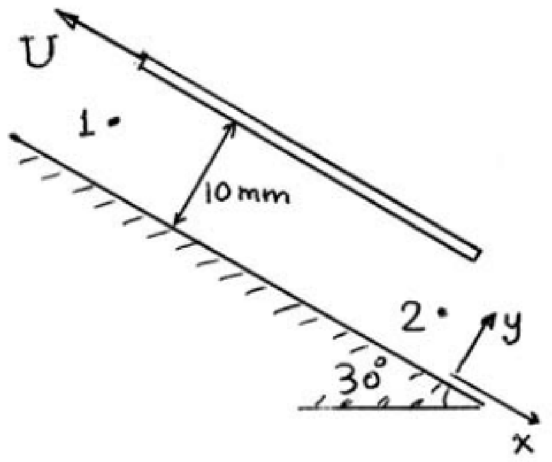
By using eq.(ii):

$$\tau = \frac{2\mu}{R_2^2 - R_1^2}\frac{R_1^2 R_2^2}{r^2}(\omega_2 - \omega_1)$$

5- Example:

1- Using the Navier-Stokes equation in the flow direction, calculate the power required to pull (1m × 1m) flat plate at speed (1 m/s) over an inclined surface. The oil between the surfaces has ($\rho = 900\ kg/m^3$, $\mu = 0.06$ Pa.s).The pressure difference between points 1 and 2 is (100 kN/m^2) .

Solution:



The Navier-Stokes equation in x- direction

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]$$

We have: Acceleration =0 , v=0 , w=0 , $\frac{\partial^2 u}{\partial x^2}=0$, $\frac{\partial^2 u}{\partial z^2}$

The equation reduces to:

$$\frac{d^2u}{dy^2}=\frac{1}{\mu}\frac{dp}{dx}-\frac{\rho}{\mu}g_x$$

Integration

$$\frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y-\frac{\rho}{\mu}g_x y+A$$

$$u=\frac{1}{2\mu}\frac{dp}{dx}y^2-\frac{\rho}{2\mu}g_x y^2+Ay+B$$

B.C (b=10 mm)

$y=0 \quad u=0 \quad \Rightarrow \quad B=0$

$y=b \quad u=-U \quad \Rightarrow \quad A=\frac{-U}{b}-\frac{b}{2\mu}\frac{dp}{dx}+\frac{\rho b}{2\mu}g_x$

$$\therefore \quad \frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y-\frac{\rho}{\mu}g_x y-\frac{U}{b}-\frac{b}{2\mu}\frac{dp}{dx}+\frac{\rho b}{2\mu}g_x$$

The shearing force on the moving plate:

$F=\tau_o\times area$

$F=\mu\cdot\left.\frac{du}{dy}\right|_{y=b}\times area$

area=1 m^2

$$F=-\frac{\mu U}{b}+\frac{b}{2}\frac{dp}{dx}-\frac{b}{2}\rho g_x$$

We have $g_x=g\cdot\sin\theta$, $\frac{dp}{dx}=\frac{-\Delta p}{l}$

$$F=\frac{-0.06\times 1}{0.01}-\frac{0.01}{2}\left(\frac{100\times 10^3}{1}\right)-\frac{0.01}{2}\times 900\times 9.81\times\sin 30$$

$F=-528\ N$

$Power=F\cdot U$

$Power=528\times 1=528\ W$ (Ans)

1- Using the Navier-Stokes equations, determine the pressure gradient along flow, the average velocity, and the discharge for an oil of viscosity 0.02 N.s/m^2 flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s. [-3200 N/m^2 per m ; 1.33 m/s ; 0.0133 m^3/s]

2- An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as shown in figure. The two plates move in opposite directions with constant velocities U_1 and U_2. The pressure gradient in the x-direction is zero. Use the Navier-Stokes equations to derive expression for the velocity distribution between the plates. Assume laminar flow.

$$[u = \frac{y}{b}(U_1 + U_2) - U_2]$$

3- Two parallel plates are spaced 2 mm apart, and oil (μ = 0.1 N.s/m^2 , S = 0.8) flows at a rate of 24×10^{-4} m^3/s per m of width between the plates. What is the pressure gradient in the direction of flow if the plates are inclined at 60° with the horizontal and if the flow is downward between the plates? [-353.2 kPa/m]

4- Using the Navier-Stokes equations, find the velocity profile for fully developed flow of water ($\mu = 1.14\times10^{-3}$ Pa.s) between parallel plates with the upper plate moving as shown in figure. Assume the volume flow rate per unit depth for zero pressure gradient between the plates is 3.75×10^{-3} m^3/s. Determine:

a- the velocity of the moving plate.

b- the shear stress on the lower plate.

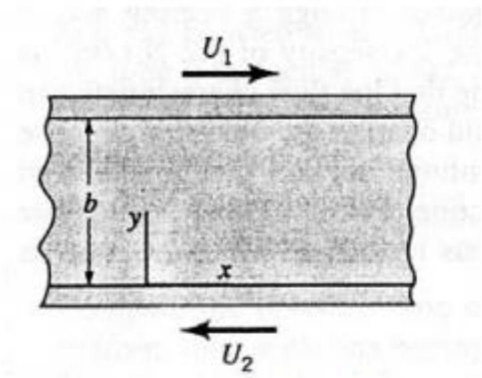
c- the pressure gradient that will give zero shear stress at y = 0.25b. (b = 2.5 mm)

d- the adverse pressure gradient that will give zero volume flow rate between the plates.

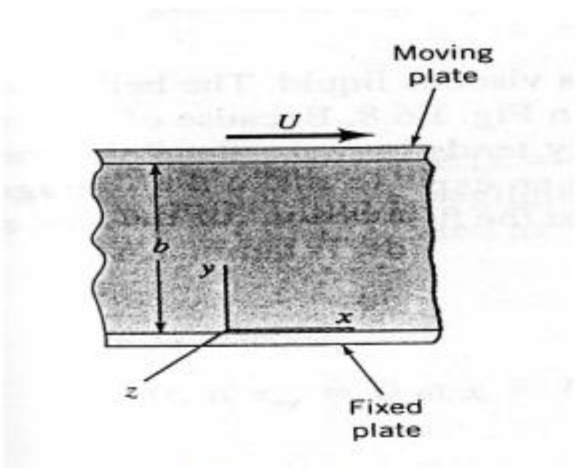
[3 m/s ; 1.37 N/m^2 ; 2.19 kN/m^2 per m ; -3.28 kN/m^2 per m]

5- A vertical shaft passes through a bearing and is lubricated with an oil (μ = 0.2 Pa.s) as shown in figure. Estimate the torque required to overcome viscous resistance when the shaft is turning at 80 rpm. (Hint: The flow between the shaft and bearing can be treated as laminar flow between two flat plates with zero pressure gradient). [0.355 N.m]

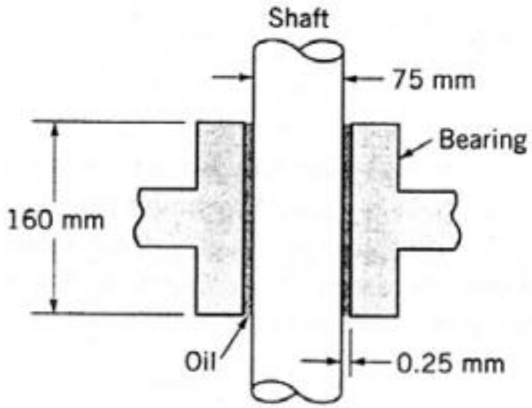
6- Determine the force on the piston of the figure due to shear, and the leakage from the pressure chamber for U = 0. [295.1 N ; 1.636×10^{-8} m^3/s]



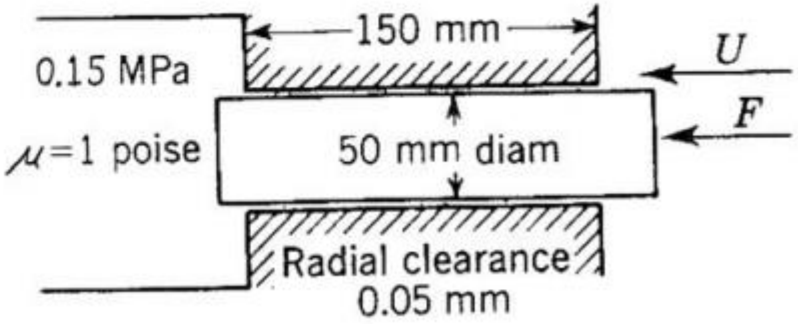
Problem No. 2



Problem No. 4

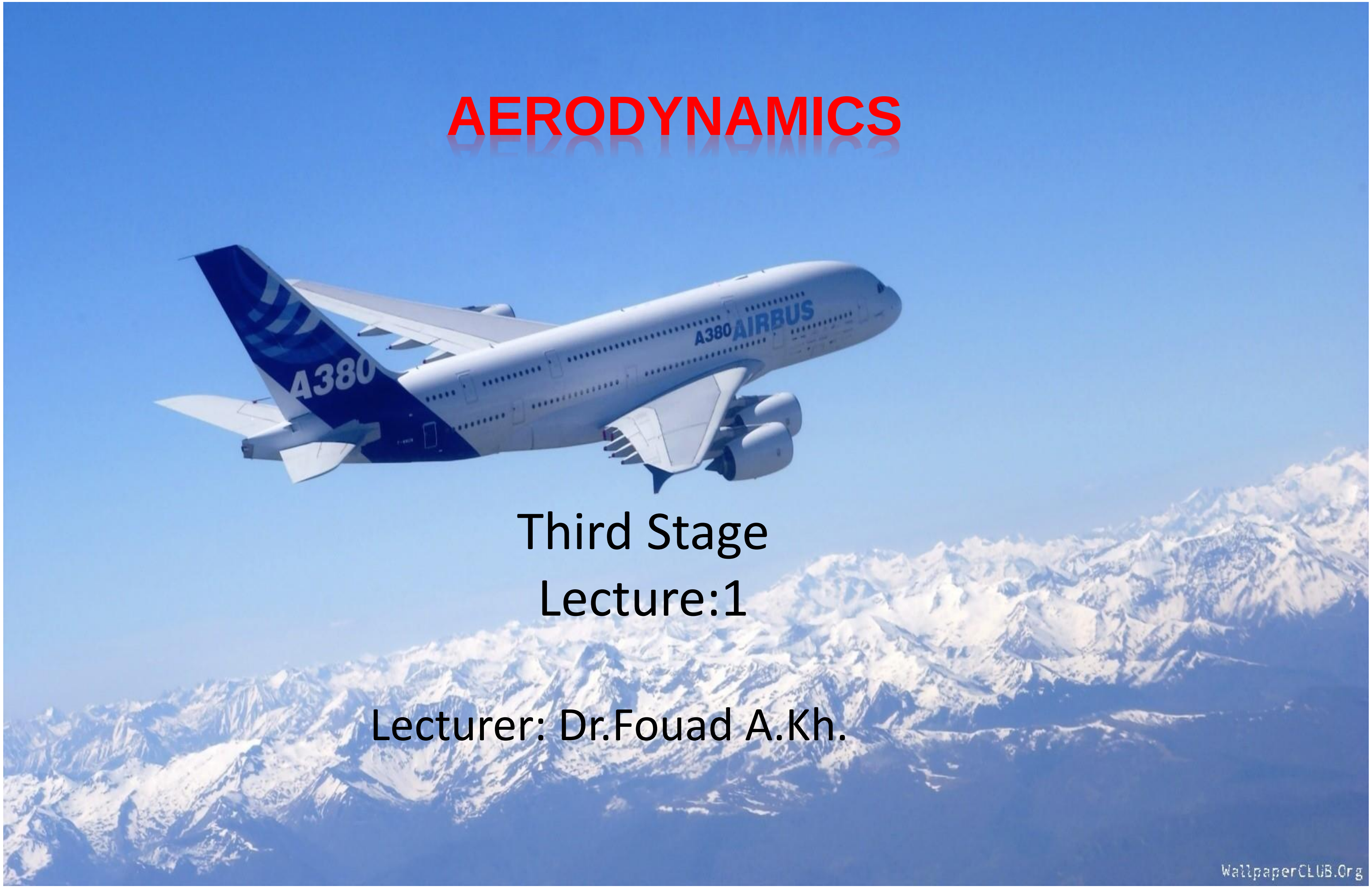


Problem No. 5



Problem No. 6

AERODYNAMICS



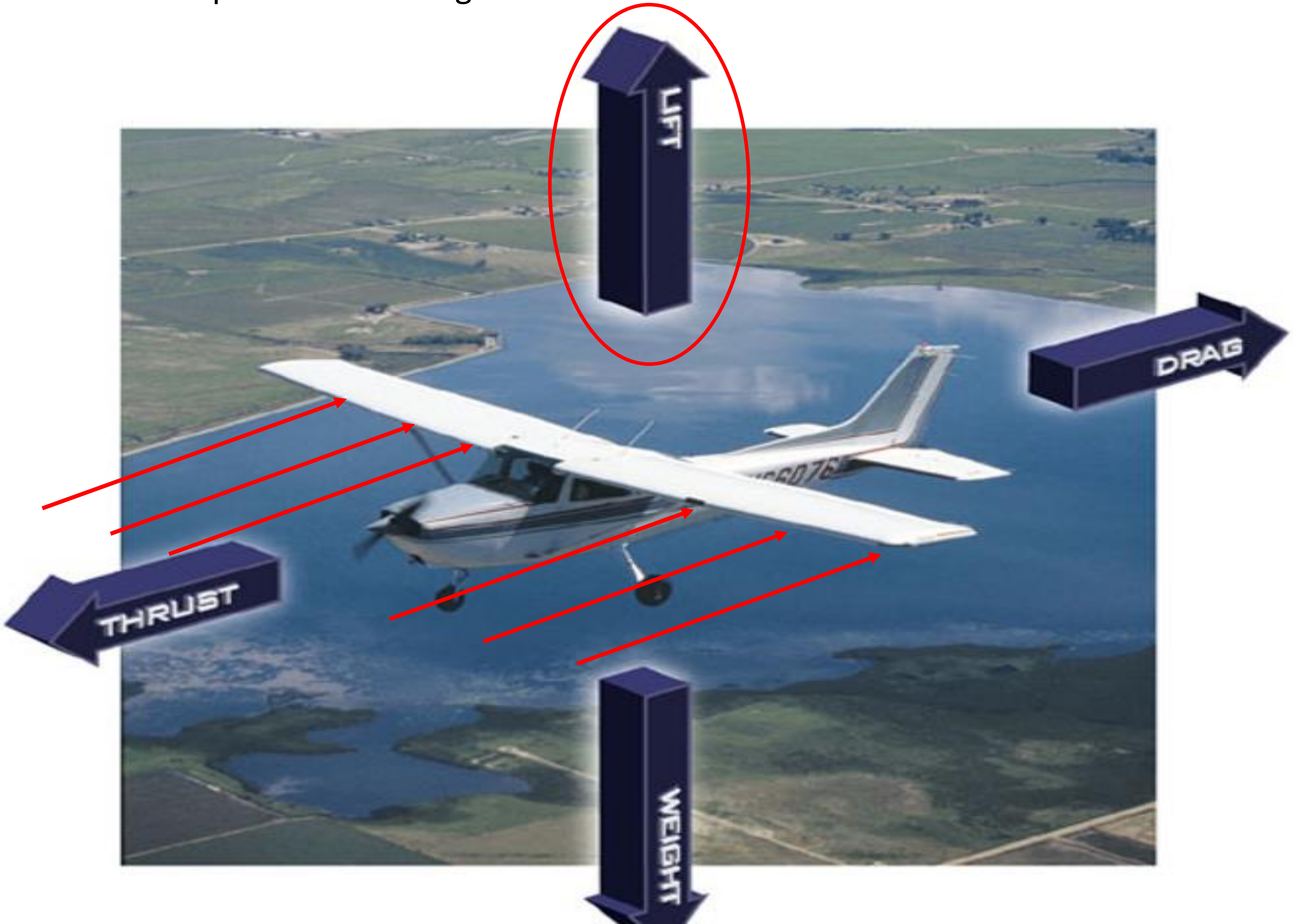
Third Stage

Lecture:1

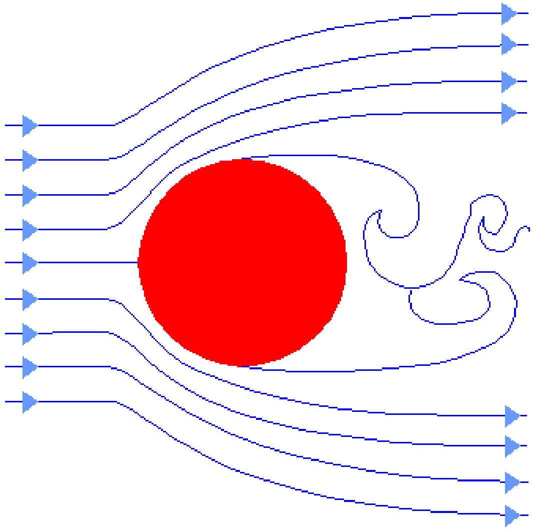
Lecturer: Dr.Fouad A.Kh.

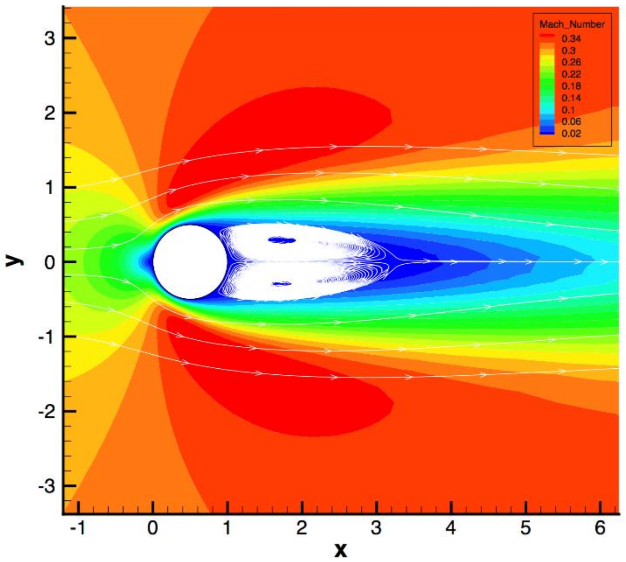
Definition

Aerodynamics is the science that study of objects in motion through the air and the forces that produce or change such motion.



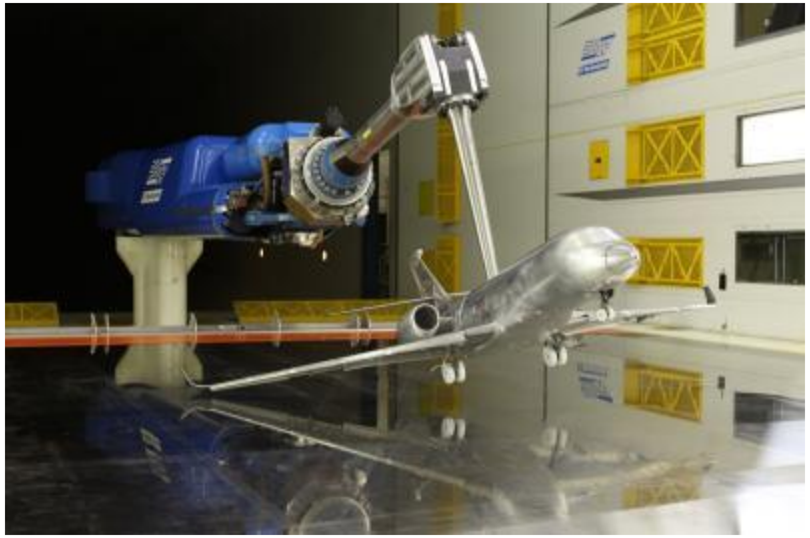
Flow around cylinder





Flow around aero foil

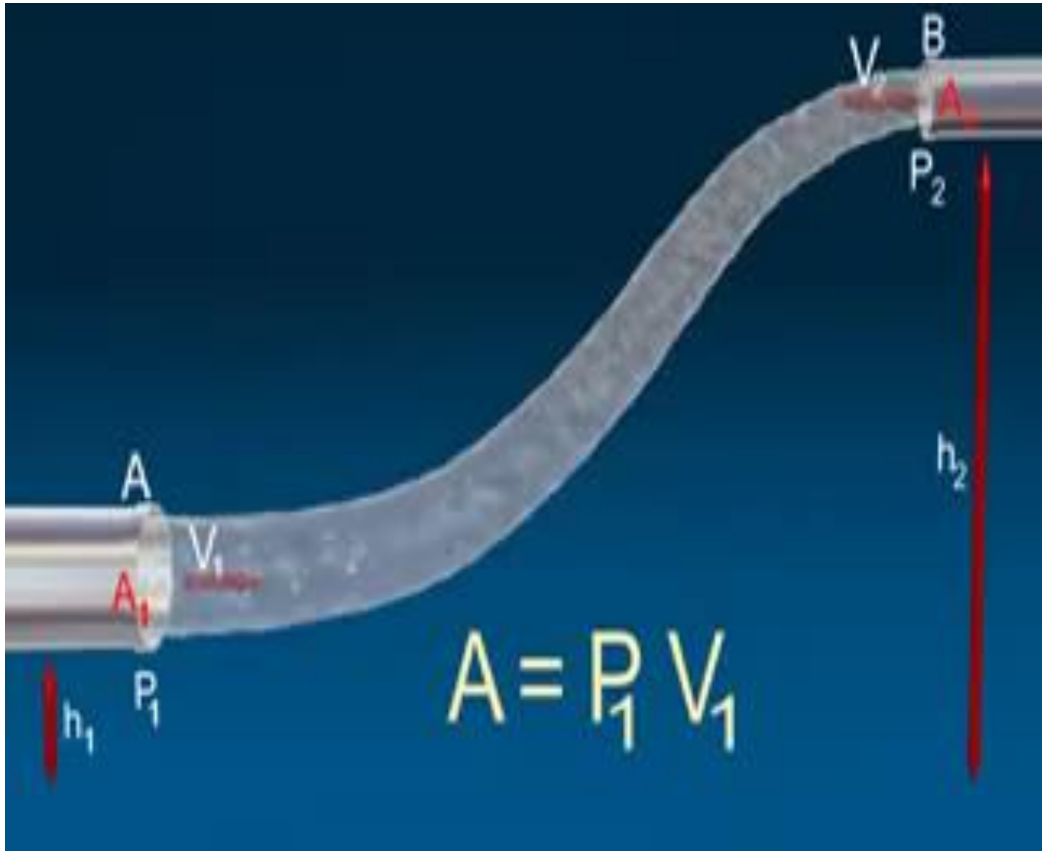
Flow around aircraft



Navier-Stokes Equations

-An important equation which give us mathematical description for the fluid flow (internal or external flow)

-fundamental partial differentials equations that describe the flow of fluids. Using the rate of stress and rate of strain



The Navier-Stokes Equations

2-Momentum Equation:

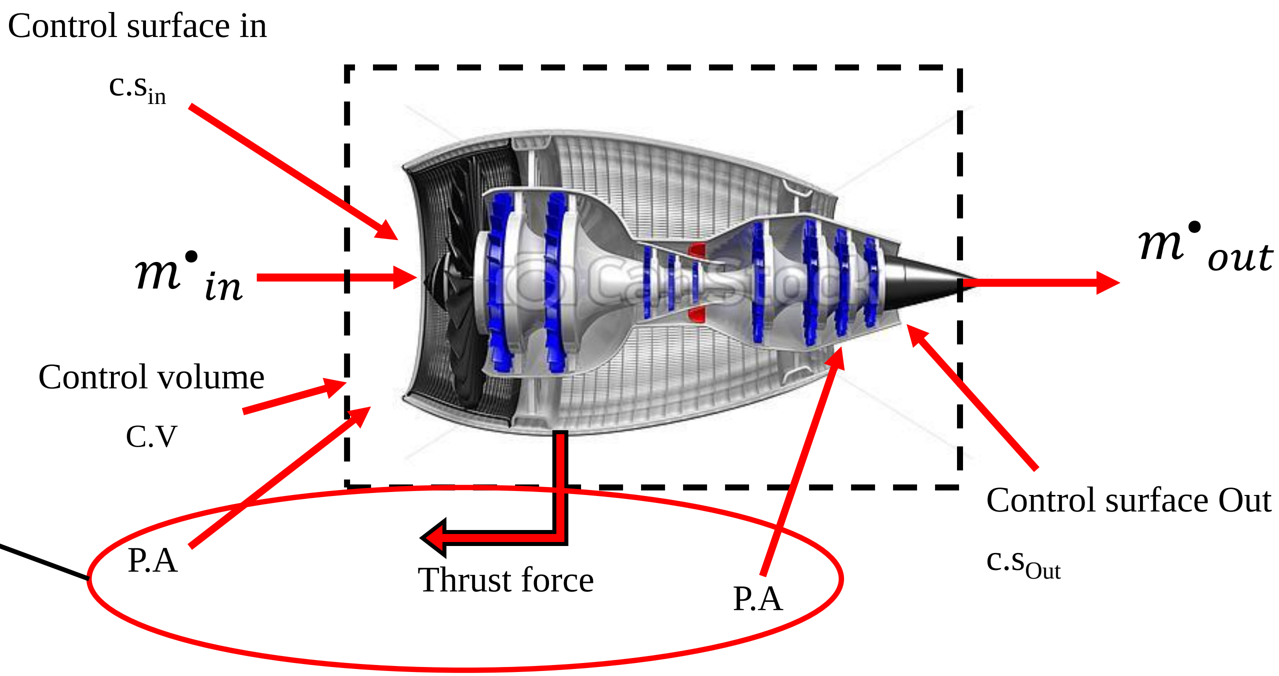
Newton's second law of motion: the resultant of force applied to particle which may be at rest or in motion is equal to rate of change of momentum of the particle in the direction of the resultant force

$$\sum F_x = \frac{d(mV_x)}{dt}$$

$$\sum F_x = m.a$$

$$a = \frac{dV}{dt}$$

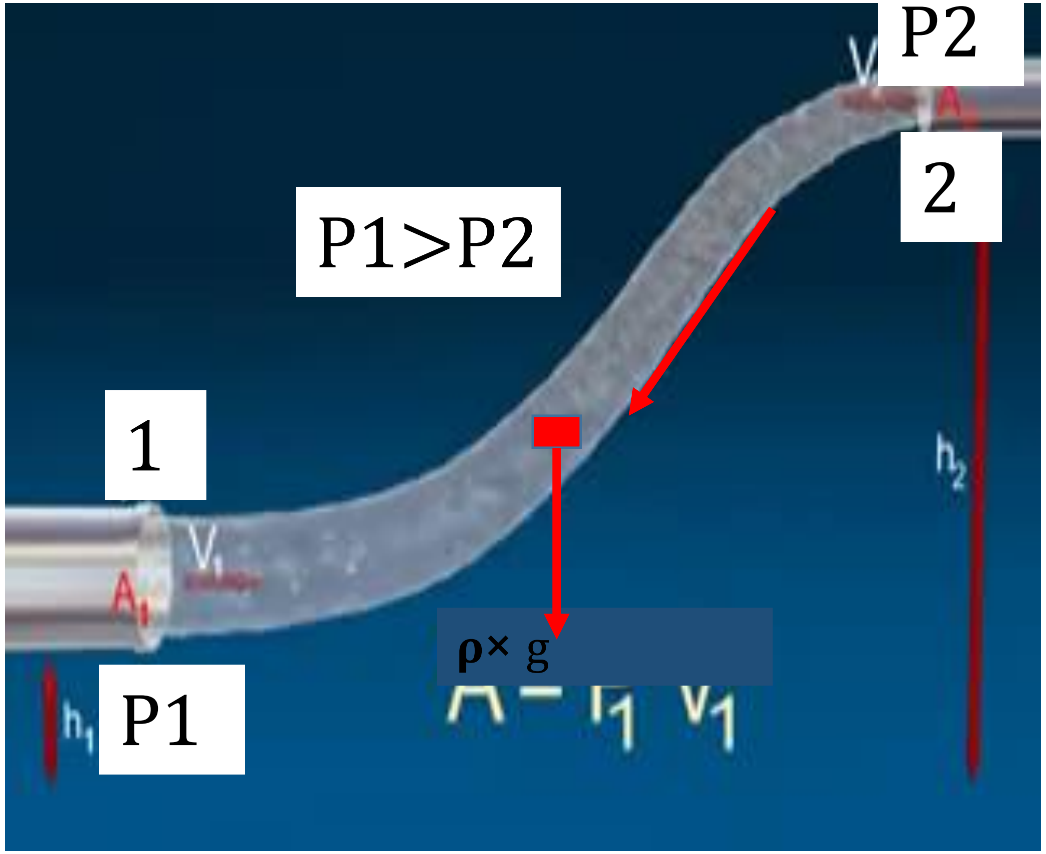
$$\sum F_x = \frac{d(mV_x)}{dt} = m.a$$



When the momentum equation is applied to an infinitesimal control volume (c.v.), it can be written in the form:

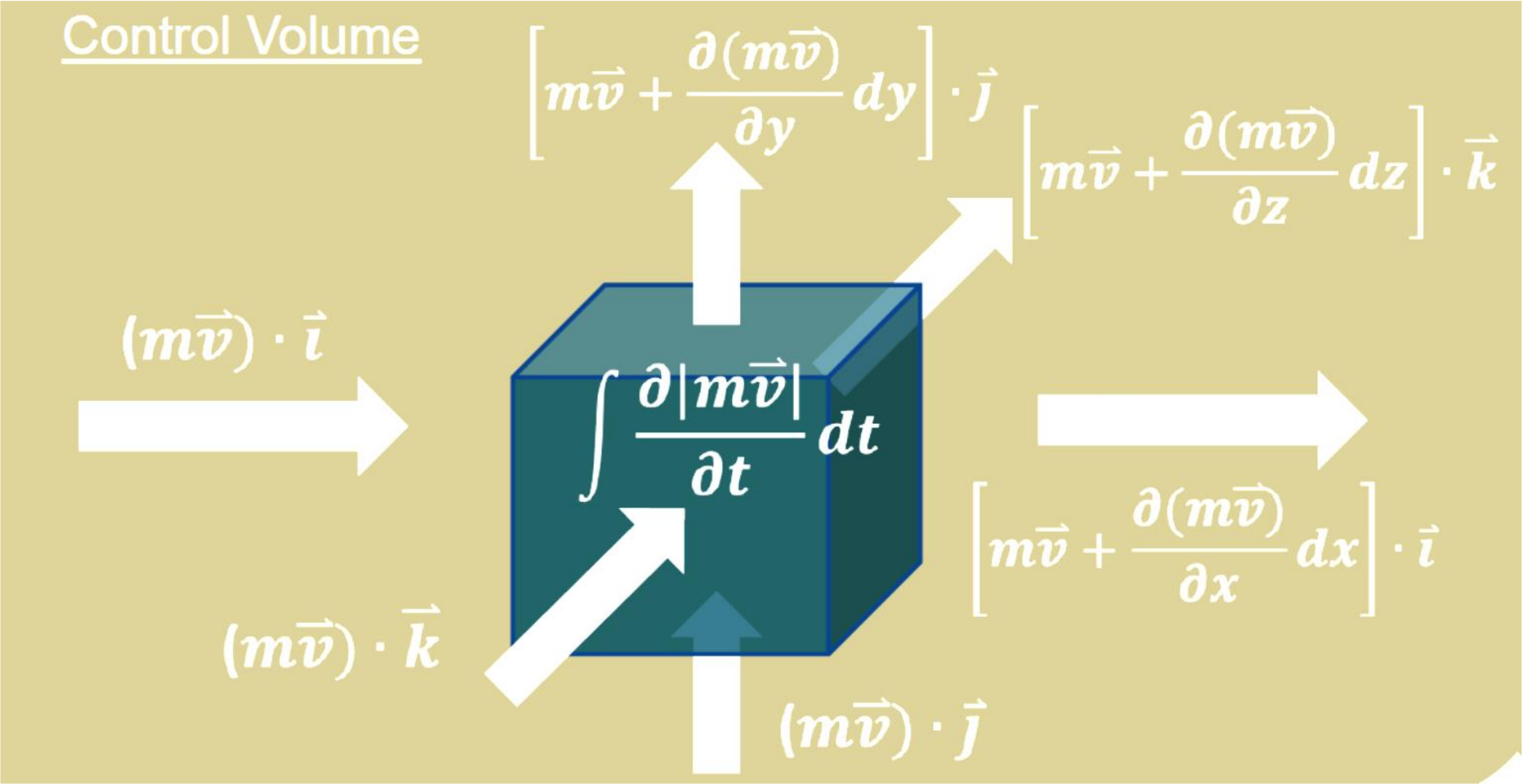
Rate of increase of momentum within the C.V. + Net rate at which momentum leaves the C.V.=

= Body force + pressure force + viscous force



❖ The second law is based on the conservation of momentum

$$\Sigma\vec{F} = \frac{D(m\vec{v})}{Dt} = \frac{\partial(m\vec{v})}{\partial t} + \frac{\partial(m\vec{v})}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial(m\vec{v})}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial(m\vec{v})}{\partial z}\frac{\partial z}{\partial t}$$



$$m \cdot \frac{D\ \vec{V}}{Dt} = \sum \vec{F} \qquad (Eq\ 1)$$

We express the total force as the sum of body forces and surface forces

$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface}$. Thus $(Eq\ 1)$ can be written as

$$m \cdot \frac{D\ \vec{V}}{Dt} = \sum \vec{F}_{body} + \sum \vec{F}_{surface} \qquad (Eq\ 2)$$

Body forces: Gravity force, Electromagnetic forse, Centrifugal force

Surface forces: Pressure forces, Viscous forces

We cosider the x-component of (*Eq* 2).

Since $m = \rho dxdydz$ and $\vec{V} = (u, v, w)$ we have

$$\rho dxdydz \cdot \frac{Du}{Dt} = \sum \vec{F}_{x,body} + \sum \vec{F}_{x,surface} \qquad (Eq\ 3)$$

x component:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

Convective Term
Rate of change
Momentum Term

Pressure Term

Body force Term
Gravity

Viscous Term

y- *component:*

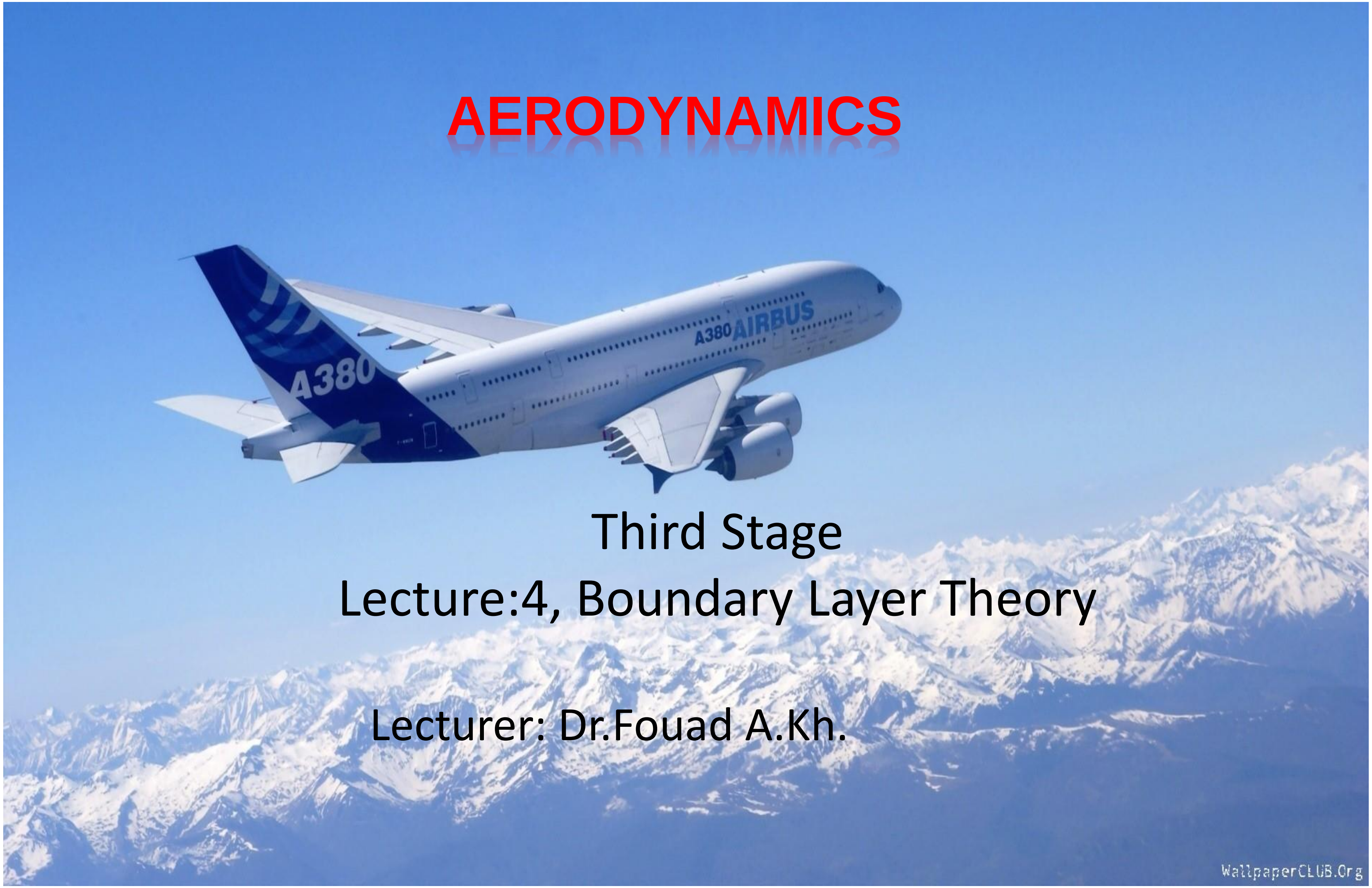
$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

z component:

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

[The vector form for these equations: $\rho\frac{D\vec{V}}{Dt}=-\nabla P+\rho\vec{g}+\mu\nabla^2\vec{V}$]

AERODYNAMICS



Third Stage

Lecture:4, Boundary Layer Theory

Lecturer: Dr.Fouad A.Kh.

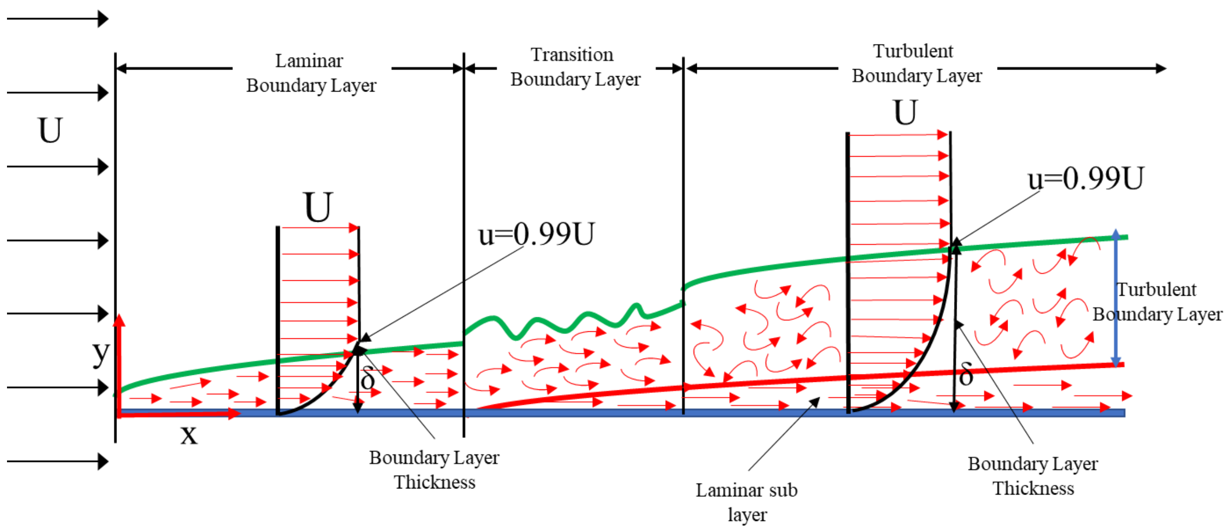
Momentum Equation:

For

1. 2D flow 2• Steady flow 3. $\frac{dp}{dx} = 0$ 4. $\frac{du}{dx} = 0$

2. $\tau o = p\frac{d}{dx}\int_0^{\delta}(U-u)u * dy \rightarrow \tau o = \rho U^2 \frac{d}{dx}\int_0^{\delta}\frac{u}{U} * (1-\frac{u}{U})dy$

For dimensionless



$\frac{u}{U} = f\left(\frac{y}{\delta}\right) = f(\eta)$, Where $\eta = \frac{y}{\delta}$

B.C At y=0, u=0, gives $\frac{u}{U} = 0$, $\eta = 0$

At $y = \delta, u = U$, gives $\frac{u}{U} = 1, f(\eta) = 1, \eta = 1$

Where $\eta = \frac{y}{\delta}$

$$\eta = \frac{y}{\delta} \qquad \eta.\delta = y \qquad \delta.d\eta = dy$$

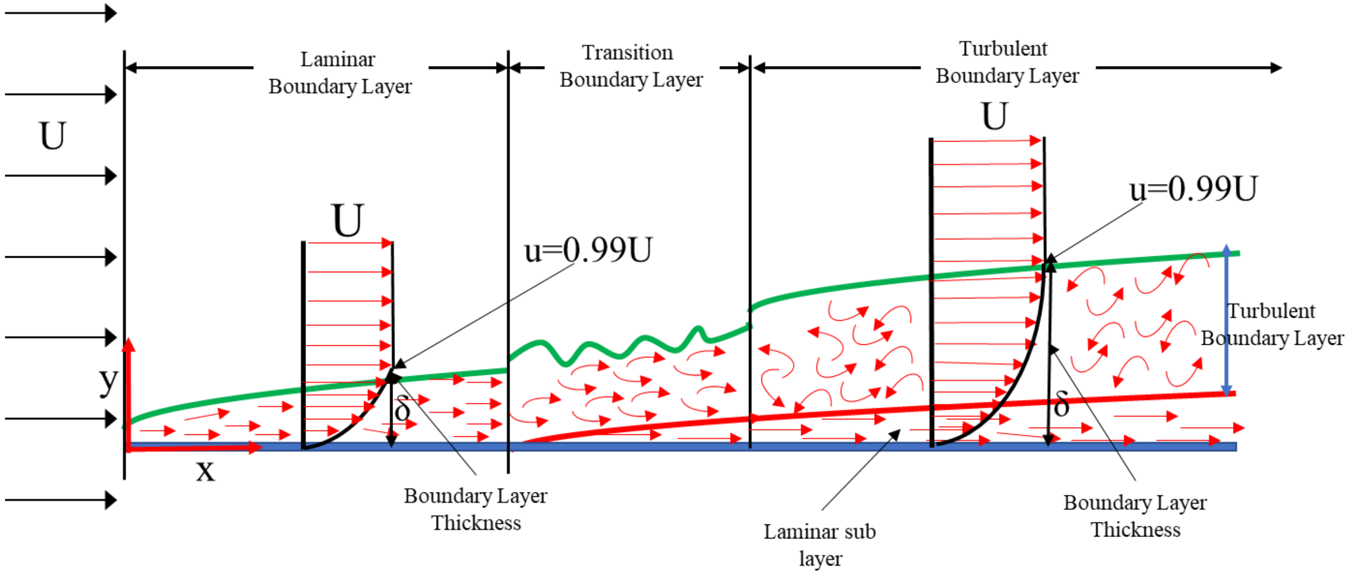
$$\tau o = \rho U^2 \frac{d\delta}{dx} \int_0^1 f(\eta)(1 - f(\eta))\, d\eta$$

$$\text{let } A = \int_0^1 f(\eta)(1 - f(\eta))\, d\eta$$

$$\tau o = \rho U^2 \frac{d\delta}{dx} A$$

$$\tau o = \mu \frac{du}{dy} |_{y=0}$$

$$\frac{u}{U} = f(\eta) \rightarrow u = Uf(\eta)$$



$$du = U\,df(\eta) \qquad \eta = \frac{y}{\delta} \qquad \delta\eta = y \qquad \delta.\,d\eta = dy$$

$$\frac{du}{dy} = \frac{U}{\delta} = \frac{df(\eta)}{d\eta}\Big|_{\eta=0} \qquad \text{Because}: \delta\eta = y$$

When y=0 $\rightarrow \eta = 0 \rightarrow \delta \neq 0$

$$\tau o = \frac{\eta}{\delta} U \frac{df(\eta)}{d\eta}\Big|_{\eta=0}$$

$$\text{let } \frac{df(\eta)}{d\eta}\Big|_{\eta=0} = B$$

$$\tau o = \frac{\eta}{\delta} UB$$

$$\tau o = \rho A U^2 \frac{d\delta}{dx} = \tau o = B * \frac{\mu U}{\delta}$$

$$\frac{d\delta}{dx} = \frac{\mu UB}{\delta \rho A U^2}$$

$$\int \delta\, d\delta = \frac{\mu UB}{\rho A U^2} dx = \frac{\mu B}{\rho A U}\, dx$$

$$\frac{\delta^2}{2} = \frac{\mu Bx}{\rho A U} \qquad * \frac{x}{x}$$

$$\delta^2 = \frac{2\,Bx^2}{A} * \frac{\mu}{\rho U x} = \frac{2\,Bx^2}{A} * \frac{1}{R_{ex}}$$

$$\delta = \sqrt{\frac{2B}{A} * \frac{x}{\sqrt{R_{ex}}}}$$

$$\frac{d\delta}{dx} = \frac{\mu UB}{\rho A U^2 \sqrt{\frac{2B}{A} * \frac{x}{\sqrt{R_{ex}}}}} = \sqrt{\frac{\mu B}{2\rho U A X}}$$

$$\frac{d\delta}{dx} = \sqrt{\frac{\mu B}{2\rho U A X}}$$

$$\tau o = \rho A U^2 \sqrt{\frac{\mu B}{2\rho U A X}} = \rho U^2 \sqrt{\frac{A\mu B}{2\rho U X}} = \rho U^2 \sqrt{\frac{BA}{2R_{ex}}}$$

$$FD = b\int_0^L \tau o \, dx \quad , b = 1 \text{ , width}$$

$$FD = \int_0^L \rho U^2 \sqrt{\frac{A\mu B}{2\rho U X}} \, dx = \int_0^L \rho U^2 \sqrt{\frac{A\mu B}{2\rho U}} * x^{\frac{1}{2}} \, dx$$

$$FD = \sqrt{\frac{\rho^2 U^4 AB\mu}{2\rho U}} * \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \Big|_0^L$$

$$= \sqrt{\frac{\rho U^3 AB\mu * 4 * L}{2}}$$

$$FD = \sqrt{2\rho U^3 AB\mu L} \quad , L = \textit{length of the plates}$$

$$CD = \frac{FD}{\frac{1}{2}\rho L U^2 * 1}$$

$$CD = \frac{\sqrt{2\rho U^3 AB\mu L}}{\frac{1}{2}\rho L U^2}$$

$$CD = 2 * \sqrt{\frac{2AB\mu}{\rho U L}} = CD = 2 * \sqrt{\frac{2AB}{R_e \, L}}$$

$$Cf = \frac{\tau o}{\frac{1}{2}\rho U^2} = f = \frac{4\tau o}{\frac{1}{2}\rho U^2} = Cf = \frac{4}{f}$$

$$\tau o = \frac{1}{2}\rho U^2 Cf$$

$$FD = \tau o * Area$$

$$= \int_0^L \tau o * b * dx \quad , b = 1 \text{ , width}$$

$$FD = \frac{1}{2}\rho U^2 Cf * Area$$

$$Cf = \frac{FD}{\frac{1}{2}\rho U^2 * Area}$$

$$FD = \sqrt{2\rho U^3 AB \mu L} = \sqrt{2\rho U^3 * 0.139 * \frac{3}{2} \mu L}$$

$$FD = 0.644 * \sqrt{2\rho U^3 \mu L}$$

$$CD = 2 * \sqrt{\frac{2AB\mu}{\rho UL}} = 2 * \sqrt{\frac{2 * 0.139 * \frac{3}{2}}{R_e L}}$$

$$= \frac{1.288}{\sqrt{R_e L}}$$

$$\delta = \sqrt{\frac{2B}{A}} * \frac{X}{\sqrt{R_{ex}}}$$

$$\frac{d\delta}{dx} = \sqrt{\frac{B\mu}{2\rho UAx}}$$

$$\tau o \ = \rho U^2 \frac{d\delta}{dx} A = \rho U^2 \sqrt{\frac{BA}{2R_{ex}}}$$

$$FD = \int_0^L \tau o \; dx = \int_0^L \tau o \; dx = \rho U^2 \sqrt{\frac{BA}{2R_{ex}}}$$

$$dx = \sqrt{2\rho U^3 AB \mu L}$$

$$CD = \frac{FD}{\frac{1}{2}\rho L U^2} = 2\sqrt{\frac{2AB}{R_e L}}$$

Turbulent B.L

$$\frac{u}{U} = (\frac{y}{\delta})^{\frac{1}{n}} = (\eta)^{\frac{1}{n}} = f(\eta) = (\eta)^{\frac{1}{n}}$$

$$\tau o = \mu \frac{du}{dy}|_{y=0}$$

is equal to ∞ because

$$\left(\frac{d(\eta)^{\frac{1}{n}}}{d\eta}\right) = \frac{1}{n}(\eta)^{\frac{1}{n}-1}|_{\eta=0}$$

$$\tau o = \frac{1}{2}\rho U^2 Cf$$

in pipe= f= 0.316$* R_e^{\frac{-1}{4}}$

$$R_e = \frac{\rho DV}{\mu}$$

$$U = 1.235V$$

$$\text{Vaverge} = \frac{1}{1.235}U = 0.809 * U$$

$$R_e = \frac{\rho DV}{\mu} \quad , D = 2\delta$$

$$\tau o = \frac{1}{2}\rho U^2 Cf = \frac{1}{2} * \frac{F}{4} * \frac{\rho U^2}{1.235^2} = f$$
$$= 0.316 * R_e^{\frac{-1}{4}}$$

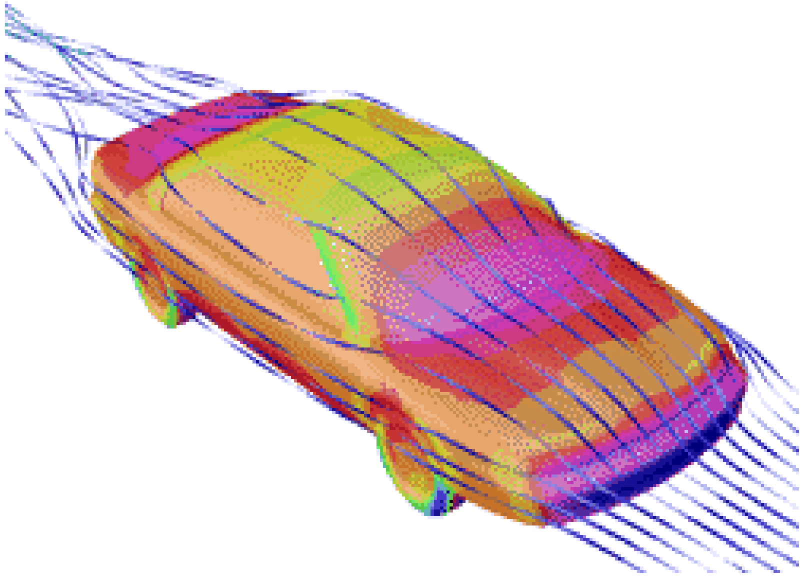
$$\tau o = \frac{1}{8} * 0.316 * \frac{\rho U^2}{1.235^2} R_e^{\frac{-1}{4}}$$
$$= \frac{0.316}{8 * 1.235^2}\rho U^2 * (\frac{\rho U * 2}{\mu * 1.235})^{\frac{-1}{4}}$$

$$\tau o = 0.0228 * \rho U^2 * R_e^{\frac{-1}{4}} * \delta \text{ where } D = 2\delta$$

Introduction

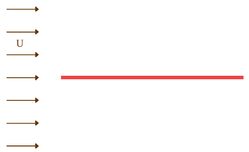
The drag on a body passing through a fluid may be considered to be made up of two components: Form drag and Skin friction drag.

Form drag: which is dependent on the pressure forces acting on the body; and the ***skin friction drag*** , which depends on the shearing forces acting between the body and the fluid.



Shear Force and Pressure Force

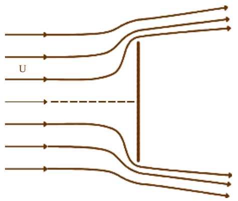
- **Shear forces:**



Major losses in pipes

  - viscous drag, frictional drag, or skin friction
  - caused by shear between the fluid and the solid surface
  - function of Surface area and Length of object

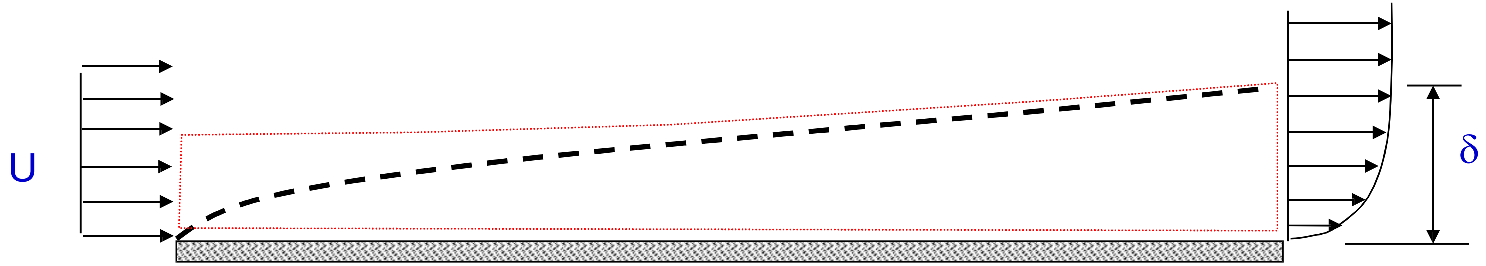
- **Pressure forces**



Flow expansion losses

  - pressure drag or form drag
  - caused by Flow separation from the body
  - function of area normal to the flow Projected area

Description of Boundary Layer



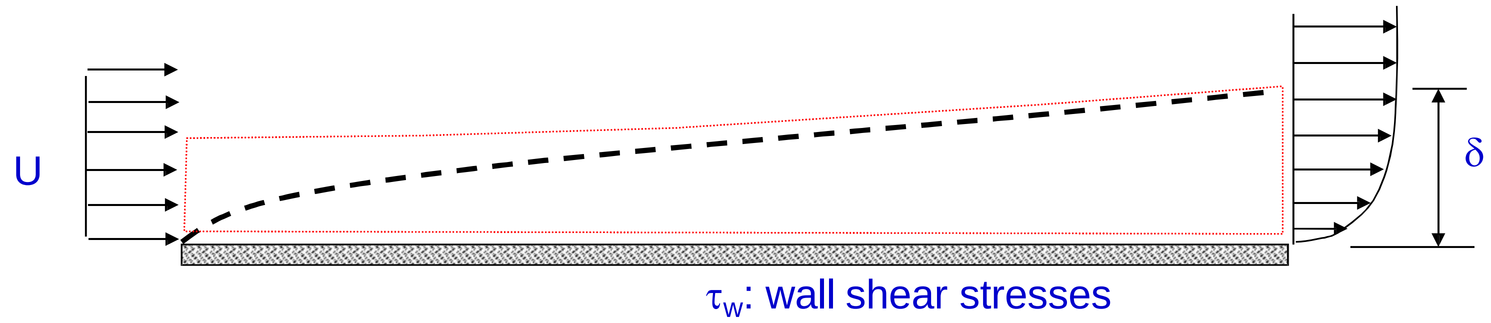
τ_w: wall shear stresses

In the immediate vicinity of the boundary surface, the velocity of the fluid increases gradually from zero at boundary surface to the velocity of the mainstream. This region is known as **BOUNDARY LAYER**.

Large velocity gradient leading to appreciable shear stress: $\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$

The nominal thickness of BOUNDARY LAYER is defined as the distance from the boundary where the velocity of fluid is 99 % of free stream velocity

Description of Boundary Layer



Consists of two layers:

CLOSE TO BOUNDARY : large velocity gradient, appreciable viscous forces.

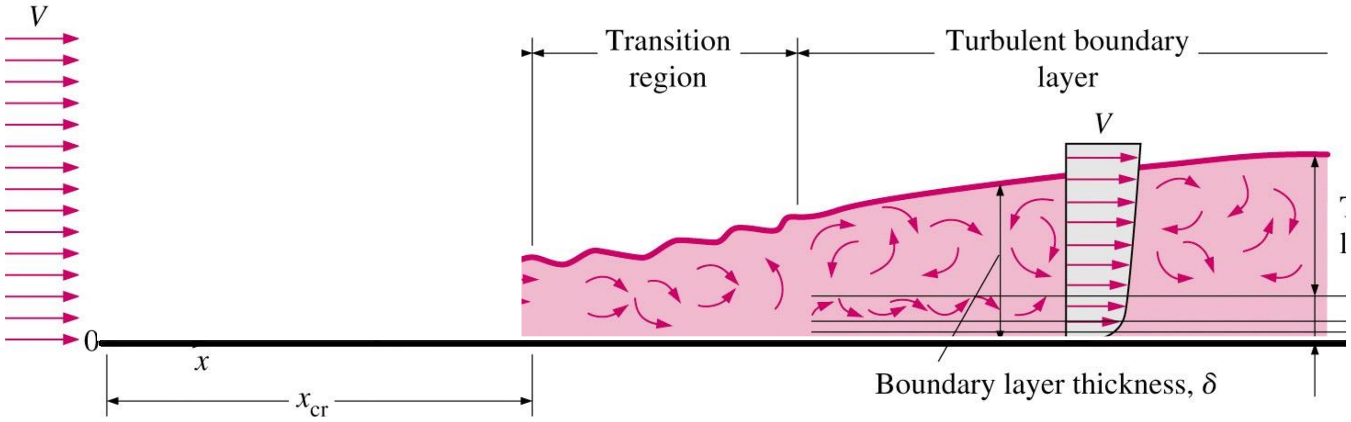
OUTSIDE BOUNDARY LAYER: viscous forces are negligible, flow may be treated as non-viscous or inviscid.

shear stress: $$\tau = \mu \left(\frac{\partial u}{\partial y} \right)$$

Shear stress acting at the plate surface sets up a shear force which opposes the fluid motion, and fluid close to the wall is decelerated.

Theoretical understanding on Boundary layer development is very important to determine the velocity gradient and hence shear forces on the surface.

Development of

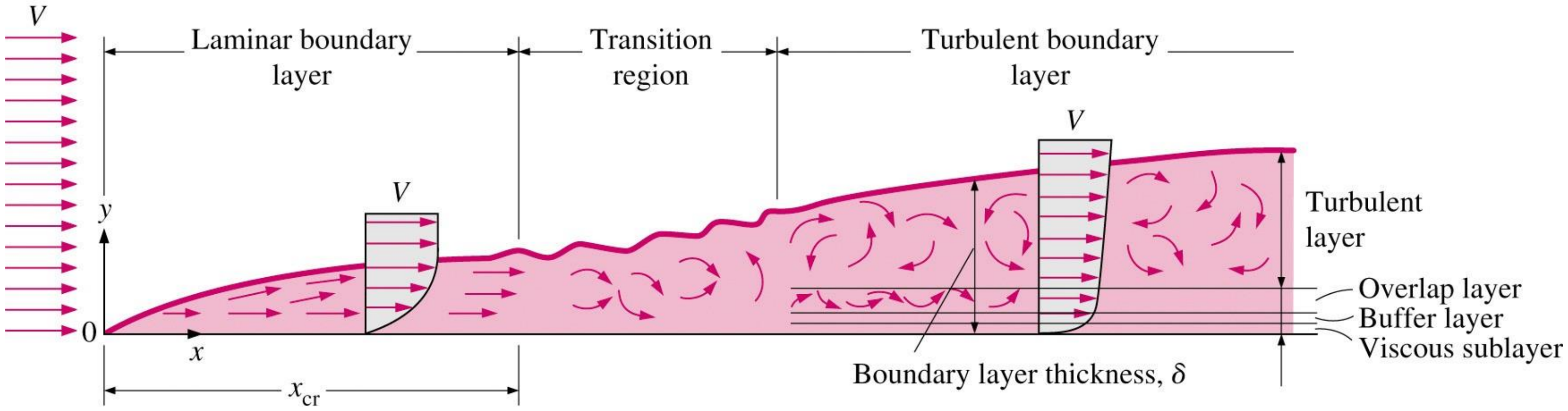


The boundary layer thickness increases as the distance x from leading edge is increases. This is because of viscous forces that dissipate more and more energy of fluid stream as the flow proceeds and large group of particles are slow downed.

In laminar boundary layer the particles are moving along stream lines.

The disturbance in fluid flow in boundary layer is amplified and the flow become unstable and the fluid flow undergoes transition from laminar to turbulent flow. **This regime is called transition regime**.

Development of

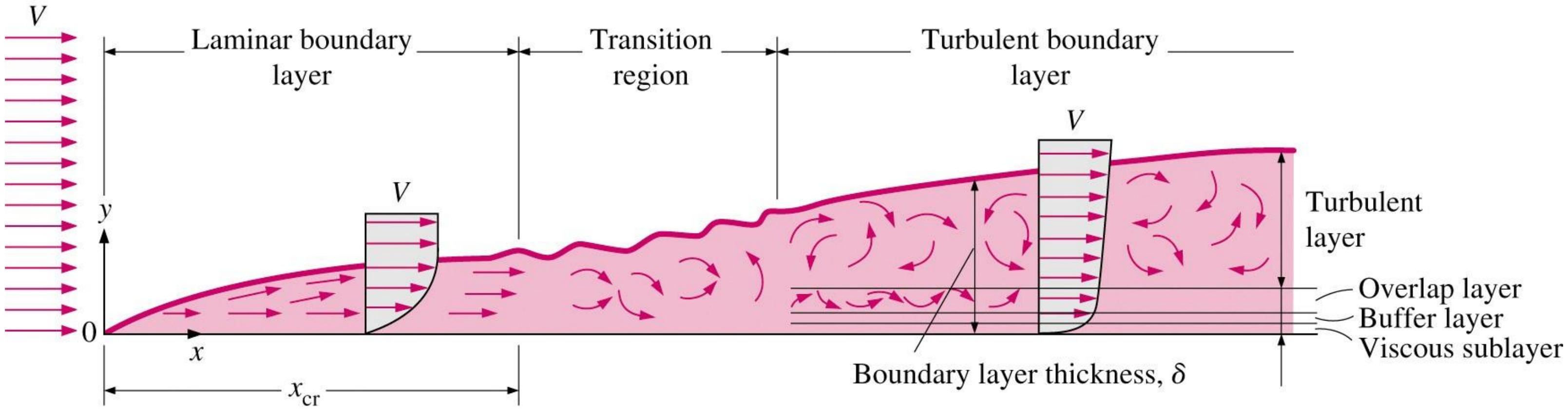


After going through transition zone of finite length the flow becomes completely turbulent which is characterized **by three dimensional, random motion of fluctuation induced bulk motion parcel of fluid.**

LAMINAR BOUNDARY LAYER PROFILE – PARABOLIC

TURBULENT BOUNDARY LAYER – PROFILE BECOMES LOGARITHMIC

Development of



BL depends on Reynold’s number & also on the surface roughness. Roughness of the surface adds to the disturbance in the flow & hastens the transition from laminar to turbulent.

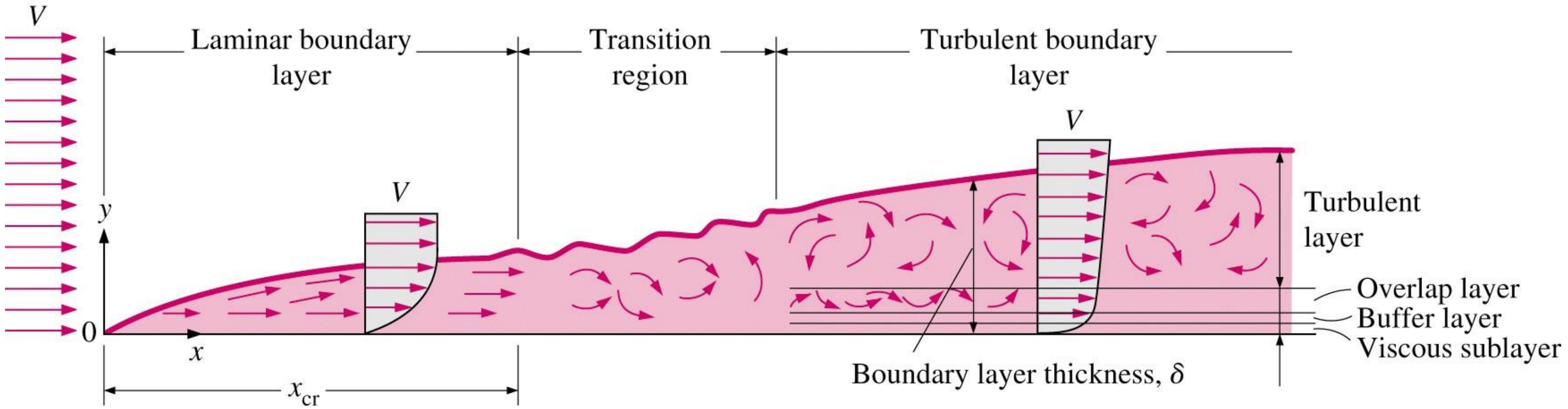
For laminar flow $\tau = \mu \left(\frac{\partial u}{\partial y} \right)$

For Turbulent flow $\tau = (\mu + \varepsilon) \frac{\partial u}{\partial y}$

Where ε is the *eddy viscosity* and is often much larger than μ

.

Boundary Layer Thickness for Laminar and Turbulent



The boundary layer thickness is governed by parameters like incoming velocity, kinematic viscosity of fluid etc.

For laminar flow

$$\delta_{lam} = \frac{5.0x}{\sqrt{Re_x}}$$

Pohlhausen (Exact solution)

$$\delta_{lam} = \frac{5.835x}{\sqrt{Re_x}}$$

Blassius (Approximate solution)

For Turbulent flow

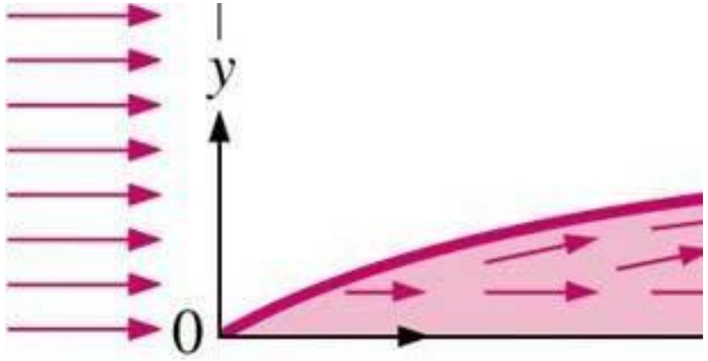
$$\delta_{tur} = \frac{0.377x}{Re^{1/5}}$$

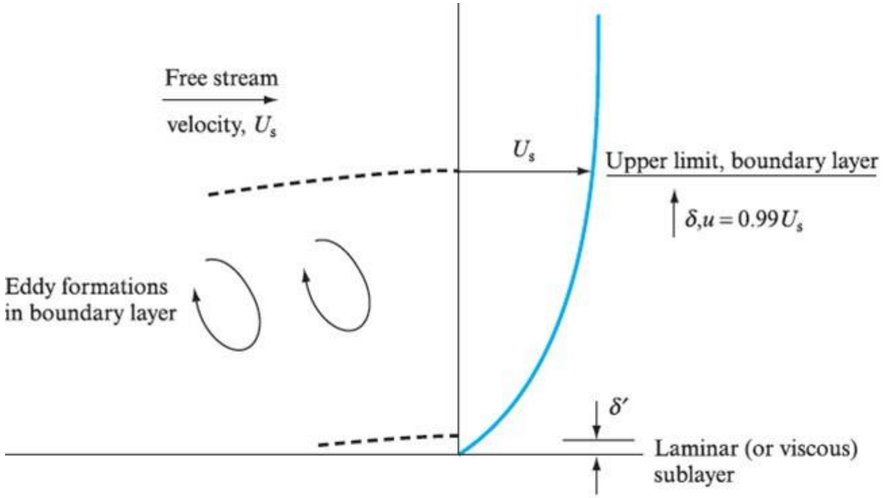
Flow Patterns and Regimes within Laminar and Turbulent Boundary Layer

As mentioned above, very close to the plane surface the flow remains laminar and a linear velocity profile may be assumed.

In this region, the velocity gradient is governed by the fluid viscosity

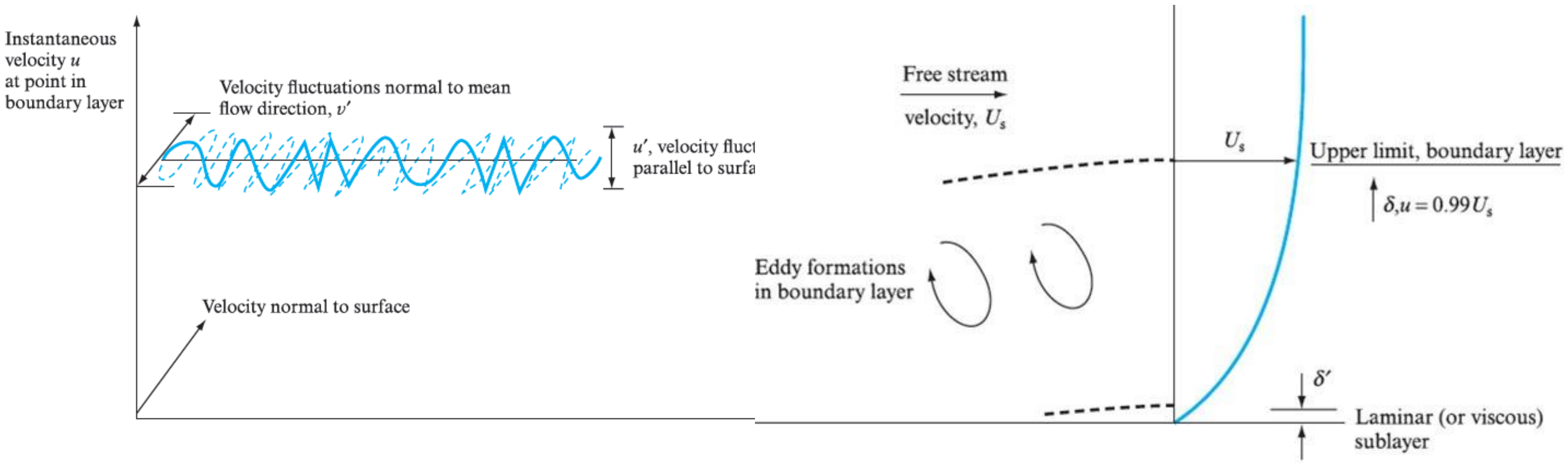
$$\left(\frac{\partial u}{\partial y} \right) = \frac{\tau}{\mu}$$





Flow Patterns and Regimes within Laminar and Turbulent Boundary Layer

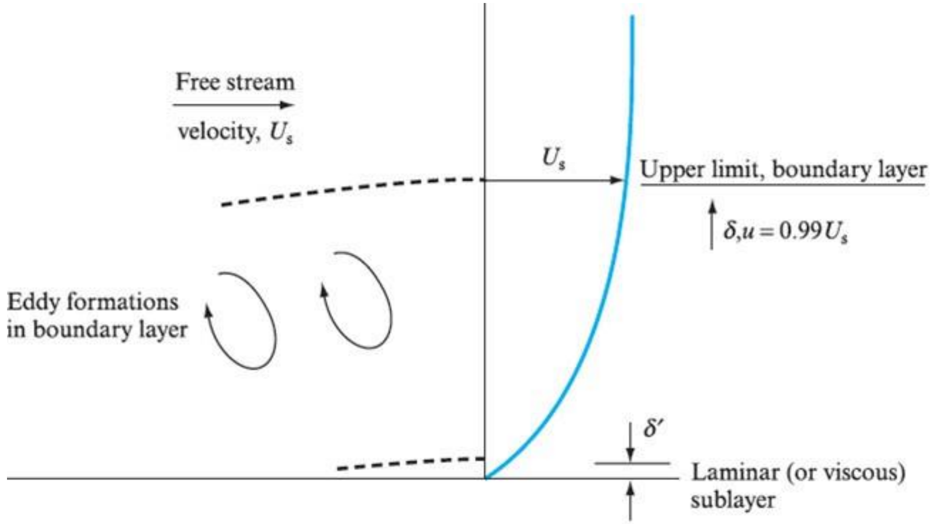
In turbulent flow, owing to the random motion of the fluid particles, eddy patterns are set up in the boundary layer which sweep small masses of fluid up and down through the boundary layer, moving in a direction perpendicular to the surface and the mean flow direction.



Flow Patterns and Regimes within Laminar and Turbulent Boundary Layer

Conversely, slow-moving fluid is lifted into the upper levels, slowing down the fluid stream and, by doing so, effectively thickening the boundary layer, explaining the more rapid growth of the turbulent boundary layer compared with the laminar one.

Owing to these eddies, fluid from the upper higher-velocity areas is forced into the slower-moving stream above the laminar sublayer, having the effect of increasing the local velocity here relative to its value in the laminar sublayer.



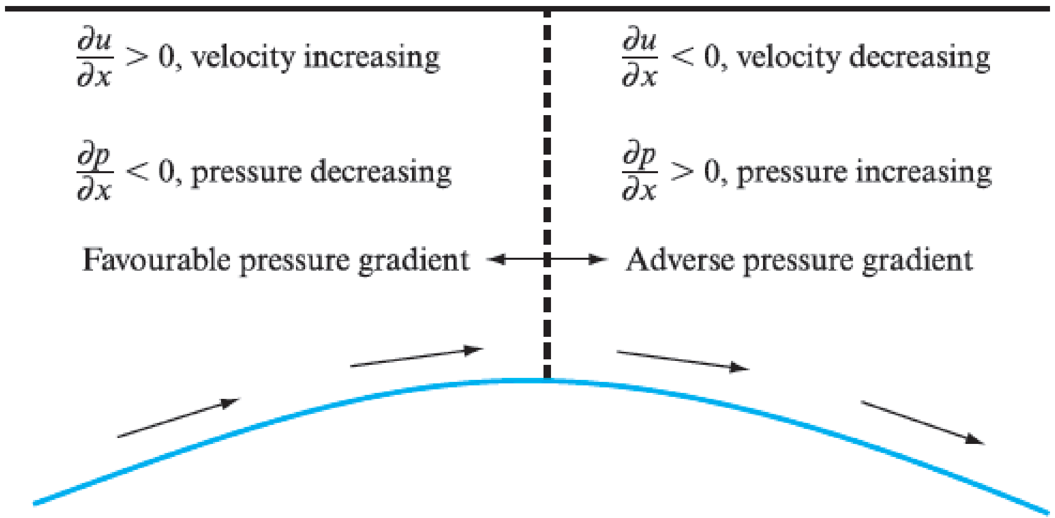
In order to explain this process, the eddy viscosity, ε should be added in Shear stress formulation.

$$\tau = (\mu + \varepsilon)\frac{\partial u}{\partial y}$$

Effect of Pressure Gradient on Boundary Layer Development

The presence of a pressure gradient $\partial p/\partial x$ *effectively means a* $\partial u/\partial x$ *term, i.e. the flow stream velocity changes across the surface.*

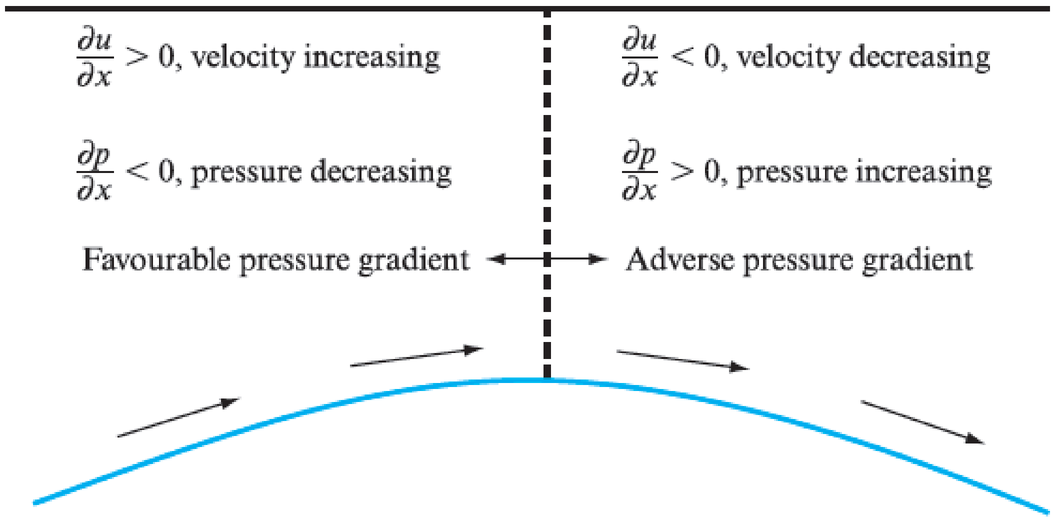
for example, consider a curved surface, then the velocity variation can be shown as:



Effect of Pressure Gradient on Boundary Layer Development

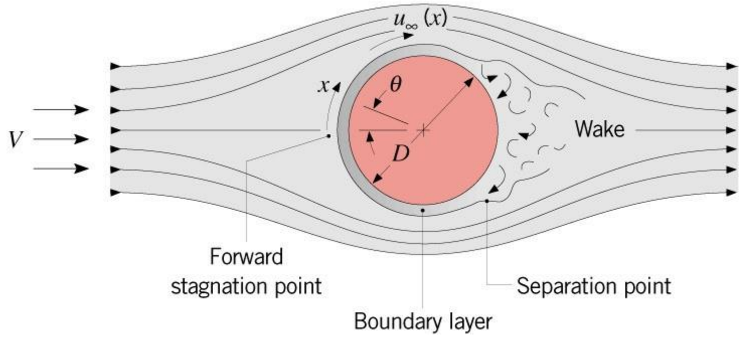
If the pressure *decreases in the downstream direction, then the boundary layer tends* to be reduced in thickness, and this case is termed a favorable pressure gradient.

If the pressure *increases in the downstream direction, then the boundary layer* thickens rapidly; this case is referred to as an adverse pressure gradient.



Cylinder in a Cross Flow

Conditions depend on special features of boundary layer development, including onset at a stagnation point and separation, as well as transition to turbulence.



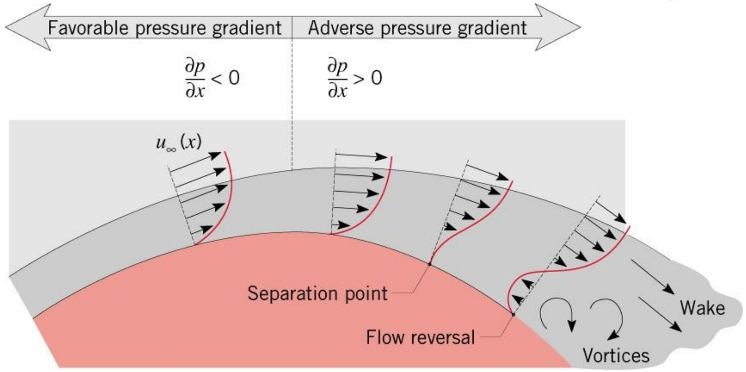
– Stagnation point: Location of zero velocity $(u_\infty = 0)$ and maximum pressure.

– Followed by boundary layer development under a favorable pressure gradient $(dp/dx < 0)$ and hence acceleration of the free stream flow $(du_\infty/dx > 0)$.

– As the rear of the cylinder is approached, the pressure must begin to increase. Hence, there is a minimum in the pressure distribution, *p(x)*, after which boundary layer development occurs under the influence of an adverse pressure gradient $(dp/dx > 0,\ du_\infty/dx < 0)$.

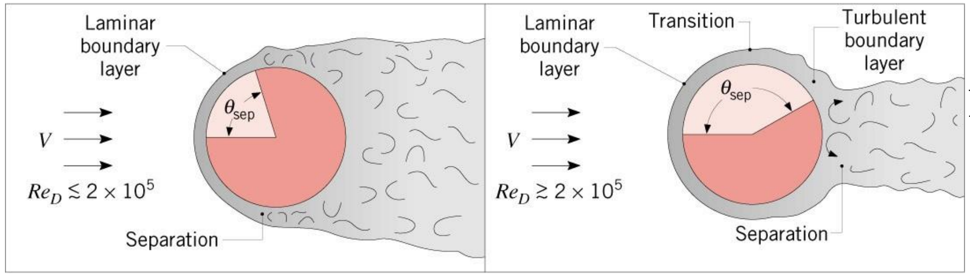
Cylinder in a Cross Flow

– Separation occurs when the velocity gradient $du/dy\,\big|_{y=0}$ reduces to zero.

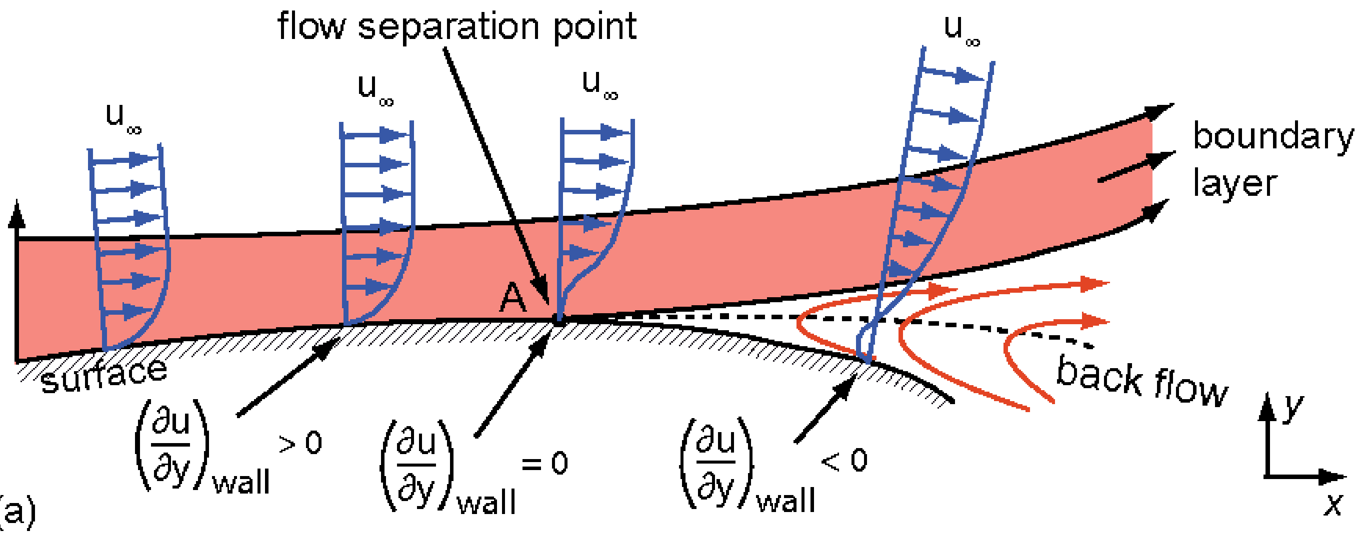


and is accompanied by flow reversal and a downstream wake.

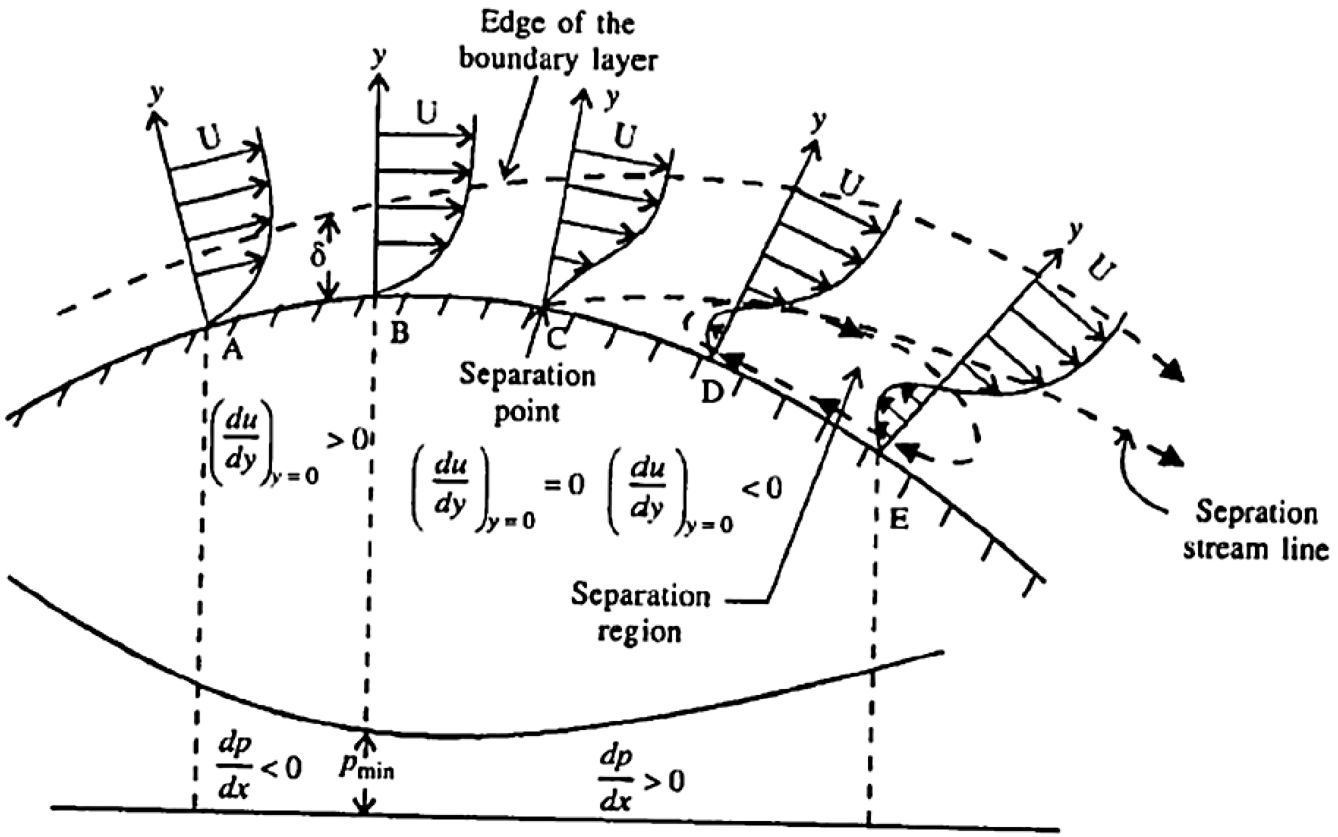
– Location of separation depends on boundary layer transition.



$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$


flow separation point
u_∞
u_∞
u_∞
u_∞
boundary layer
A
surface
back flow
$\left(\frac{\partial u}{\partial y}\right)_{wall} > 0$
$\left(\frac{\partial u}{\partial y}\right)_{wall} = 0$
$\left(\frac{\partial u}{\partial y}\right)_{wall} < 0$
y
x

(a)


Edge of the boundary layer
y
U
δ
A
B
C
D
E
Separation point
$\left(\frac{du}{dy}\right)_{y=0} > 0$
$\left(\frac{du}{dy}\right)_{y=0} = 0$
$\left(\frac{du}{dy}\right)_{y=0} < 0$
Sepration stream line
Separation region
$\frac{dp}{dx} < 0$
p_{min}
$\frac{dp}{dx} > 0$

Boundary Layer History

❖ 1904 Prandtl

Fluid Motion with Very Small Friction

2-D boundary layer equations

❖ 1908 Blasius

The Boundary Layers in Fluids with Little Friction

Solution for laminar, 0-pressure gradient flow

❖ 1921 von Karman

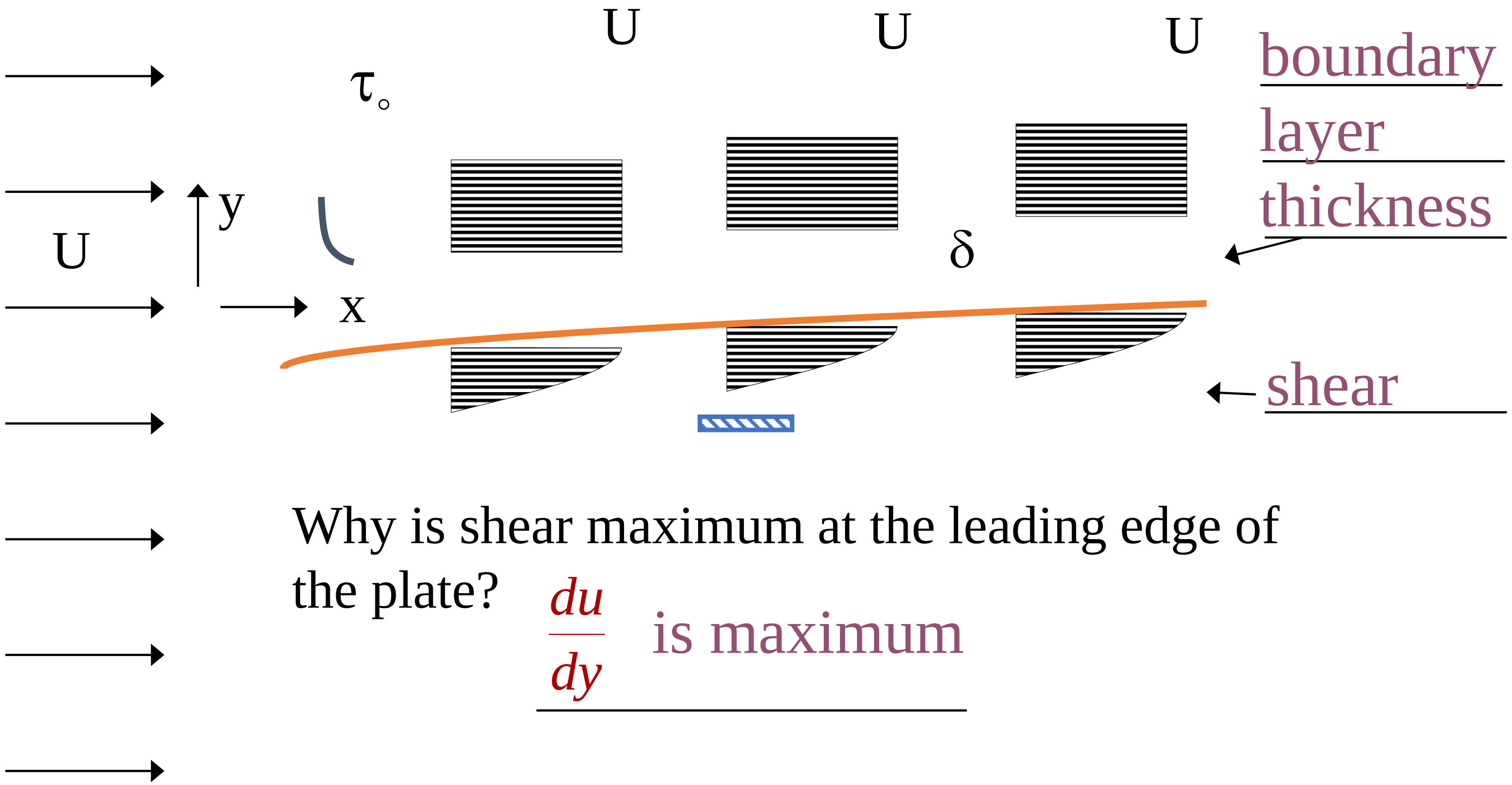
Integral form of boundary layer equations

THANK YOU

HAVE A NICE DAY

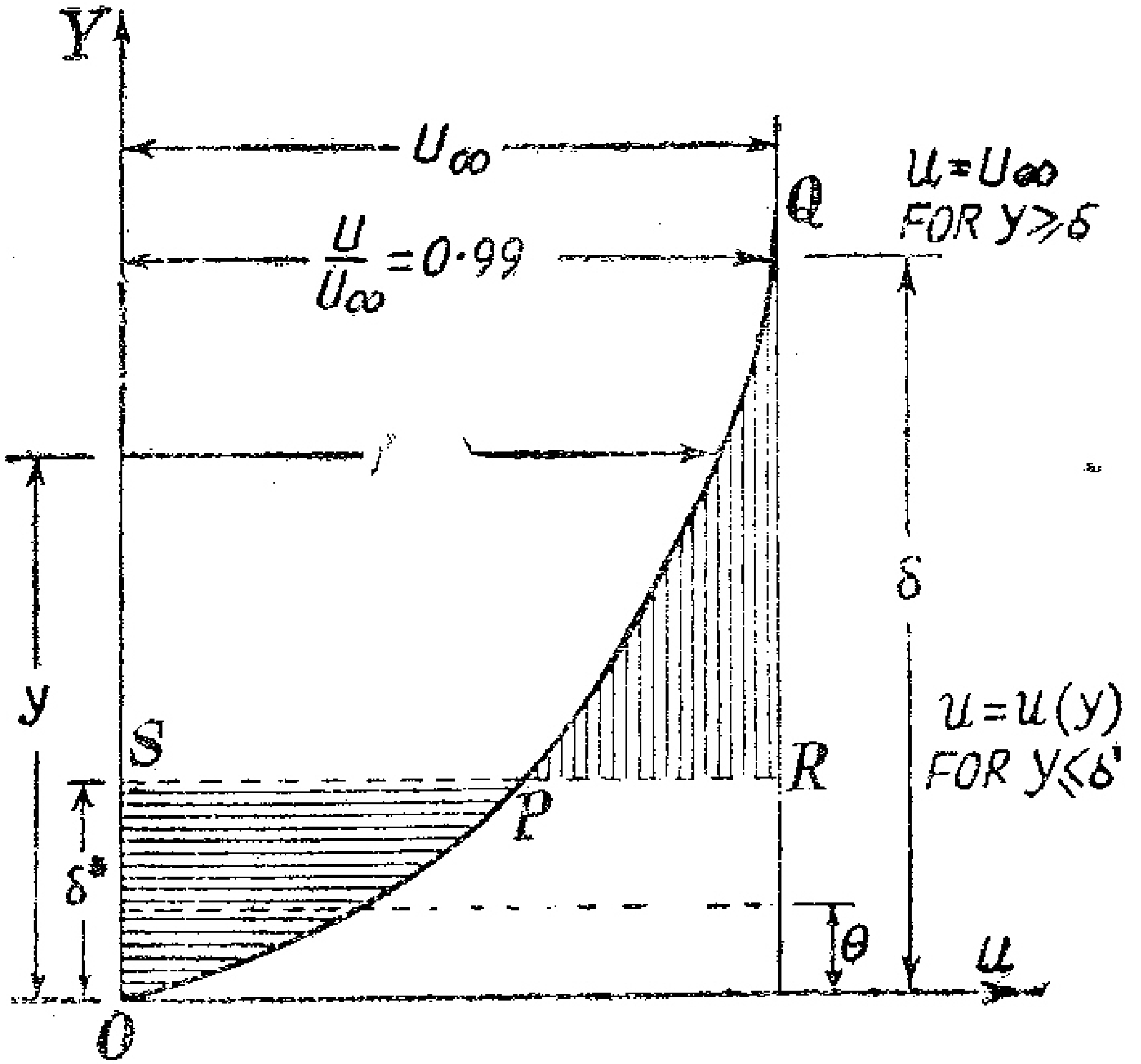
Flat Plate: Parallel to Flow

$\delta = \; y_{(u=0.99U_s)}$



Why is shear maximum at the leading edge of the plate? $\frac{du}{dy}$ is maximum

Graphical Representation




AERODYNAMICS
A380
A380 AIRBUS
Third Stage
Lecture:5, Ideal Fluid Flow
Lecturer: Dr.Fouad A.Kh.
WallpaperCLUB.Org

Contents

1. Ideal Fluids

An *ideal fluid* is one which is *incompressible*, and has *zero viscosity*.

Though no real fluid satisfies these criteria, there are situations in which viscosity of gases and liquids, and compressibility effects in gases, have little effect, and the theory of the ideal fluid can give an accurate prediction of the real flow.

For example, for a real fluid flowing past and around a stationary object, ideal theory works well *outside the boundary layer*.

2. Steady Two-Dimensional Flow

In this unit we shall consider only *steady, two-dimensional* flow, in which:

• the fluid velocity at any point remains constant with time;

• the direction and magnitude of the fluid velocity will in general vary in the x- and y-directions, but not in the z-direction.

3. Streamlines, Pathlines and Streaklines

A *streamline* is a line in the flow such that the velocity of each particle on the line is tangential to the line.

A *pathline* is the path traced out by one particle of the fluid.

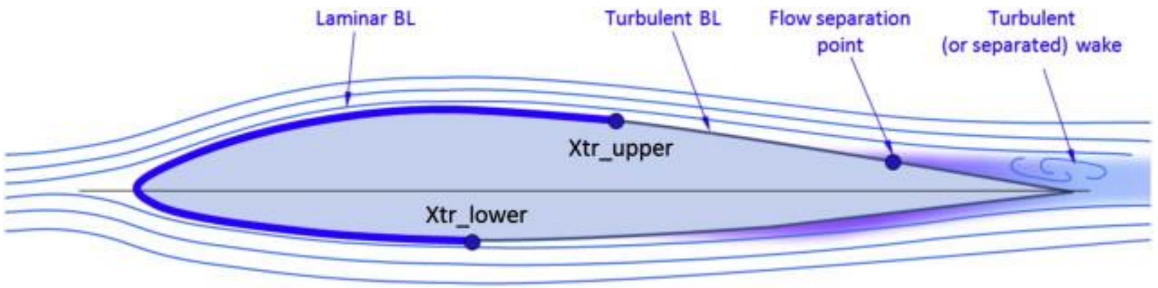
A *streakline* joins all the particles which have passed a particular fixed point in the flow.

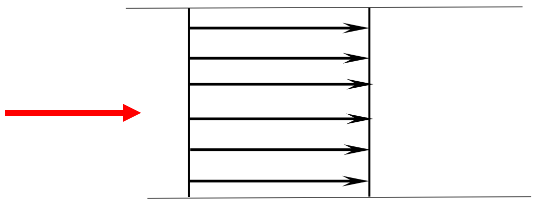
For steady, ideal flow, all the above are the same, and we need use only the term *streamline*.

Types of fluid Flow

1. Real and Ideal Flow:

If the fluid is considered frictionless with zero viscosity it is called ideal.
In real fluids the viscosity is considered and shear stresses occur causing conversion of mechanical energy into thermal energy

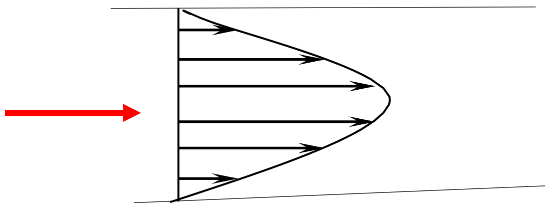




Ideal

Friction = 0

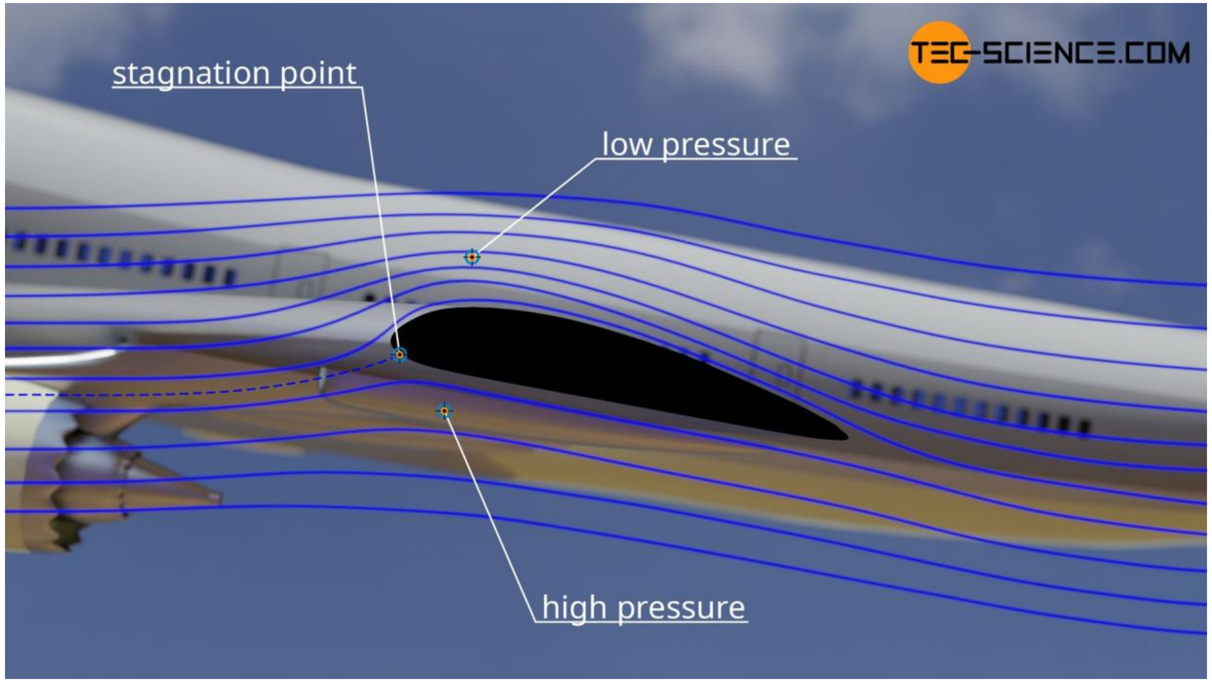
Ideal Flow ($\mu = 0$)
Energy loss =0



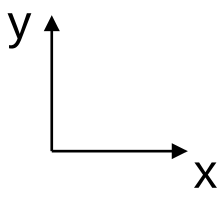
Real

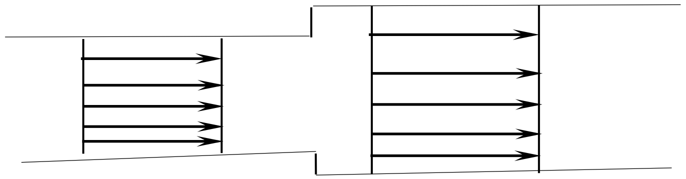
Friction ≠ o

Real Flow ($\mu \neq 0$)
Energy loss ≠ 0



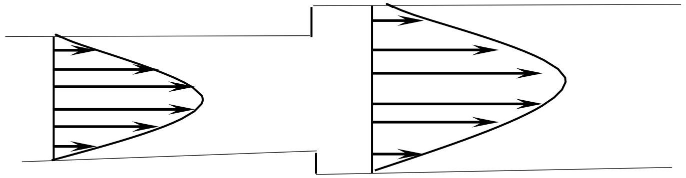
One , Two and three Dimensional Flow :(cont.)





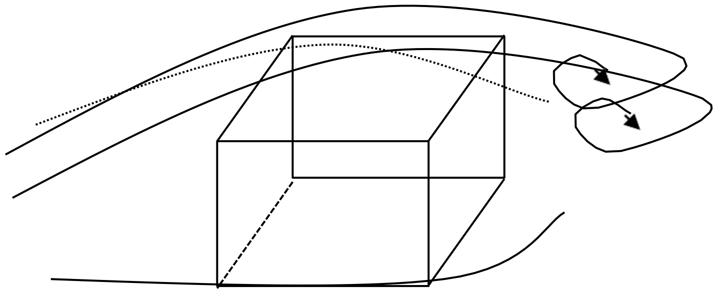
One dimensional flow means that the flow velocity is function of one coordinate

V = f(X or Y or Z)



Two dimensional flow means that the flow velocity is function of two coordinates

V = f(X,Y or X,Z or Y,Z)



Three dimensional flow means that the flow velocity is function of there coordinates

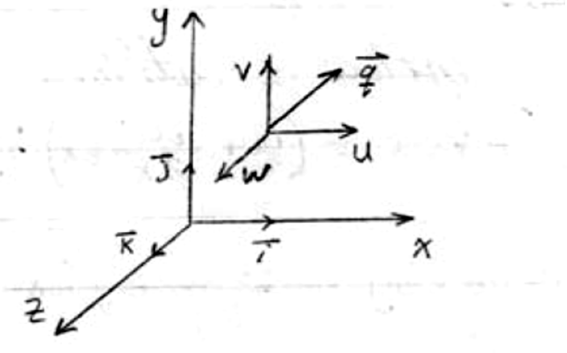
V = f(X,Y,Z)

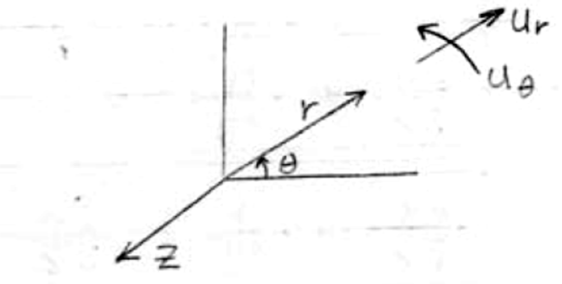
1- Introduction

Velocity vector

$\vec{q} = u\vec{i} + v\vec{j} + w\vec{k}$ In Cartesian coordinates

$\vec{q} = u_r\vec{r} + u_\theta\vec{\theta} + w\vec{k}$ In Polar coordinates





- The gradient operator ∇ is given by:

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

$$u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} = \mathbf{V}\cdot\nabla$$

Where q=V

Divergence of $\vec{q} = \nabla\cdot\vec{q}$

$$\nabla\cdot\vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Continuity equation

$$\nabla\cdot\vec{q} = 0$$

Or

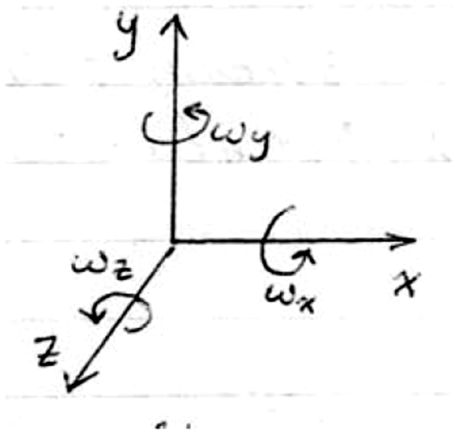
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Curl of $\vec{q} = \nabla \times \vec{q}$

Vorticity equation

$$\nabla \times \vec{q} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

$$\nabla \times \vec{q} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$



If $\nabla \times \vec{q} \neq 0$ the flow is called rotational

If $\nabla \times \vec{q} = 0$ the flow is called irrotational

2- Requirements for ideal- fluid flow

1- non viscous.

2- incompressible.

3- $\nabla \cdot \vec{q} = 0$

4- $\nabla \times \vec{q} = 0$

- In mathematics

∇ = gradient (del or nabla operator)

In the three-dimensional Cartesian coordinate system, the gradient is given by:

$$\nabla = \bar{i}\,\frac{\partial}{\partial x} + \bar{j}\,\frac{\partial}{\partial y} + \bar{k}\,\frac{\partial}{\partial z}$$

∇ . = divergence

$\nabla \cdot \bar{q}$ = divergence of $\bar{q}$ (div $\bar{q}$)

$$\nabla \cdot \bar{q} = \left[\bar{i}\,\frac{\partial}{\partial x} + \bar{j}\,\frac{\partial}{\partial y} + \bar{k}\,\frac{\partial}{\partial z}\right] \cdot \left[u\,\bar{i} + v\,\bar{j} + \mathrm{w}\,\bar{k}\right]$$

then

$$\nabla \cdot \bar{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \qquad \dots\dots\dots\dots\dots \quad (2)$$

= Continuity equation

∇ **x** = **curl**

∇ **x** $\bar{q}$ = **curl of** $\bar{q}$

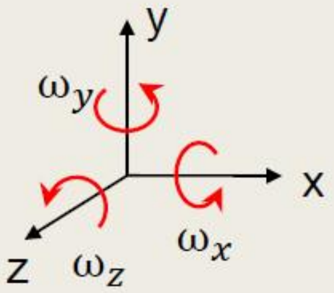
$$\nabla \times \bar{q} = \left[\bar{i}\,\frac{\partial}{\partial x} + \bar{j}\,\frac{\partial}{\partial y} + \bar{k}\,\frac{\partial}{\partial z}\right] \times \left[u\,\bar{i} + v\,\bar{j} + w\,\bar{k}\right]$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

then

$$\nabla \times \bar{q} = \bar{i}\,\underbrace{\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)}_{\omega_x} + \bar{j}\,\underbrace{\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)}_{\omega_y} + \bar{k}\,\underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}_{\omega_z} \ldots\ldots\ldots\ldots\ldots \quad (3)$$

= Vorticity equation



For two-dimensional flow:

$\omega_x = 0$, $\omega_y = 0$

If $\nabla \times \bar{q} \neq 0$ *at every point in a flow, the flow is called rotational.*

If $\nabla \times \bar{q} = 0$ *at every point in a flow, the flow is called irrotational.*

1.2 Requirements for ideal-fluid flow

1. Non Viscous (μ=0)

2. Incompressible (ρ=constant)

$$\frac{\partial \rho}{\partial t} = 0 \quad , \quad \frac{\partial \rho}{\partial x} = 0 \quad , \quad \frac{\partial \rho}{\partial y} = 0 \quad , \quad \frac{\partial \rho}{\partial z} = 0$$

3. The Continuity Equation:

$$\nabla . \overline{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

4. Irrotational Flow

$$\nabla \mathbf{x} \, \overline{q} = 0$$

The Acceleration Field of a Fluid

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \boldsymbol{v}\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u}{\partial t} + (\mathbf{V}\cdot\nabla)u$$

$$a_y = \frac{d\boldsymbol{v}}{dt} = \frac{\partial \boldsymbol{v}}{\partial t} + u\frac{\partial \boldsymbol{v}}{\partial x} + \boldsymbol{v}\frac{\partial \boldsymbol{v}}{\partial y} + w\frac{\partial \boldsymbol{v}}{\partial z} = \frac{\partial \boldsymbol{v}}{\partial t} + (\mathbf{V}\cdot\nabla)\boldsymbol{v}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + \boldsymbol{v}\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} + (\mathbf{V}\cdot\nabla)w$$

- Summing these into a vector, we obtain the total

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \underbrace{\frac{\partial \mathbf{V}}{\partial t}}_{\text{Local}} + \underbrace{\left(u\frac{\partial \mathbf{V}}{\partial x} + \boldsymbol{v}\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right)}_{\text{Convective}} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\boldsymbol{\nabla})\mathbf{V}$$

The Acceleration Field of a Fluid

- The term δV/δt is called the **local acceleration**, which vanishes if the flow is steady-that is, independent of time.
- The three terms in parentheses are called the **convective acceleration**, which arises when the particle moves through regions of spatially varying velocity, as in a nozzle or diffuser.
- The gradient operator ∇ is given by:

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

$$u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} = \mathbf{V} \cdot \nabla$$

Example 1. Acceleration field

Given the eulerian velocity vector field

$$\mathbf{V} = 3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}$$

find the total acceleration of a particle.

Solution

- *Assumptions:* Given three known unsteady velocity components, $u = 3t$, $v = xz$, and $w = ty^2$.

- *Solution step 1:* First work out the local acceleration $\partial\mathbf{V}/\partial t$:

$$\frac{\partial \mathbf{V}}{\partial t} = \mathbf{i}\frac{\partial u}{\partial t} + \mathbf{j}\frac{\partial v}{\partial t} + \mathbf{k}\frac{\partial w}{\partial t} = \mathbf{i}\frac{\partial}{\partial t}(3t) + \mathbf{j}\frac{\partial}{\partial t}(xz) + \mathbf{k}\frac{\partial}{\partial t}(ty^2) = 3\mathbf{i} + 0\mathbf{j} + y^2\,\mathbf{k}$$

Solution step 2: In a similar manner, the convective acceleration terms, are

Solution step 2: In a similar manner, the convective acceleration terms, are

$$u\frac{\partial \mathbf{V}}{\partial x} = (3t)\frac{\partial}{\partial x}(3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}) = (3t)(0\mathbf{i} + z\mathbf{j} + 0\mathbf{k}) = 3tz\,\mathbf{j}$$

$$v\frac{\partial \mathbf{V}}{\partial y} = (xz)\frac{\partial}{\partial y}(3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}) = (xz)(0\mathbf{i} + 0\mathbf{j} + 2ty\mathbf{k}) = 2txyz\,\mathbf{k}$$

$$w\frac{\partial \mathbf{V}}{\partial z} = (ty^2)\frac{\partial}{\partial z}(3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}) = (ty^2)(0\mathbf{i} + x\mathbf{j} + 0\mathbf{k}) = txy^2\,\mathbf{j}$$

- *Solution step 3:* Combine all four terms above into the single "total" or "substantial" derivative:

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} = (3\mathbf{i} + y^2\mathbf{k}) + 3tz\mathbf{j} + 2txyz\mathbf{k} + txy^2\mathbf{j}$$

$$= 3\mathbf{i} + (3tz + txy^2)\mathbf{j} + (y^2 + 2txyz)\mathbf{k} \qquad \textit{Ans.}$$

- *Comments:* Assuming that **V** is valid everywhere as given, this total acceleration vector $d\mathbf{V}/dt$ applies to all positions and times within the flow field.

Example 2. Acceleration field

- An idealized velocity field is given by the formula

$$\mathbf{V} = 4tx\mathbf{i} - 2t^2y\mathbf{j} + 4xz\mathbf{k}$$

- Is this flow field steady or unsteady? Is it two- or three dimensional? At the point $(x, y, z) = (1, 1, 0)$, compute the acceleration vector.

Solution

- The flow is unsteady because time t appears explicitly in the components.
- The flow is three-dimensional because all three velocity components are nonzero.
- Evaluate, by differentiation, the acceleration vector at (x, y, z) = (−1, +1, 0).

Example 2. Acceleration field

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 4x + 4tx(4t) - 2t^2y(0) + 4xz(0) = 4x + 16t^2x$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -4ty + 4tx(0) - 2t^2y(-2t^2) + 4xz(0) = -4ty + 4t^4y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0 + 4tx(4z) - 2t^2y(0) + 4xz(4x) = 16txz + 16x^2z$$

or: $$\frac{d\mathbf{V}}{dt} = (4x + 16t^2x)\mathbf{i} + (-4ty + 4t^4y)\mathbf{j} + (16txz + 16x^2z)\mathbf{k}$$

at $(x, y, z) = (-1, +1, 0)$, we obtain $$\frac{d\mathbf{V}}{dt} = -4(1 + 4t^2)\mathbf{i} - 4t(1 - t^3)\mathbf{j} + 0\mathbf{k}$$

Exercise 1

- The velocity in a certain two-dimensional flow field is given by the equation

$$\mathbf{V} = 2xt\hat{\mathbf{i}} - 2yt\hat{\mathbf{j}}$$

where the velocity is in m/s when *x, y,* and *t* are in meter and seconds, respectively.

1. Determine expressions for the local and convective components of acceleration in the x and y directions.
2. What is the magnitude and direction of the velocity and the acceleration at the point $x = y = 2\ m$ at the time $t = 0$?

Example 3

- Consider the steady, two-dimensional velocity field given by

$$\vec{V} = (u, v) = (1.3 + 2.8x)\vec{i} + (1.5 - 2.8y)\vec{j}$$

- Verify that this flow field is incompressible.

Solution

- **Analysis.** The flow is two-dimensional, implying no z component of velocity and no variation of u or v with z.
- The components of velocity in the x and y directions respectively are

$$u = 1.3 + 2.8x \qquad v = 1.5 - 2.8y$$

- To check if the flow is incompressible, we see if the incompressible continuity equation is satisfied:

$$\underbrace{\frac{\partial u}{\partial x}}_{2.8} + \underbrace{\frac{\partial v}{\partial y}}_{-2.8} + \underbrace{\frac{\partial w}{\partial z}}_{0 \text{ since 2-D}} = 0 \qquad \text{or} \qquad 2.8 - 2.8 = 0$$

- So we see that the incompressible continuity equation is indeed satisfied. Hence the flow field is incompressible.

Example 4

- Consider the following steady, three-dimensional velocity field in Cartesian coordinates:

$$\vec{V} = (u, v, w) = (axy^2 - b)\vec{i} + cy^3\vec{j} + dxy\vec{k}$$

where *a, b, c, and d* are constants. Under what conditions is this flow field incompressible?

Solution

Condition for incompressibility:

$$\underbrace{\frac{\partial u}{\partial x}}_{ay^2} + \underbrace{\frac{\partial v}{\partial y}}_{3cy^2} + \underbrace{\frac{\partial w}{\partial z}}_{0} = 0 \qquad ay^2 + 3cy^2 = 0$$

- Thus to guarantee incompressibility, constants *a and c* must satisfy the following relationship:

$$a = -3c$$

Example 5

- An idealized incompressible flow has the proposed three-dimensional velocity distribution

$$\mathbf{V} = 4xy^2\mathbf{i} + f(y)\mathbf{j} - zy^2\mathbf{k}$$

- Find the appropriate form of the function *f(y)* which satisfies the continuity relation.
- **Solution:** Simply substitute the given velocity components into the incompressible continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\frac{\partial}{\partial x}(4xy^2)+\frac{\partial f}{\partial y}+\frac{\partial}{\partial z}(-zy^2)=4y^2+\frac{df}{dy}-y^2=0$$

or: $\frac{df}{dy}=-3y^2$. Integrate: $f(y)=\int(-3y^2)dy=\mathbf{-y^3+constant}$ *Ans.*

Example 6

- For a certain incompressible flow field it is suggested that the velocity components are given by the equations

$$u = 2xy \quad v = -x^2y \quad w = 0$$

Is this a physically possible flow field? Explain.

Any physically possible incompressible flow field must satisfy conservation of mass as expressed by the relationship

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

For the velocity distribution given,

$$\frac{\partial u}{\partial x} = 2y \qquad \frac{\partial v}{\partial y} = -x^2 \qquad \frac{\partial w}{\partial z} = 0$$

Substitution into Eq. (1) shows that

$$2y - x^2 + 0 \neq 0$$

Thus, this is not a physically possible flow field. <u>No.</u>

Example 7

- For a certain incompressible, two-dimensional flow field the velocity component in the *y* direction is given by the equation

$$v = x^2 + 2xy$$

- Determine the velocity in the x direction so that the continuity equation is satisfied.

Example 7 - solution

To satisfy the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Since

$$\frac{\partial v}{\partial y} = 2x$$

Then from Eq. (1)

$$\frac{\partial u}{\partial x} = -2x \qquad (2)$$

Equation (2) can be integrated with respect to x to obtain

$$\int du = -\int 2x\, dx + f(y)$$

or

$$u = \underline{\underline{-x^2 + f(y)}}$$

where $f(y)$ is an undetermined function of y.

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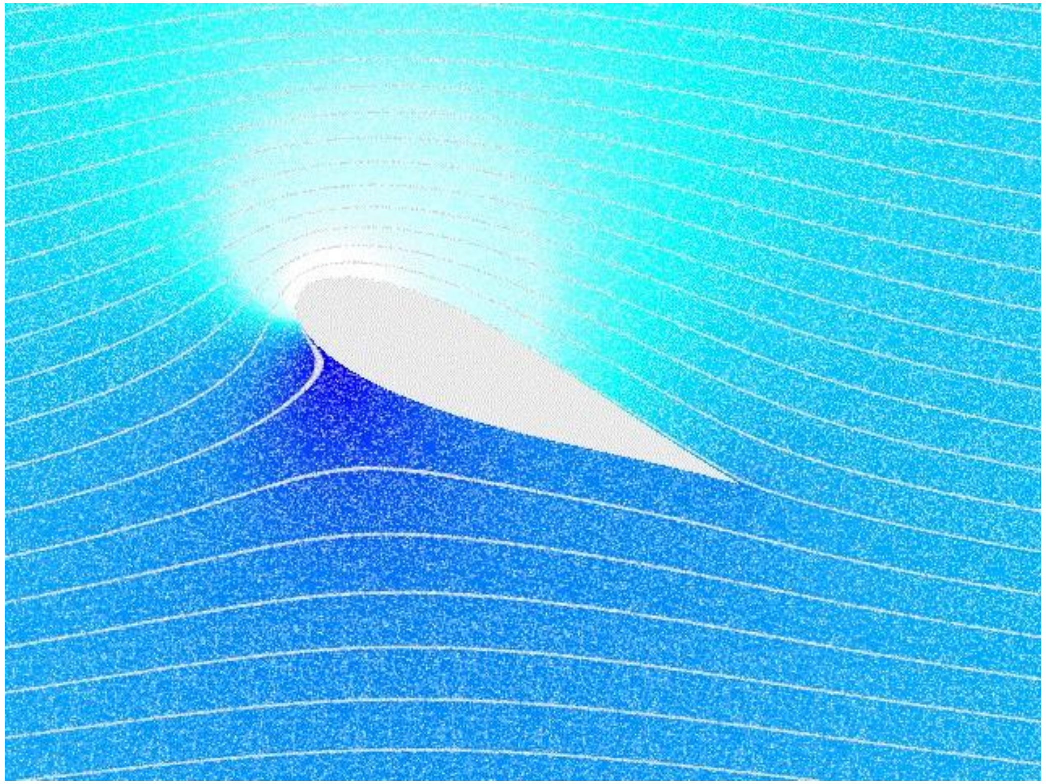

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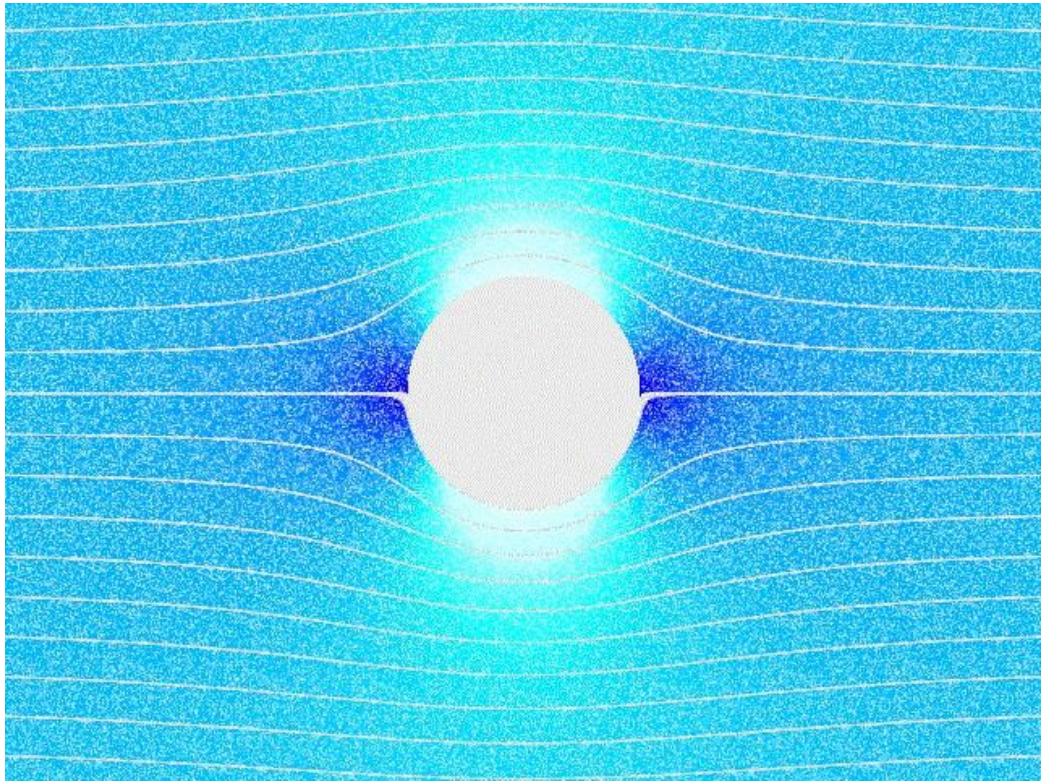

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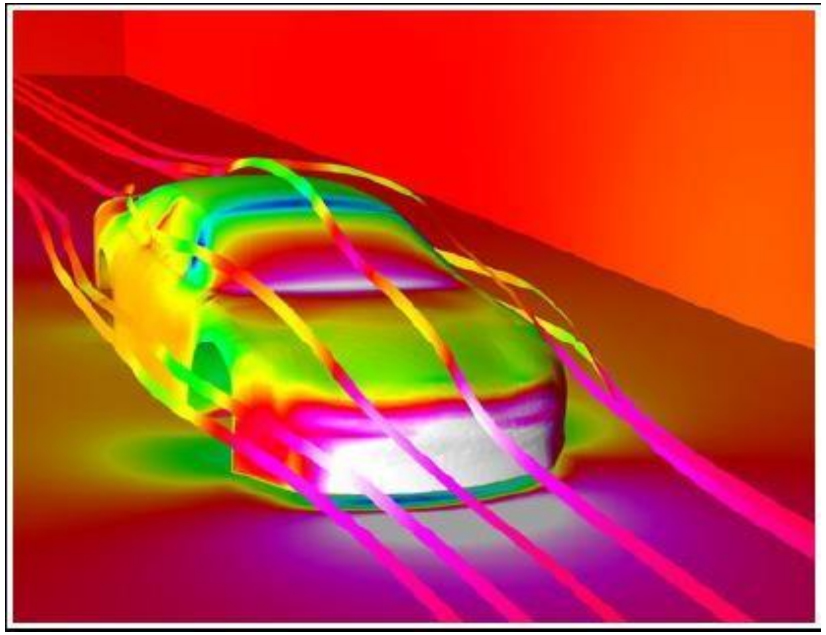

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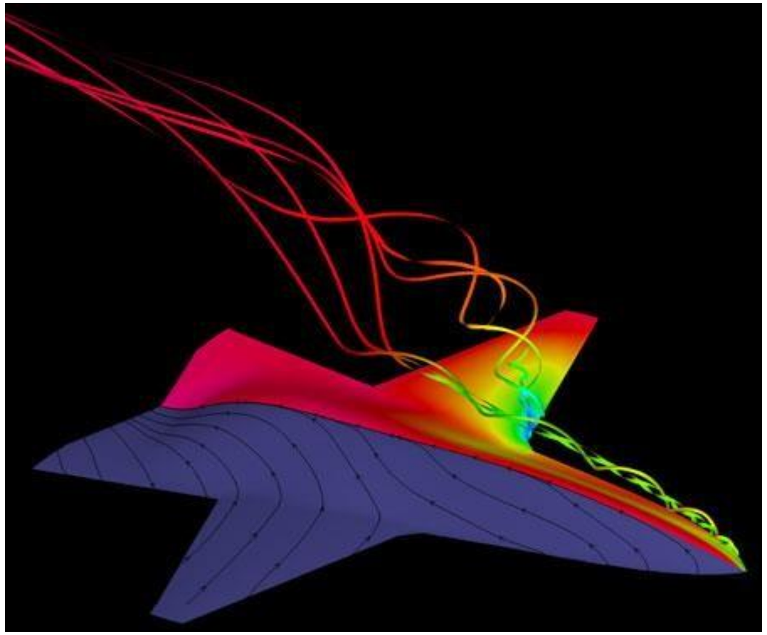

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✓A Streamline is a curve that is everywhere tangent to the *instantaneous* local velocity vector.

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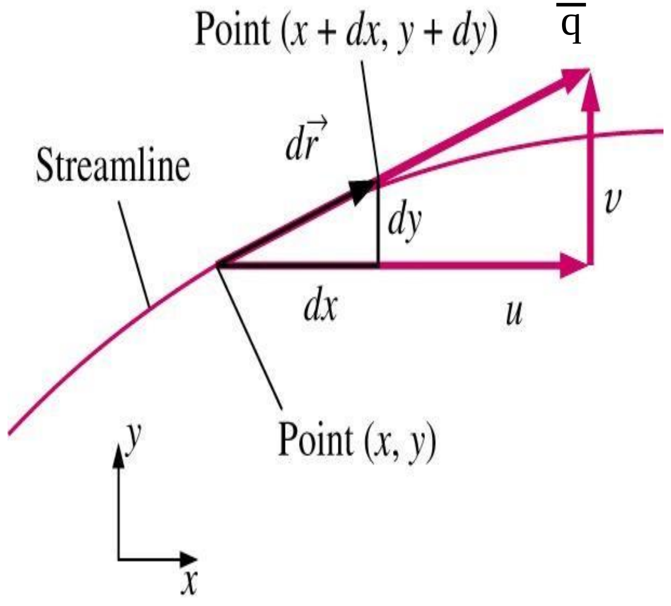

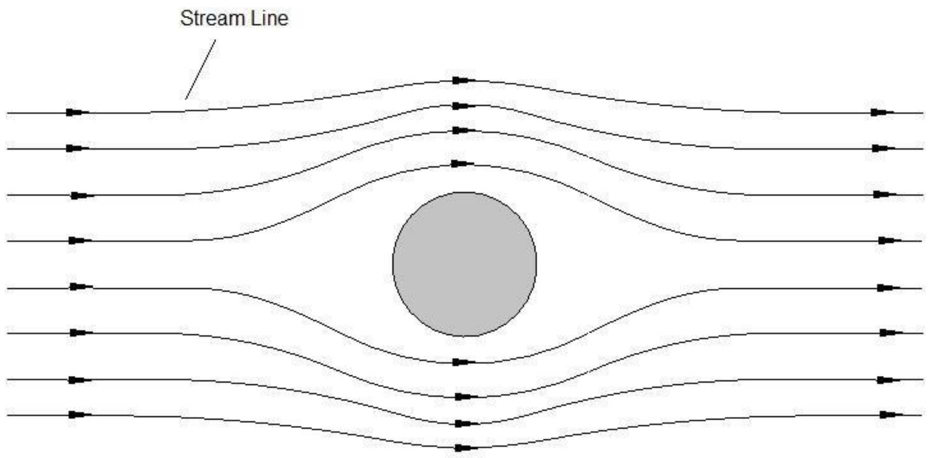

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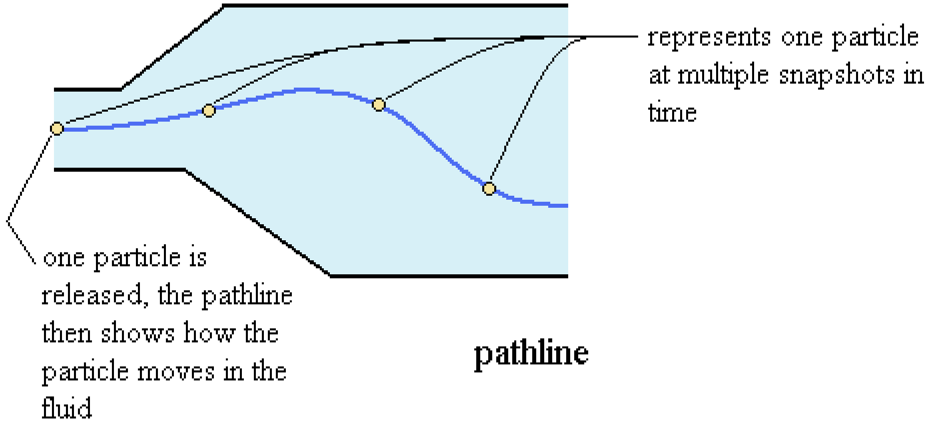

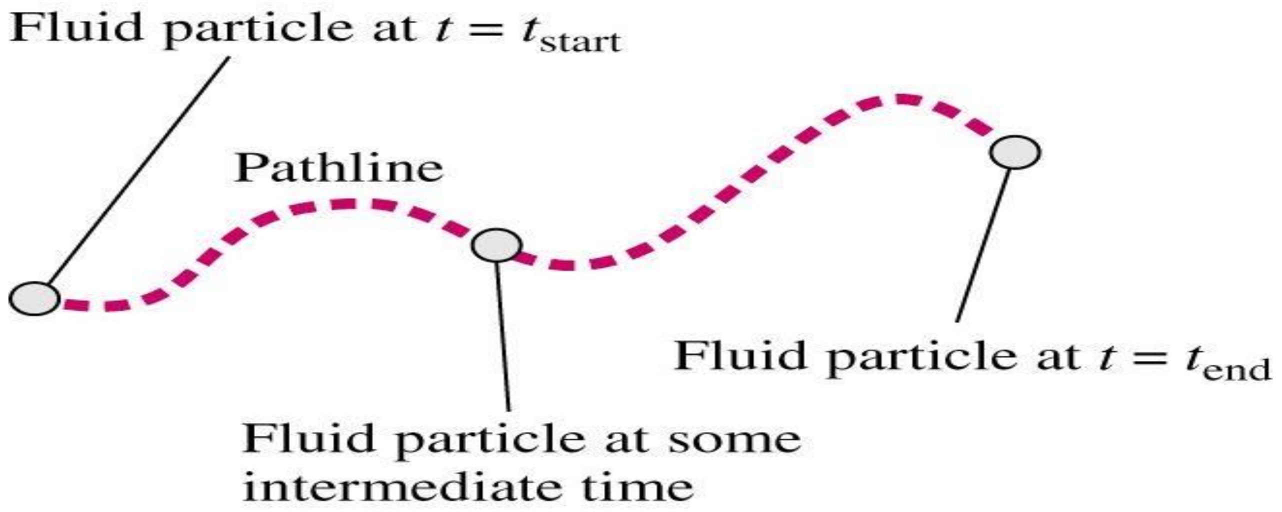

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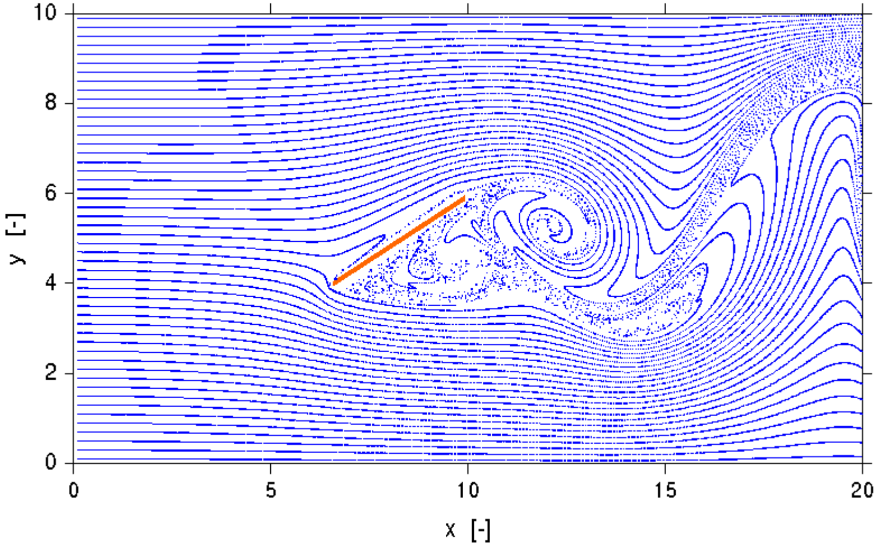

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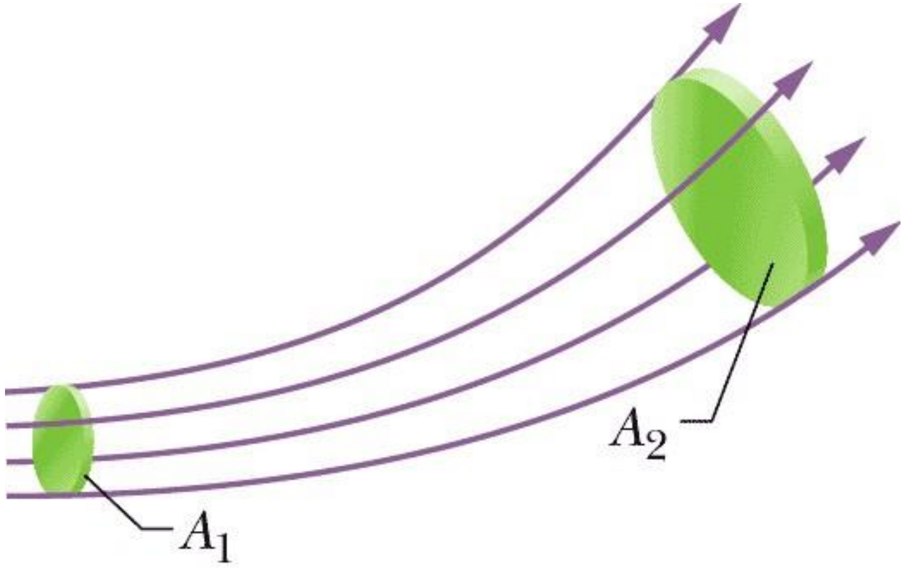

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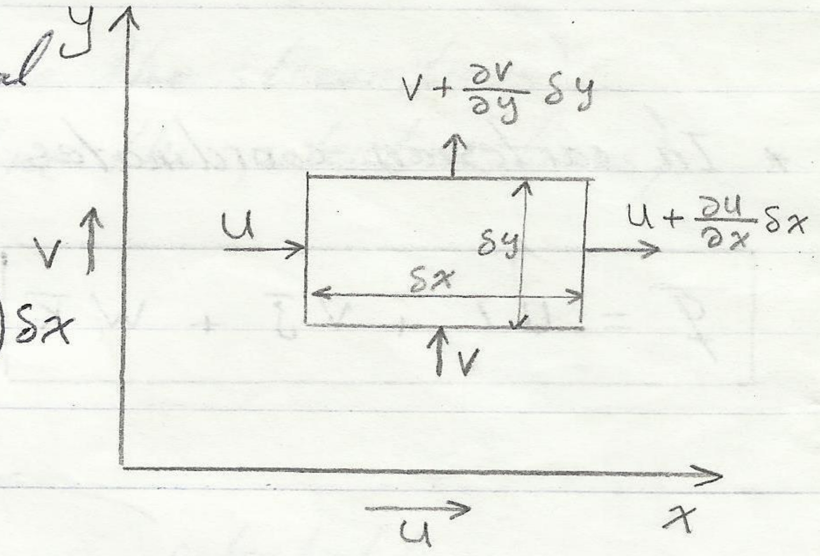

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divided by $\delta x \delta y$

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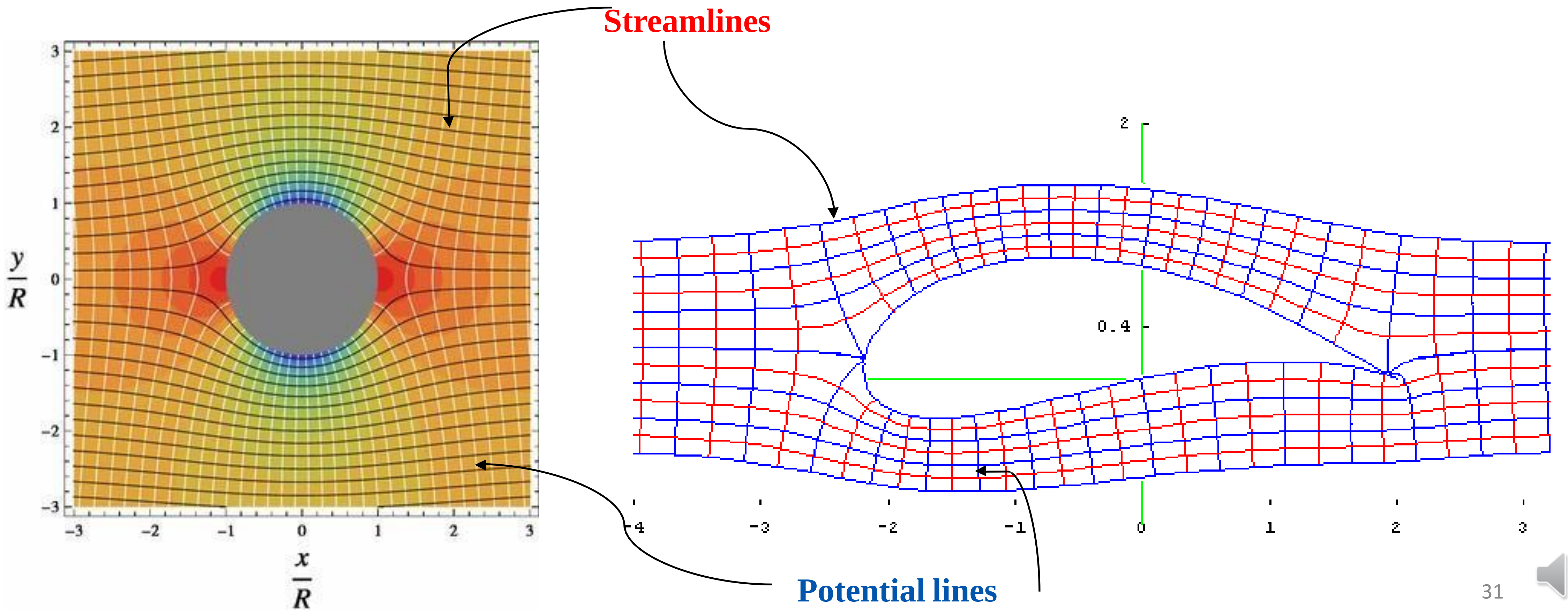

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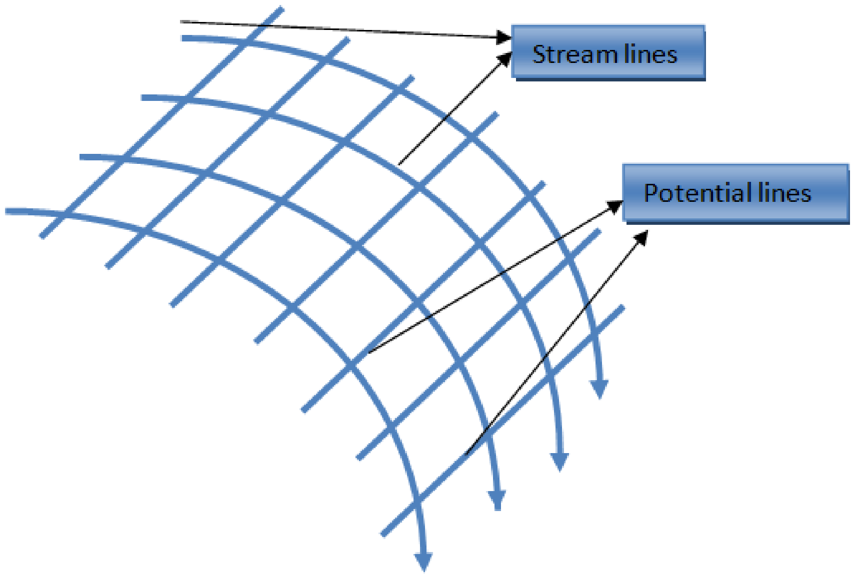

Flow Net

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- **Streamline in a fluid flow is an imaginary curve in which the tangent at any point represent the velocity vector at that point.**

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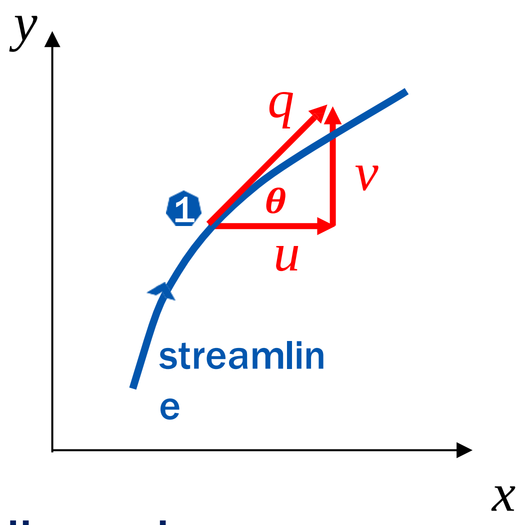

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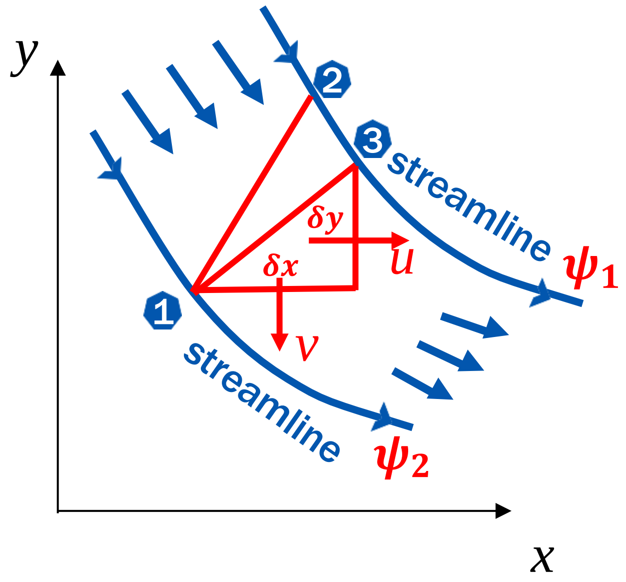

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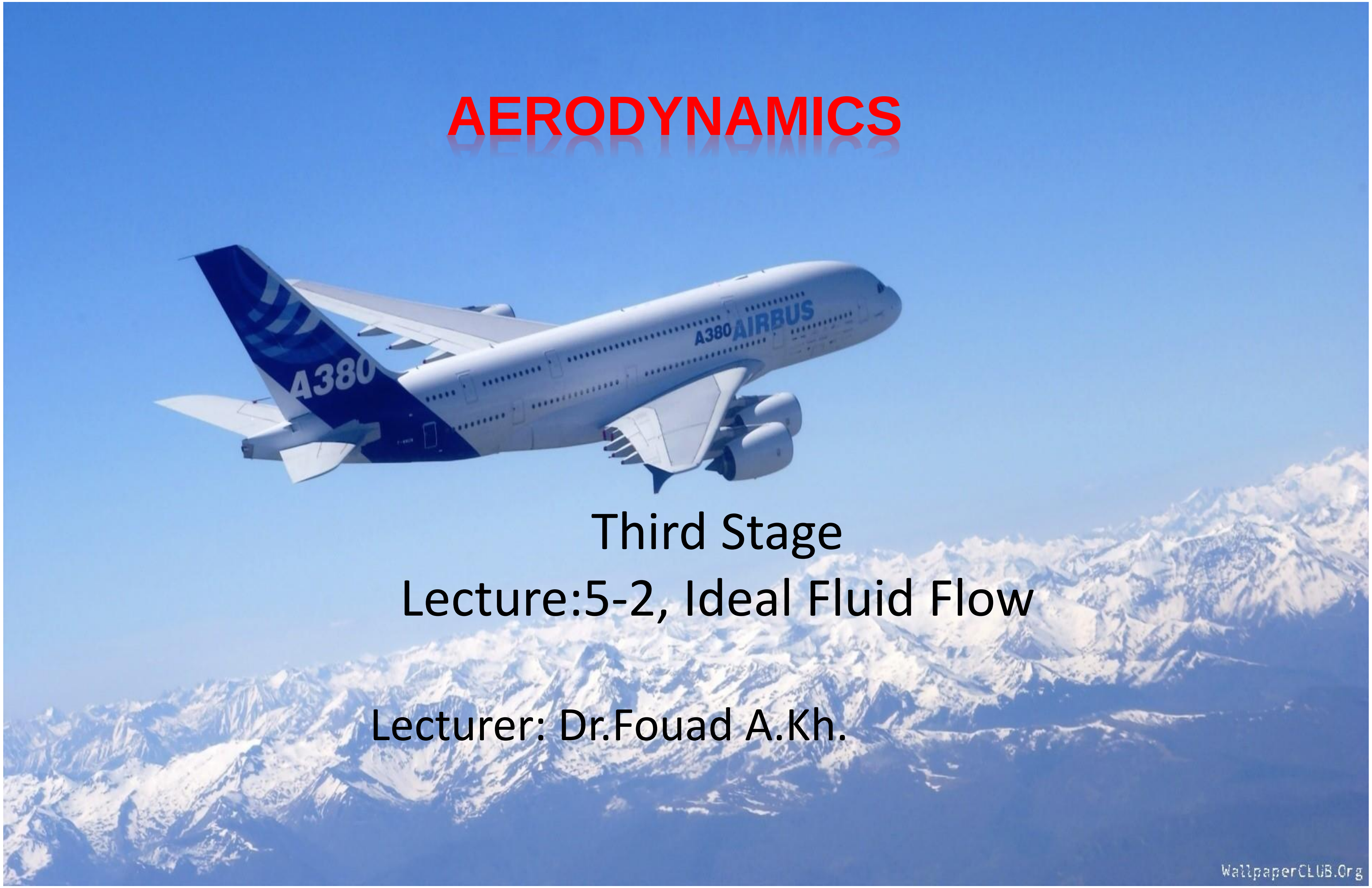
AERODYNAMICS



Third Stage

Lecture:5-2, Ideal Fluid Flow

Lecturer: Dr.Fouad A.Kh.

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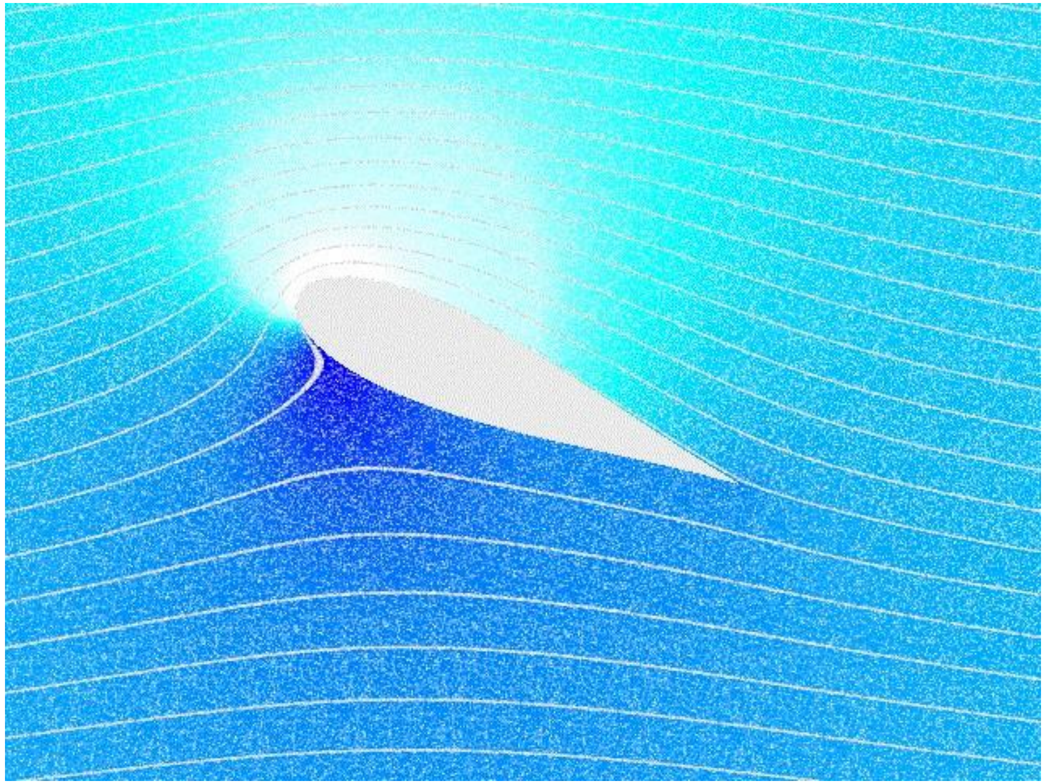

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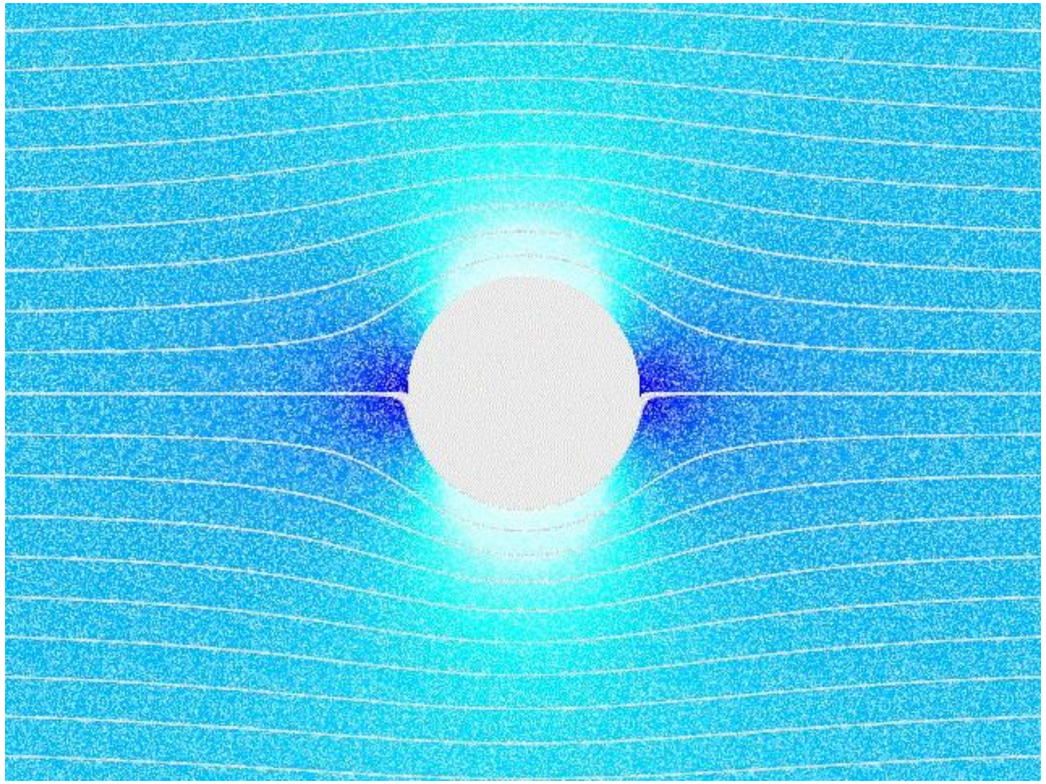

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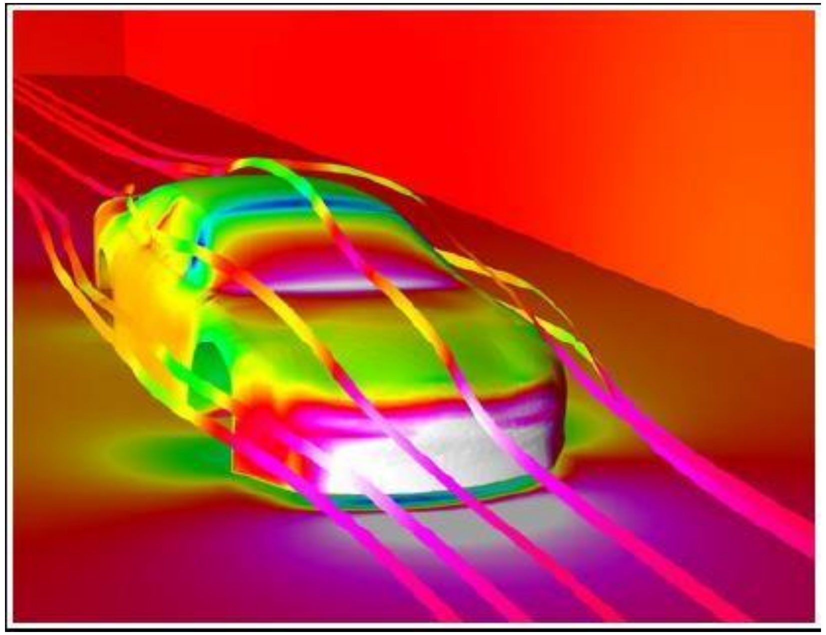

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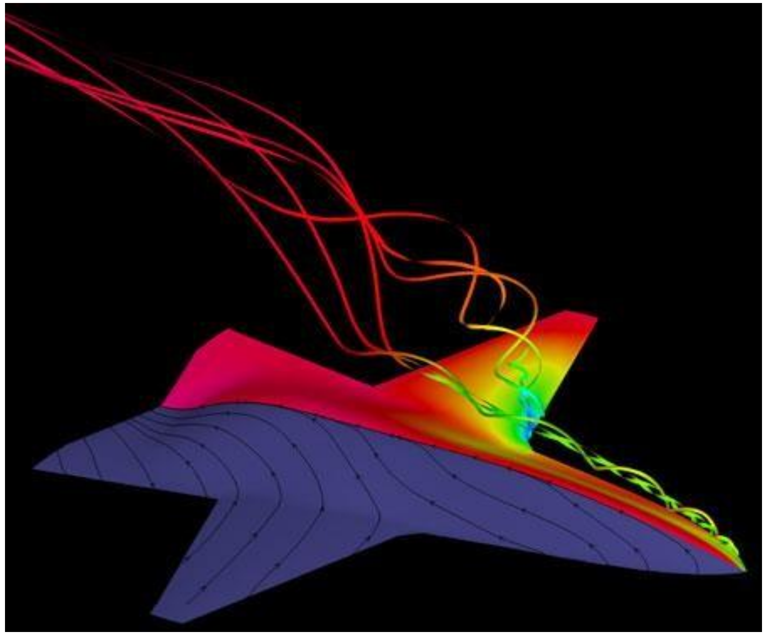

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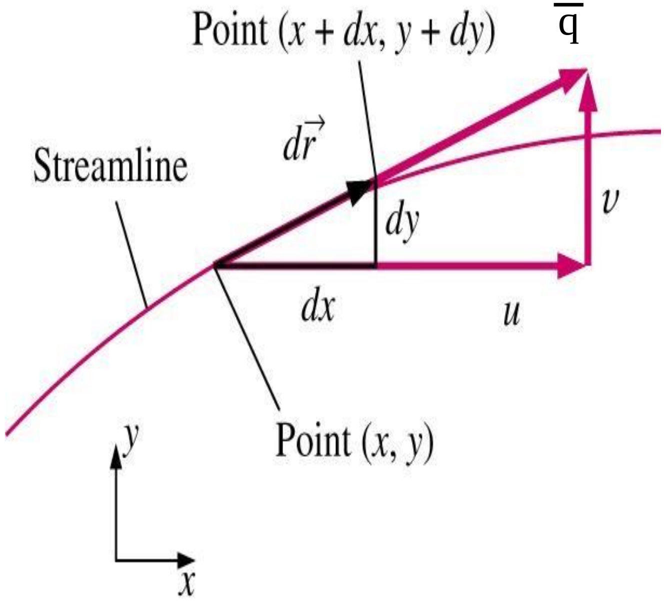

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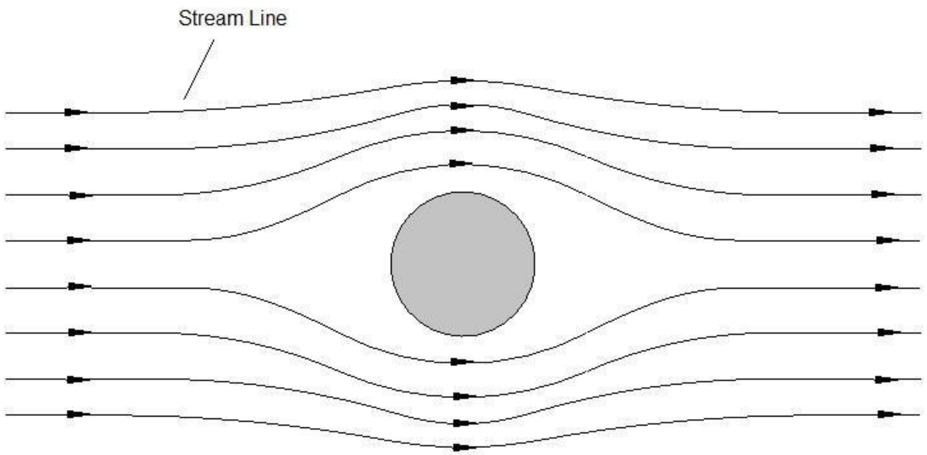

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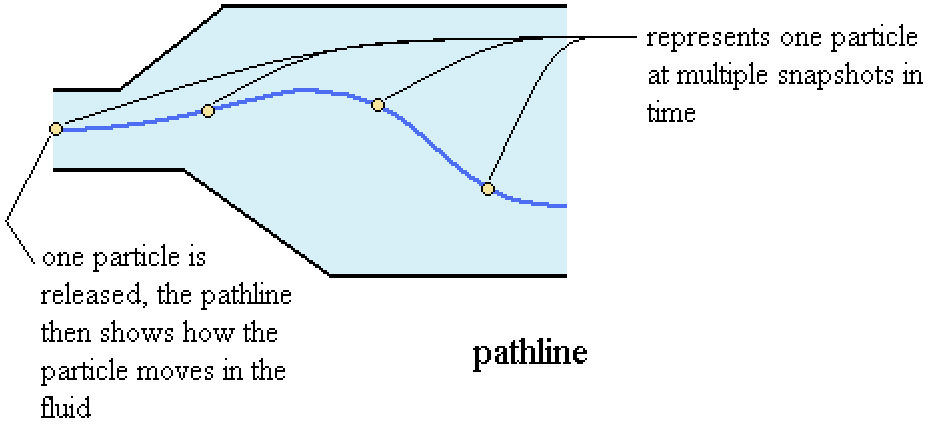

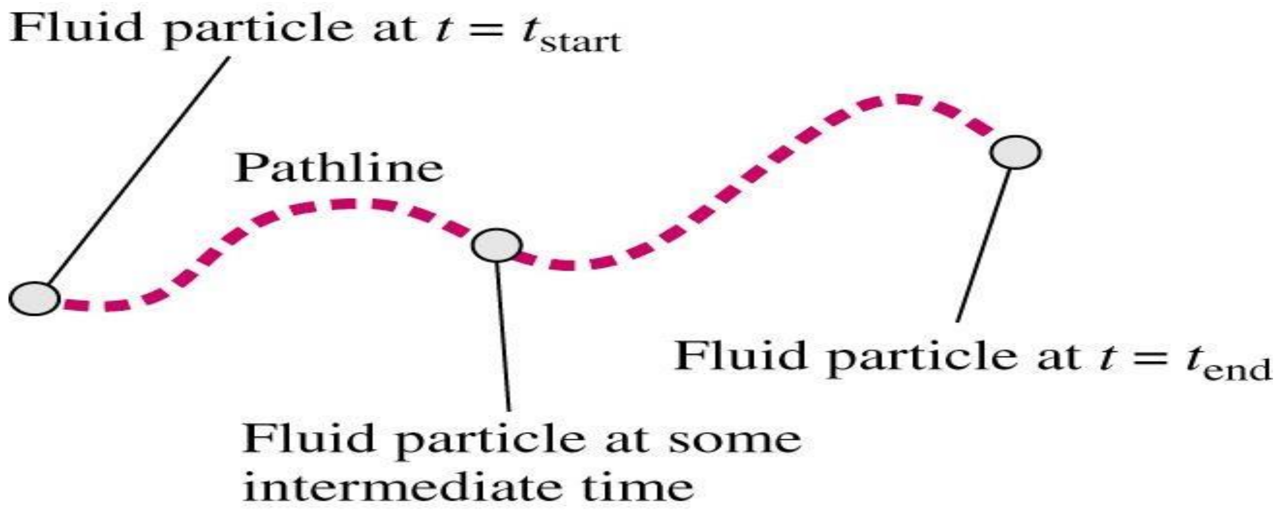

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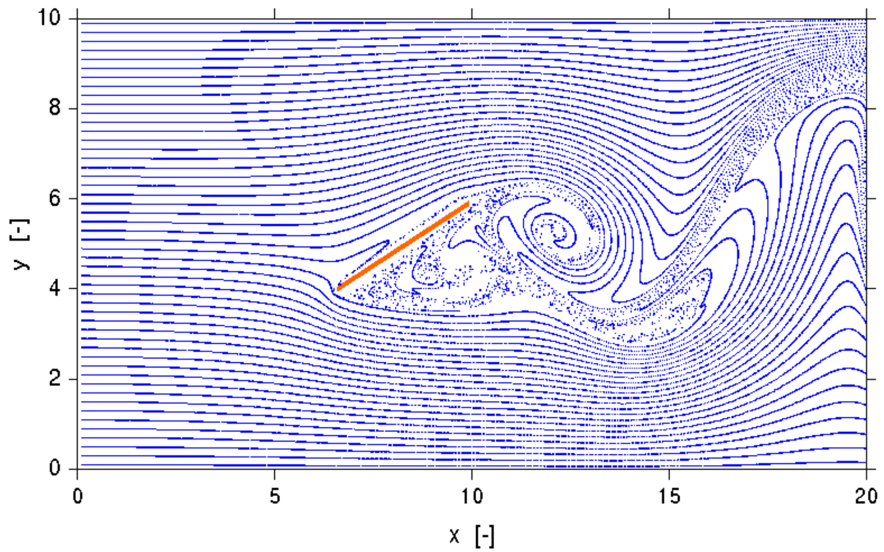

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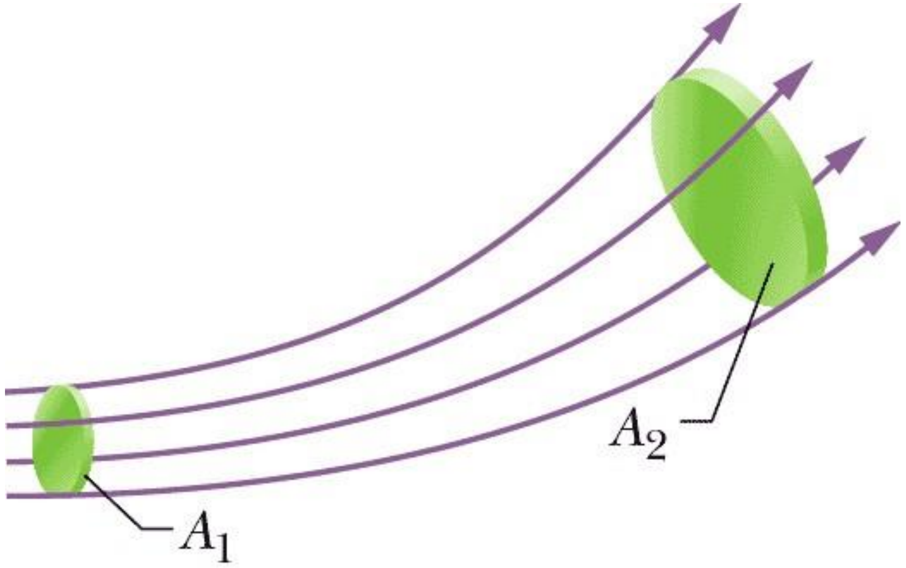

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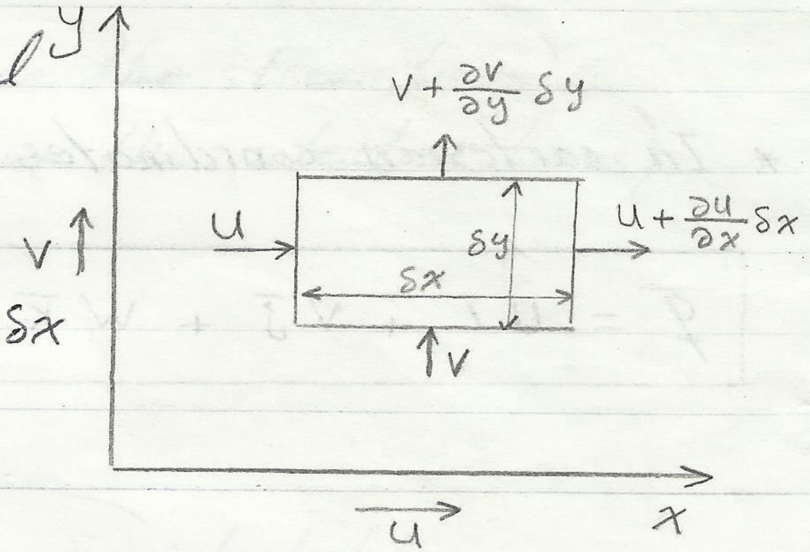

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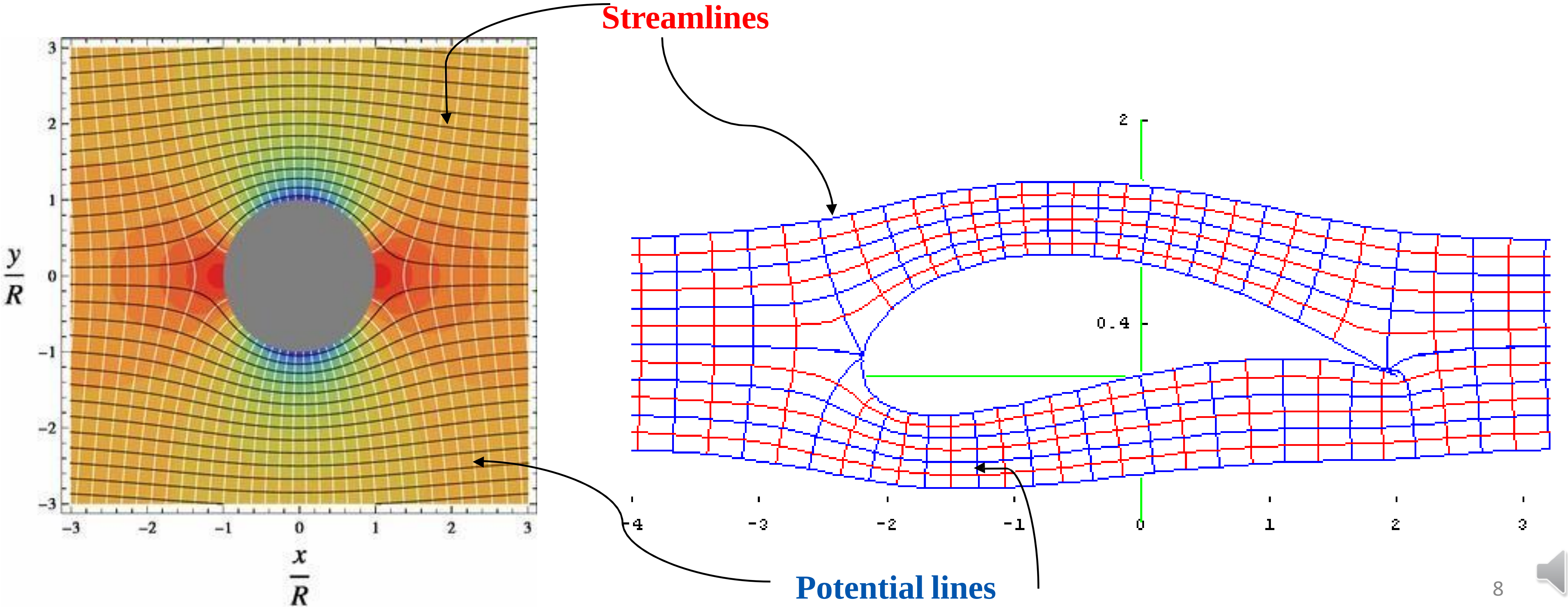

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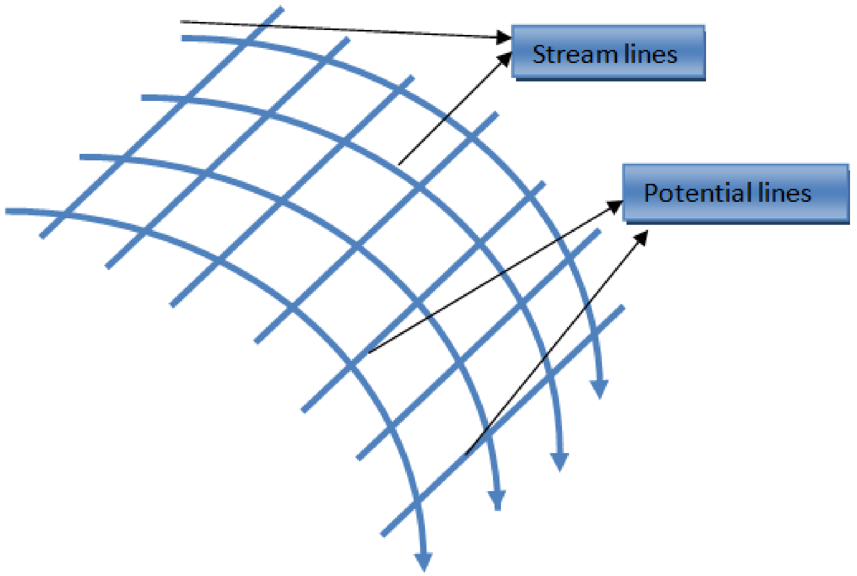

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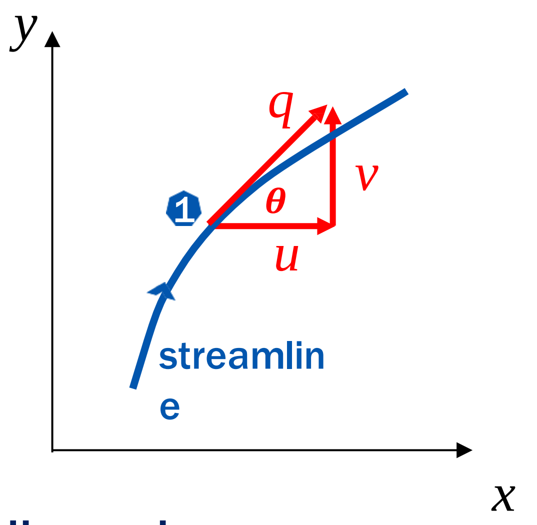

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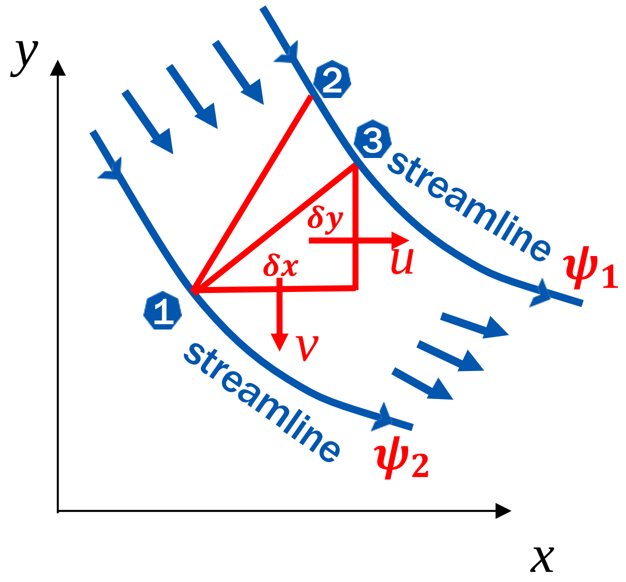

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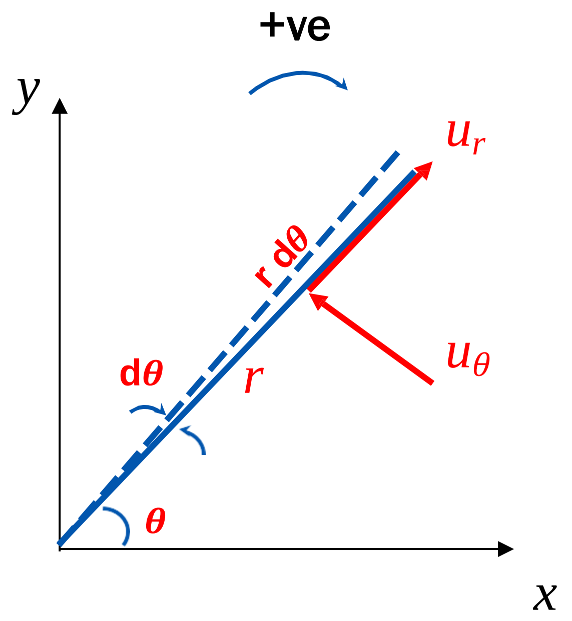
Comparing eq (i) & (ii) we note that:

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots.(4a)$$

In the cylindrical coordinates

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad , \quad u_\theta = -\frac{\partial \psi}{\partial r} \quad \ldots\ldots\ldots\ldots\ldots.(4b)$$



- $x = r \cos \theta$
- $y = r \sin \theta$

- **If stream function exists, it is a possible case of fluid flow which may be rotational or irrotational.**
- **If stream function satisfies the Laplace equation, it is a possible case of an irrotational flow.**

** Continuity equation in terms of stream function:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

$$\therefore \ \frac{\partial^2 \psi}{\partial x\, \partial y} - \frac{\partial^2 \psi}{\partial y\, \partial x} = 0 \qquad \ldots\ldots\ldots\ldots\ldots.(5)$$

<u>Continuity equation in terms of stream function</u>

معادلة الاستمرارية بدلالة دالة الانسياب

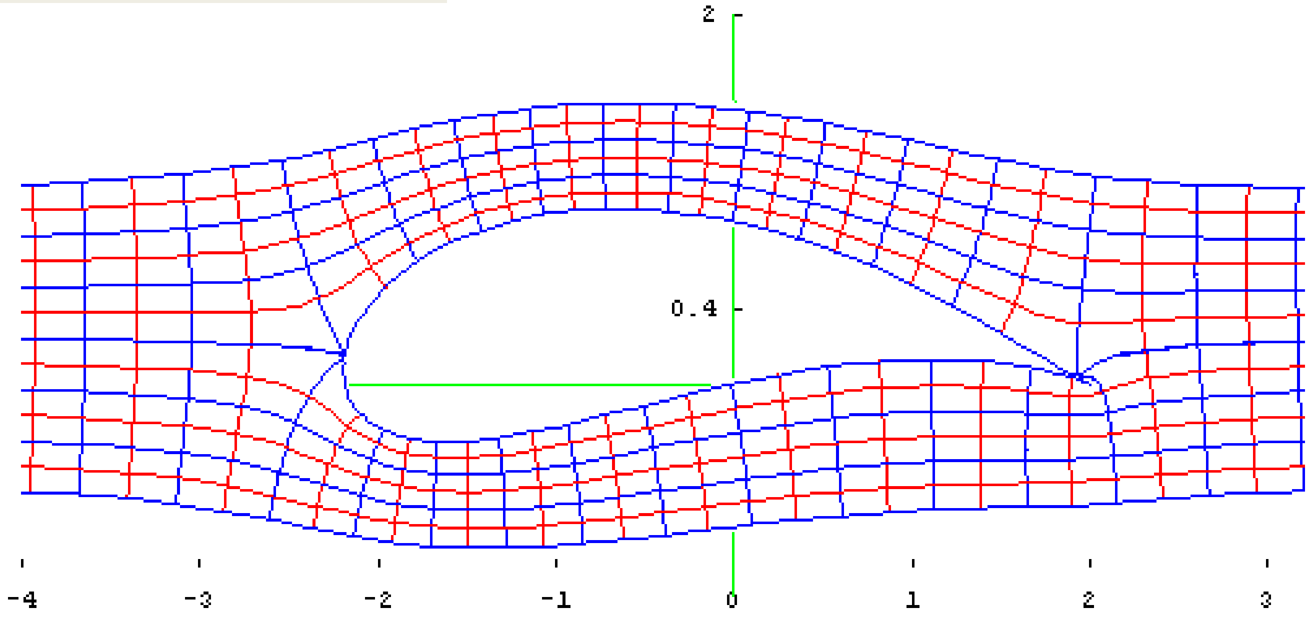
1.4 Potential Function (or Velocity Potential)

دالة الجهد

- For an irrotational flow:

$$\nabla \cdot \overline{q} = 0$$

- It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by $\emptyset(x, y)$ phi.



Velocity Potential Function

- Mathematically the velocity potential $\emptyset(x, y)$ may be defined as:

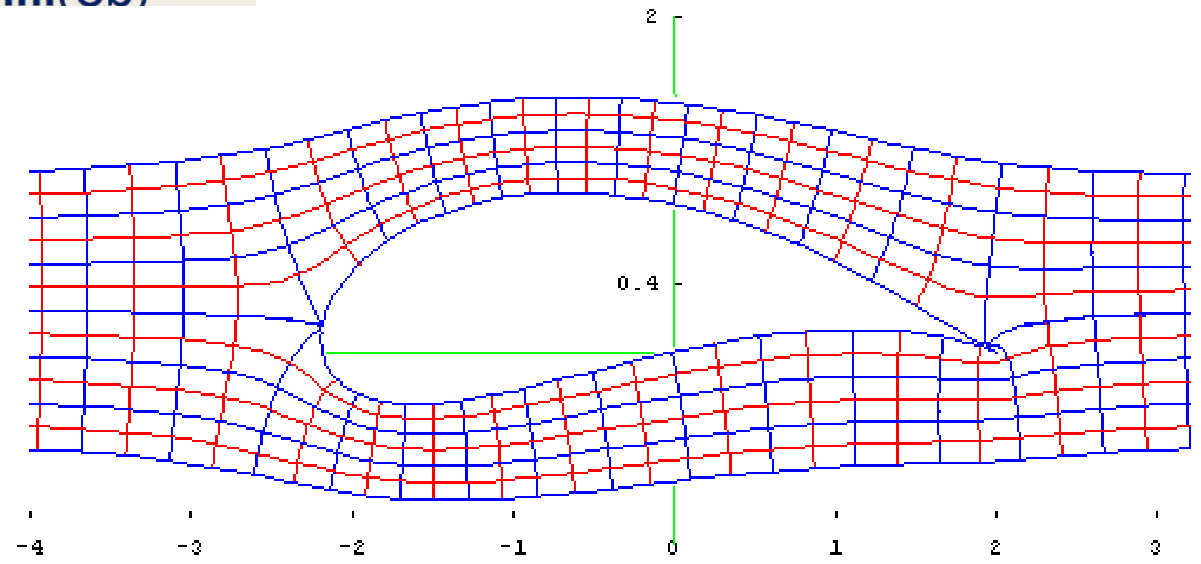
$$\overline{q} = \nabla\, \emptyset$$

- Thus

$$u = \frac{\partial \emptyset}{\partial\, x} \quad , \quad v = \frac{\partial \emptyset}{\partial\, y} \qquad \text{................(6a)}$$

- In the cylindrical coordinates:

$$u_r = \frac{\partial \emptyset}{\partial\, r} \quad , \quad u_\theta = \frac{1}{r}\,\frac{\partial \emptyset}{\partial\, \theta} \qquad \text{................(6b)}$$



- Continuity equation in terms of velocity potential:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \emptyset}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \emptyset}{\partial y}\right) = 0$$

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = 0 \qquad \text{........................(7)}$$

Or

$$\nabla^2 \emptyset = 0$$

- Equation (7) is known as Laplace equation معادلة لابلاس

- Any function $\emptyset$ that satisfies the Laplace equation is a possible irrotational fluid-flow case.

- Principle of superposition   دالة الجمع

i. If $\emptyset_1$ and $\emptyset_2$ are solutions of equation (7) then ($\emptyset_1$+ $\emptyset_2$) is also a solution.

ii. If $\emptyset_1$ is a solution of equation (7) then (C $\emptyset_1$) is also a solution, where C is constant

- Because irrotational flow can be described by the velocity potential $\emptyset$, irrotational flow is called Potential flow

1.5.1 Uniform Flow: (Rectilinear Flow) - Uniform flow in the x-direction

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \emptyset}{\partial x} \qquad \text{------------- } (i)$$

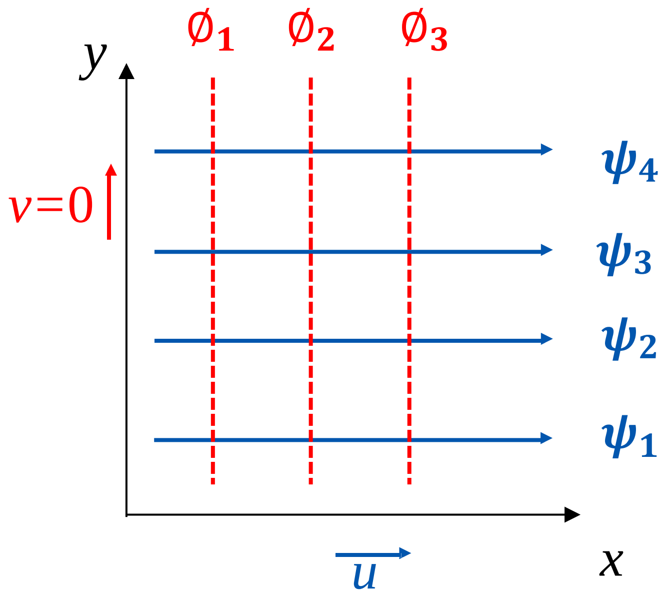
$$v = 0 \qquad \text{------------- } (ii)$$

from (i)

$$\psi = \int u\, dy = u\, y \quad \text{or} \quad \psi = u\, y$$

also from (i)

$$\emptyset = \int u\, dx = u\, x \quad \text{or} \quad \emptyset = u\, x$$



- Uniform flow in the y-direction

$$u = 0 \qquad \text{-------------------} (i)$$

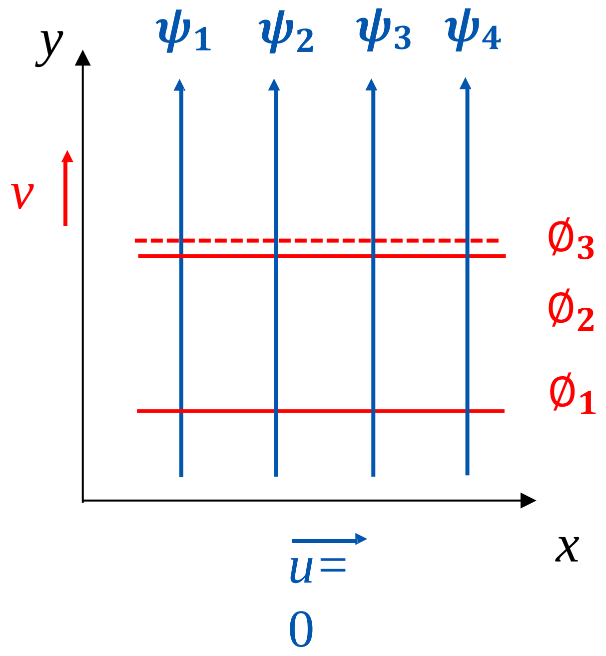
$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \emptyset}{\partial y} \text{-------------------} (ii)$$

from (ii)

$$\psi = -v\,x$$

and

$$\emptyset = v\,y$$

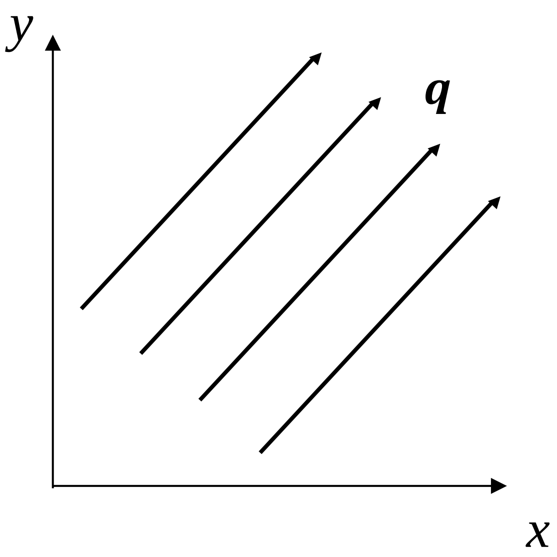


- General uniform flow

$$q = \sqrt{u^2 + v^2}$$

$$\psi = u\,y - v\,x$$

$$\emptyset = u\,x + v\,y$$



❖ Summary

❖ Relation between stream function and velocity potential function, we have:

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial\emptyset}{\partial x}$$

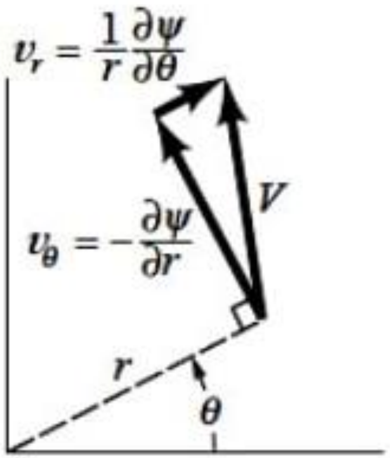
$$v = -\frac{\partial\psi}{\partial x} = \frac{\partial\emptyset}{\partial y}$$

❖ In the cylindrical coordinates:

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\emptyset}{\partial r}$$

$$u_\theta = -\frac{\partial\psi}{\partial x} = \frac{1}{r}\frac{\partial\emptyset}{\partial\theta}$$

..........................(8)



$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = -\frac{\partial\psi}{\partial r}$$

Show that the two-dimensional flow described by the equation $\psi = x + x^2 - y^2$ is irrotational. Find the velocity potential for this flow.

$$\psi = x + x^2 - y^2$$

$$u = \frac{\partial \psi}{\partial y} = -2y$$

$$v = -\frac{\partial \psi}{\partial x} = -1 - 2x$$

for two-dimensional flow

$$\omega_x = 0 \quad , \quad \omega_y = 0$$

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2 + 2 = 0$$

$\therefore$ flow is irrotational

$$\phi = \int u\,dx = \int -2y\,dx = -2yx + f(y)$$

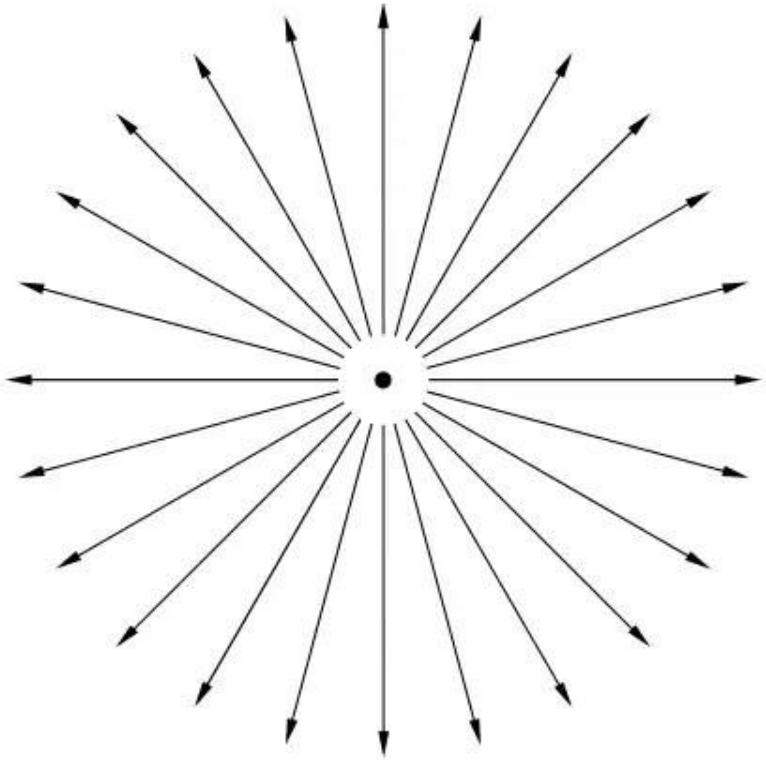
$$v = \frac{\partial \phi}{\partial y} \Rightarrow -1 - 2x = -2x + f'(y)$$

$$f'(y) = -1 \Rightarrow f(y) = -y + c$$

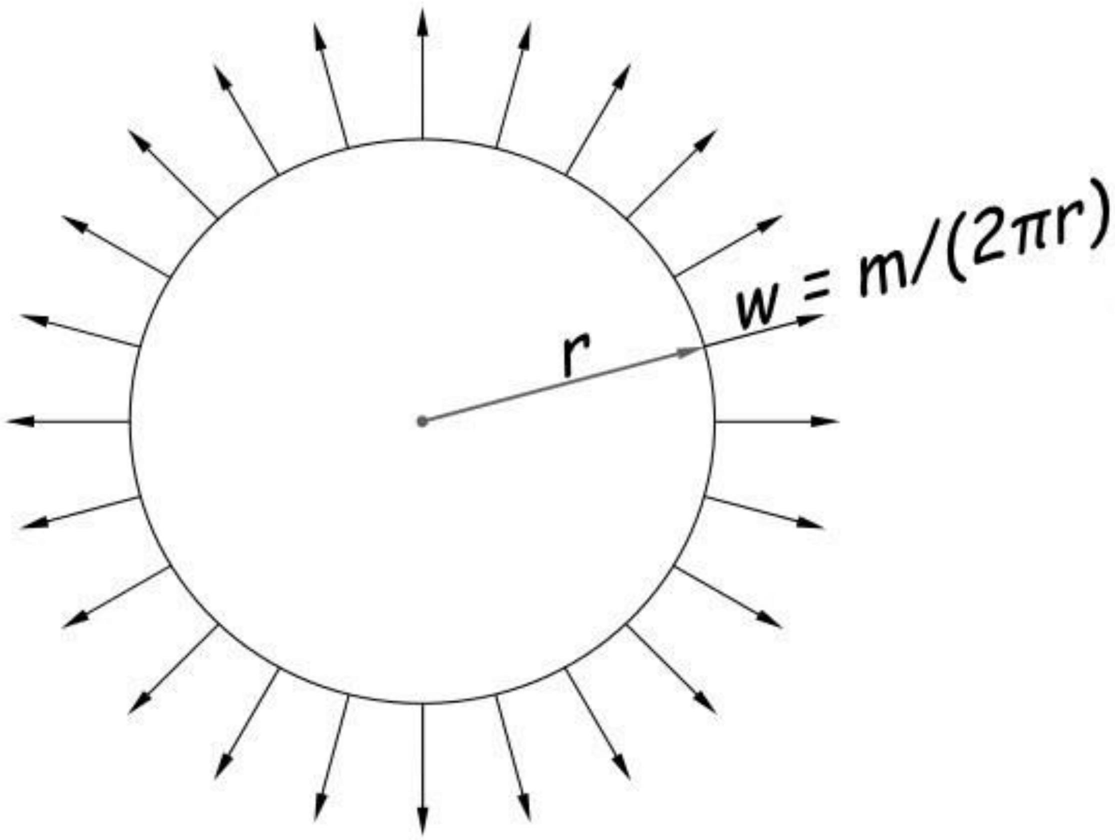
$$\phi = -y - 2yx + c$$

4.4 Line Source

Flow is radially outwards. The "line" is at right angles to the plane of flow, so is seen as a point in the diagram.



The *strength* of the source, *m*, is the discharge in m^3/s per m length of the line source: ie the units of *m* are m^2/s, the same as ψ.

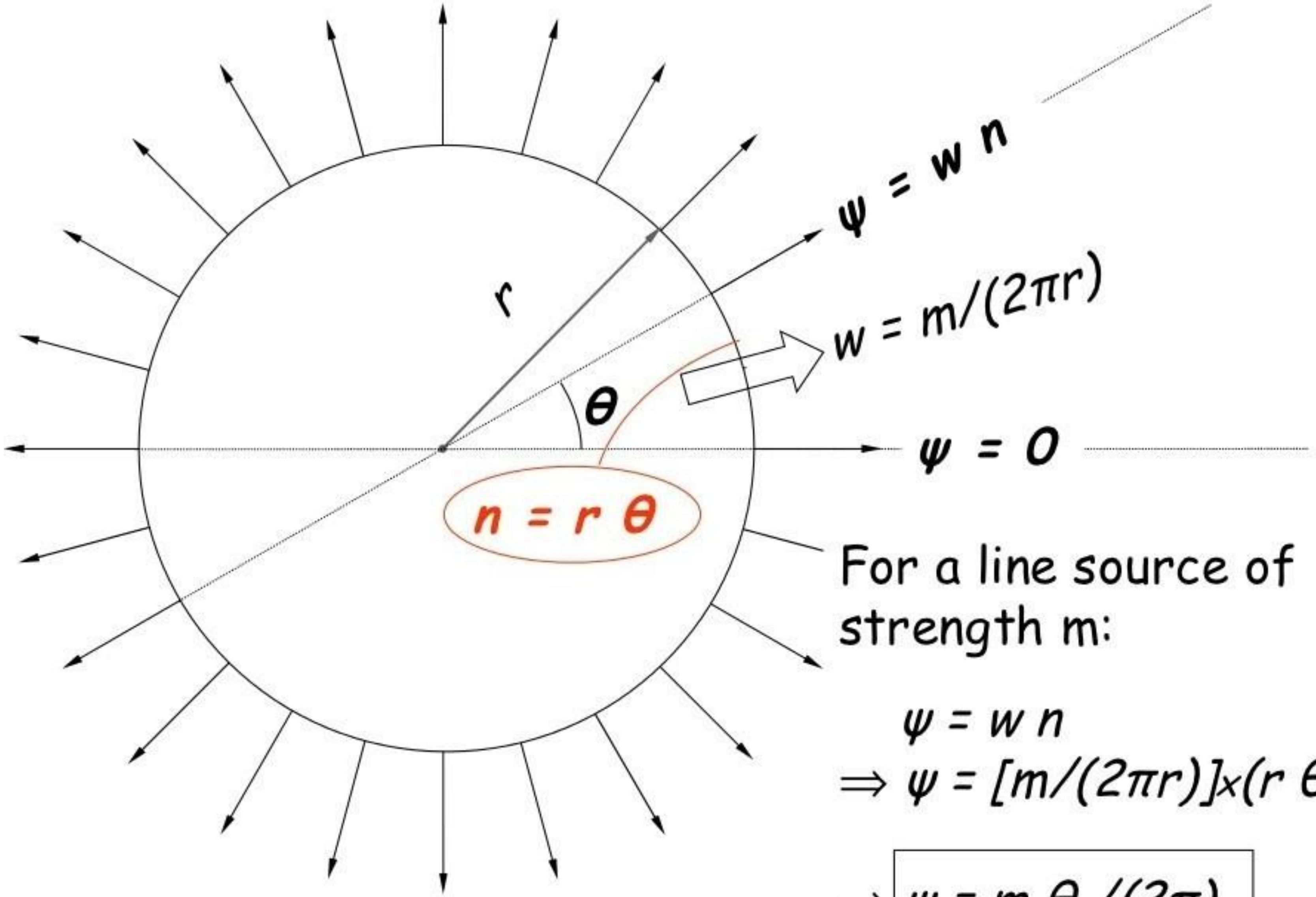


discharge per unit width = *m*

area per unit width = $2\pi r$

velocity = (discharge)/(area)

$$\Rightarrow \boxed{w = m/(2\pi r)}$$



For a line source of strength m:

$$\psi = w\,n$$

$$\Rightarrow \psi = [m/(2\pi r)] \times (r\,\theta)$$

$$\Rightarrow \boxed{\psi = m\,\theta/(2\pi)}$$

1.5.2 Source Flow المنبع

- Strength of the source, $K = \frac{Q}{2\,\pi}$ $\left[\frac{m^2}{s.m}\right]$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\,\theta} = \frac{\partial\emptyset}{\partial\,r} \qquad ---------- -(i)$$

$$u_\theta = 0 \qquad ---------- -(ii)$$

from (i)

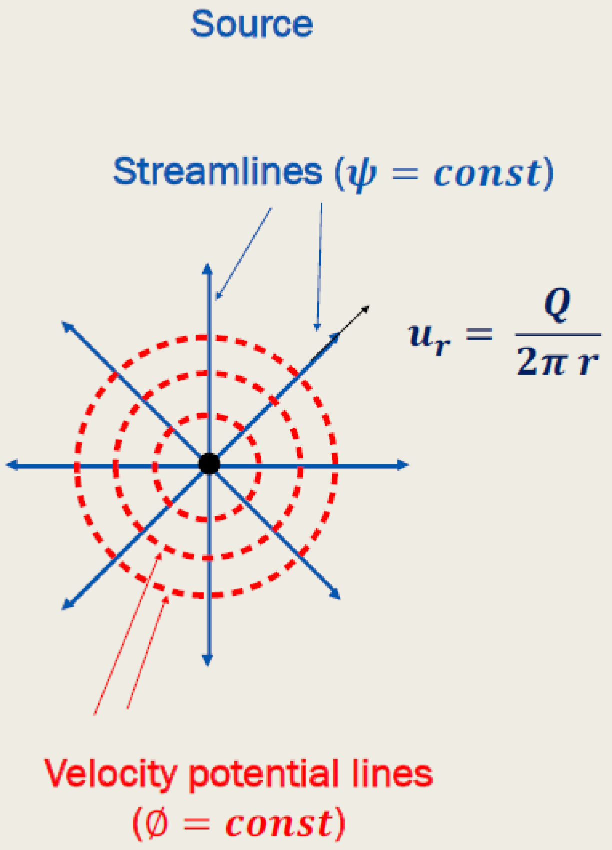
$$\psi = \int r\,u_r\,d\theta = \int r\frac{Q}{2\,\pi\,r}d\theta = \int K\,d\theta$$

$$\psi = K\,\theta$$

and

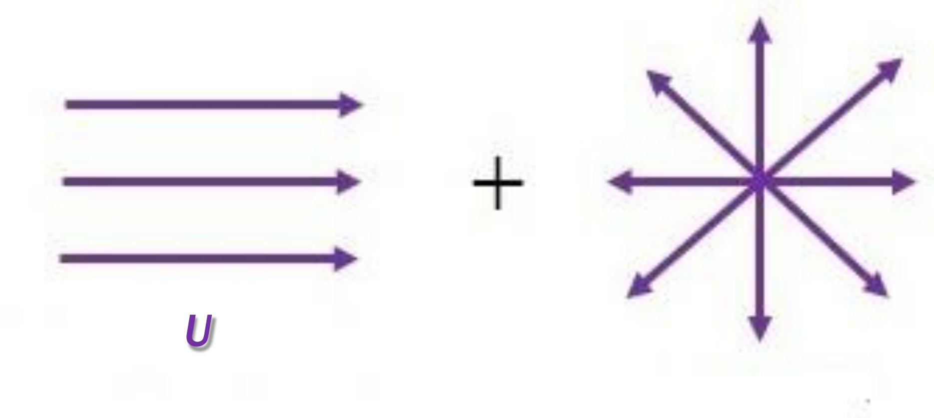
$$\emptyset = \int u_r\,dr = \int \frac{Q}{2\,\pi\,r}dr = \int K\frac{dr}{r}$$

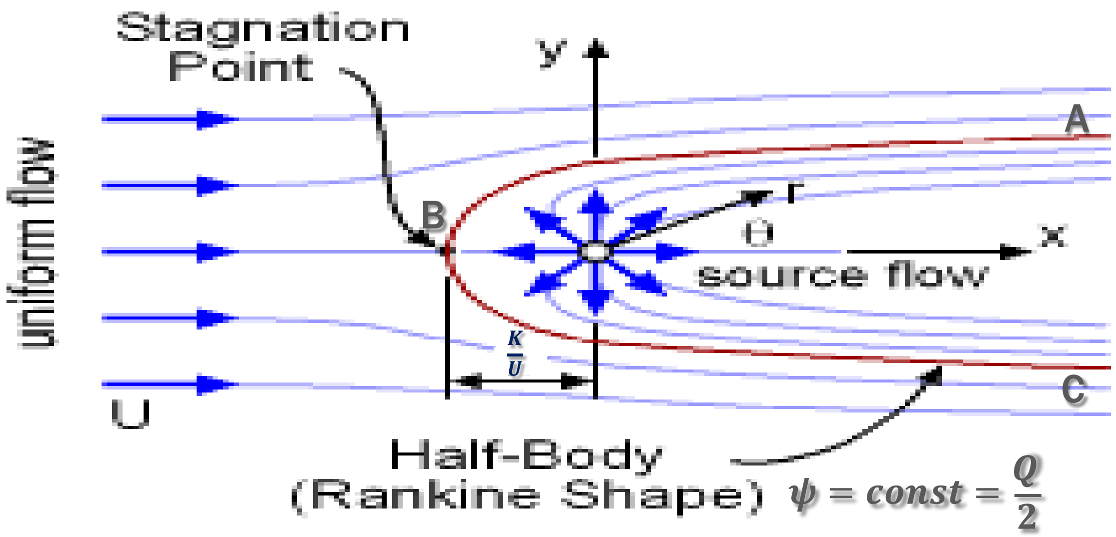
$$\emptyset = K\ln r$$



1.6 Combination of Basic Flows

1.6.1 Uniform Flow and Source





Uniform flow	Source	Resulting
$\psi = U\,y$	$\psi = K\,\theta$	$\psi = U\,y + K\,\theta$
$\emptyset = U\,x$	$\emptyset = K\ln r$	$\emptyset = U\,x + K\ln r$

The resulting ψ is:

$$\psi = U\,y + K\,\theta$$

$$\psi = U\,r\,\sin\theta + K\,\theta = constant$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U\cos\theta + \frac{K}{r}$$

$$\psi = U\, r\, \sin\theta + K\,\theta$$

and

$$u_\theta = -\frac{\partial \psi}{\partial r} = -U\sin\theta$$

Points B is the stagnation point and can be located by setting the equation for

u_r and u_θ equal to zero

$$U\cos\theta_B + \frac{K}{r_B} = 0 \qquad \dots\dots\dots\dots(i)$$

$$U\sin\theta_B = 0 \qquad \dots\dots\dots\dots(ii)$$

from (ii): $\quad \sin\theta_B = 0 \quad \Rightarrow \quad \theta_B = \pi$

sub in (i): $\quad U = \frac{K}{r_B} \quad \Rightarrow \quad r_B = \frac{K}{U}$

$$\therefore\ coordinates\ of\ B\ (r_B, \theta_B) = \left(\frac{K}{U}, \pi\right)$$

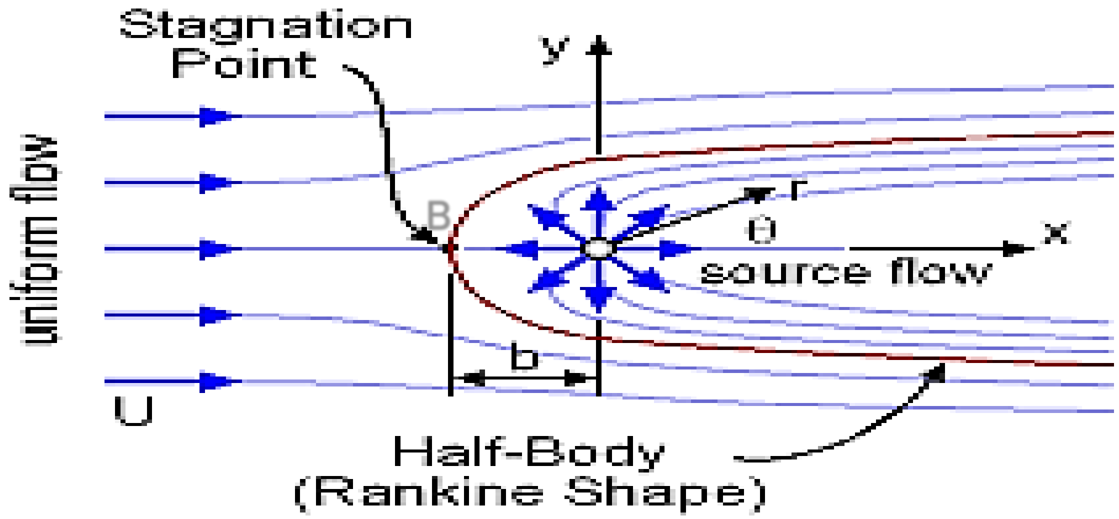
***sub the coordinates of B in the equ of ψ yields*:**

$$\psi = U\,r\,sin\theta + K\,\theta$$

$$\psi = U\,\frac{K}{U}\,sin\,\pi + K\,\pi = const$$

$$\psi = K\,\pi$$

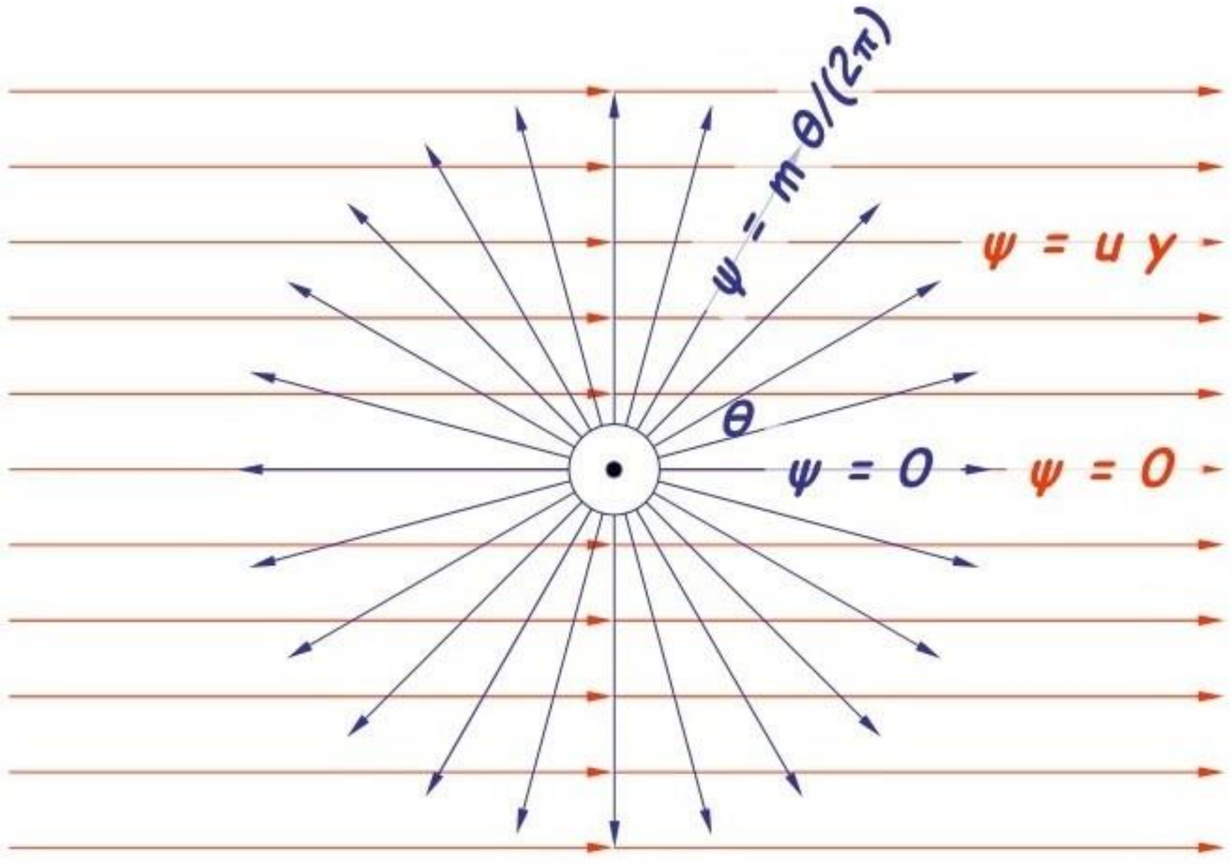
$$\therefore \psi = \frac{Q}{2\pi}\,\pi = \frac{Q}{2}$$



the streamline ABC $\left(\psi = \frac{Q}{2}\right)$ ***is a dividing streamline.***
This streamline could be replaced by a solid surface of thesame shape, forming a semi − infinite body (***half − body***) (Rankine Shape)

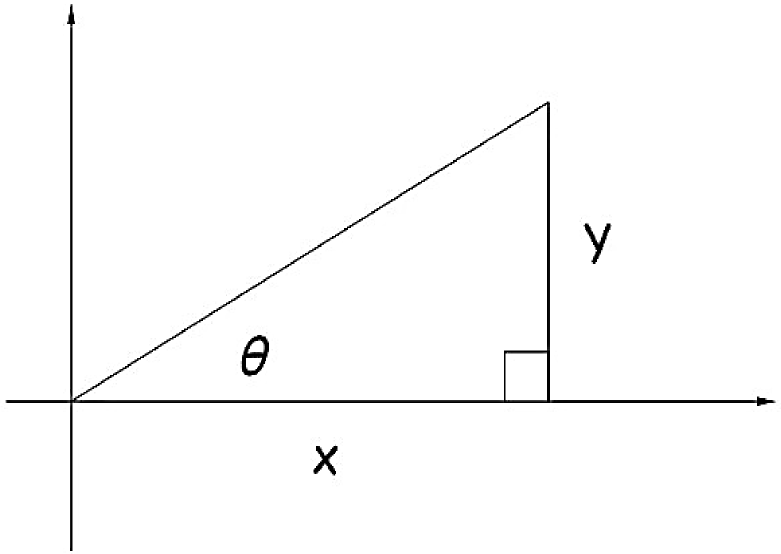
4.5 Uniform Flow + Line Source

At any point in the field, $\psi = u\,y + m\,\theta/(2\pi)$

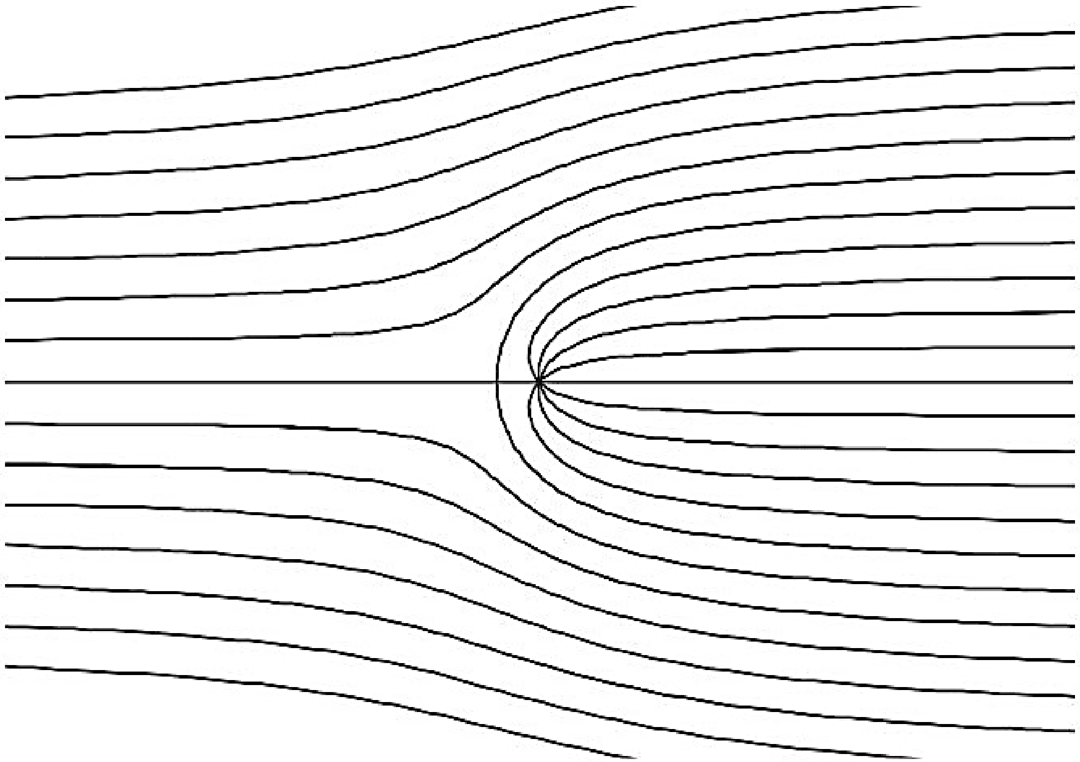


Note:

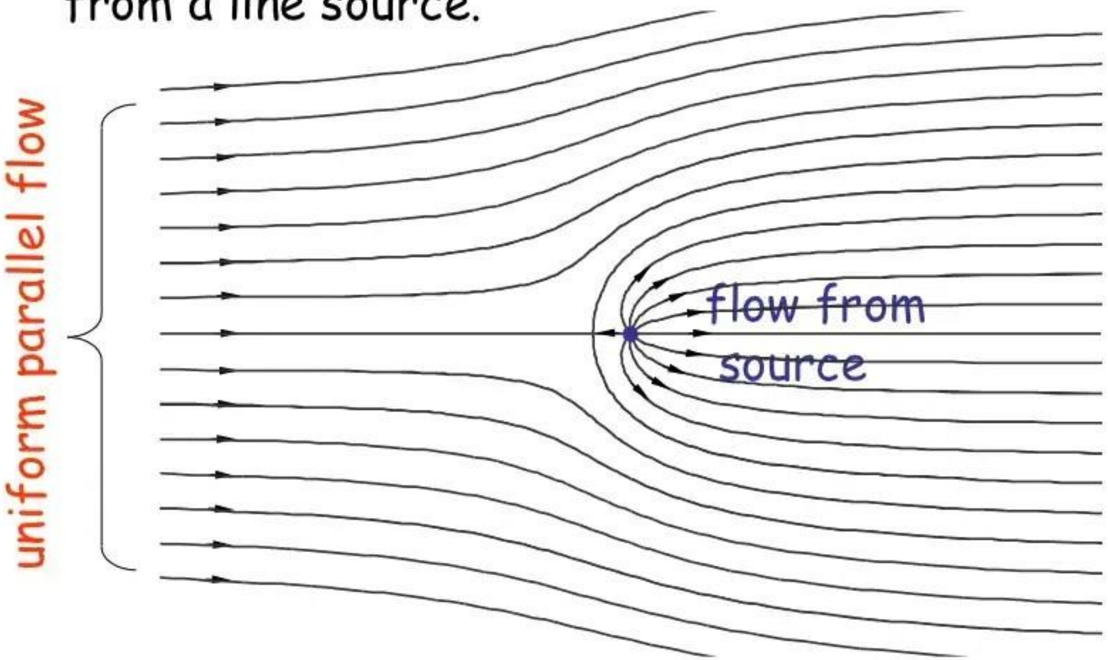
in the equation $\psi = u\,y + m\,\theta\,/(2\pi)$, $\boxed{\theta = \tan^{-1}(y/x)}$



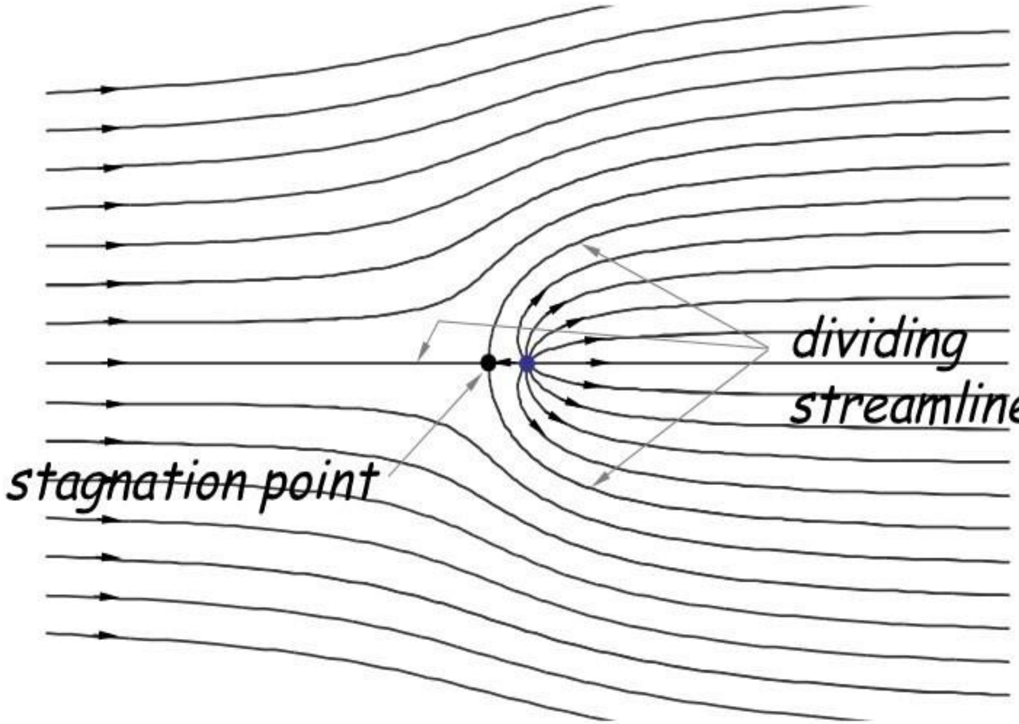
Lines of constant $\psi = u\,y + m\,\theta\,/(2\pi)$ look like this:



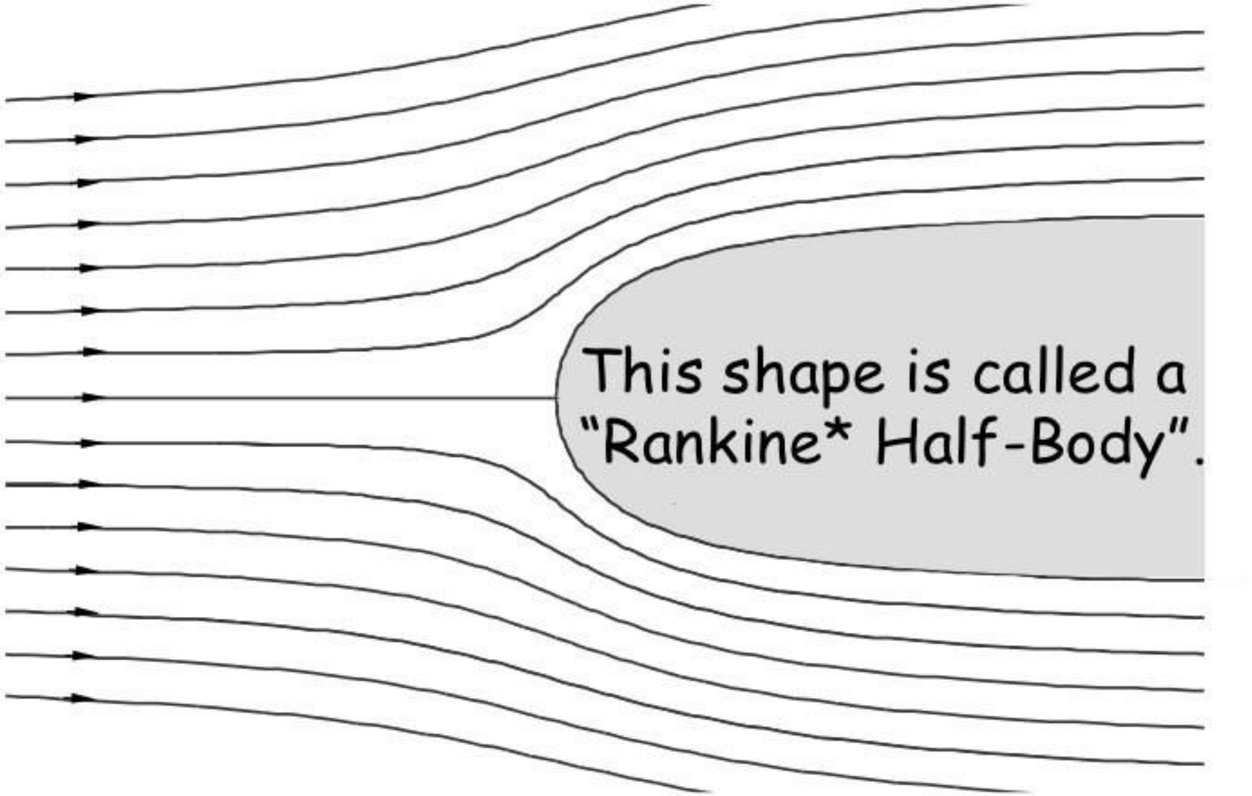
… so these streamlines represent the combination of uniform parallel flow with flow from a line source.



We can identify the *stagnation point* where the two flows cancel, and the *stagnation* or *dividing streamlines* which pass through this point.



Outside the dividing streamlines, this is a good model of flow meeting the front of a rounded body, shaped like the two dividing streamlines in the right hand half of the picture

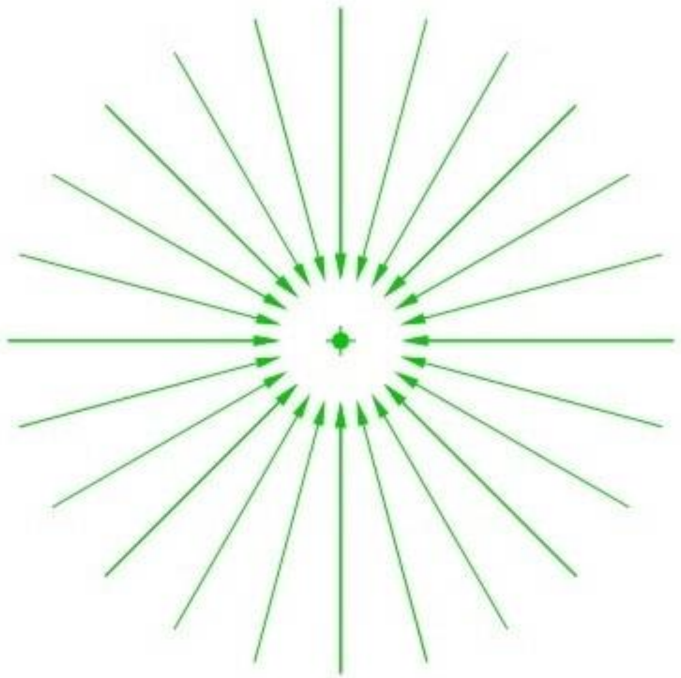


*W J M Rankine 1820-1872: professor of Engineering, University of Glasgow, from 1855

4.6 Line Sink

Flow is radially inwards. A line sink is the opposite of a line source!

For a line sink of strength m: $\psi = -m\,\theta/(2\pi)$



1.5.3 Sink Flow

- A sink is a source with negative strength, $K = -\dfrac{Q}{2\pi}$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{\partial \emptyset}{\partial r} \qquad ----------- -(i)$$

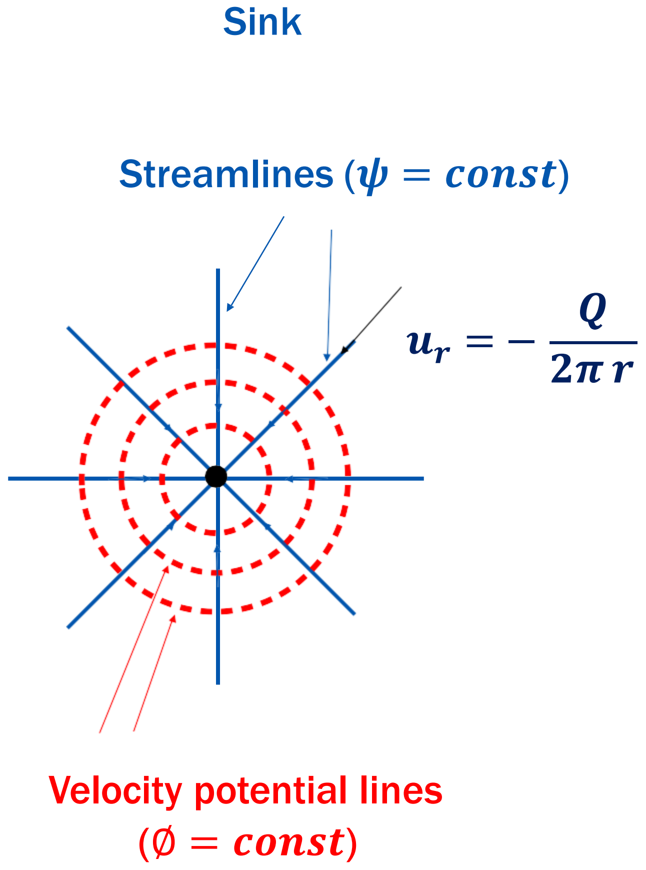
$$u_\theta = 0 \qquad ----------- -(ii)$$

from (i)

$$\psi = -K\,\theta$$

and

$$\emptyset = -K \ln r$$



1.5.4 Doublet Flow المزدوج

A doublet is a special case of a source and sink pair when the two approach each other under the limiting case of:

1. The distance $l \Longrightarrow 0$
2. The product $(Q\,.\,l)$ also called doublet strength (μ), remains constant

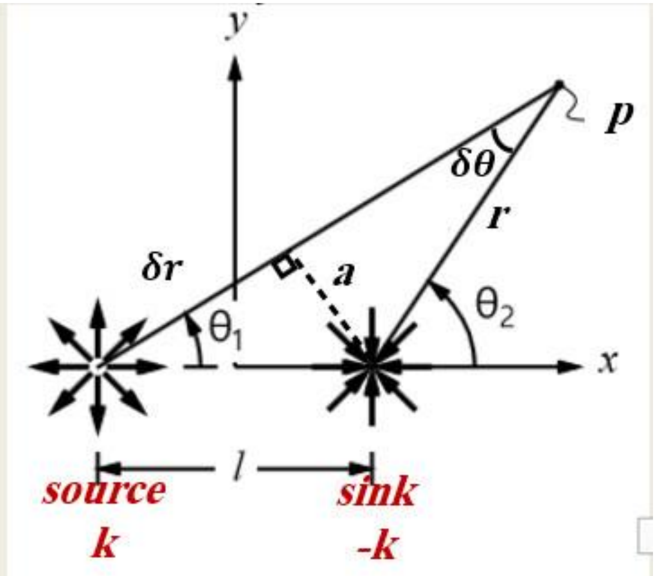
$$\psi = K\,\theta_1 - K\,\theta_2 = K\,(\theta_1 - \theta_2) = -K\,\delta\theta$$

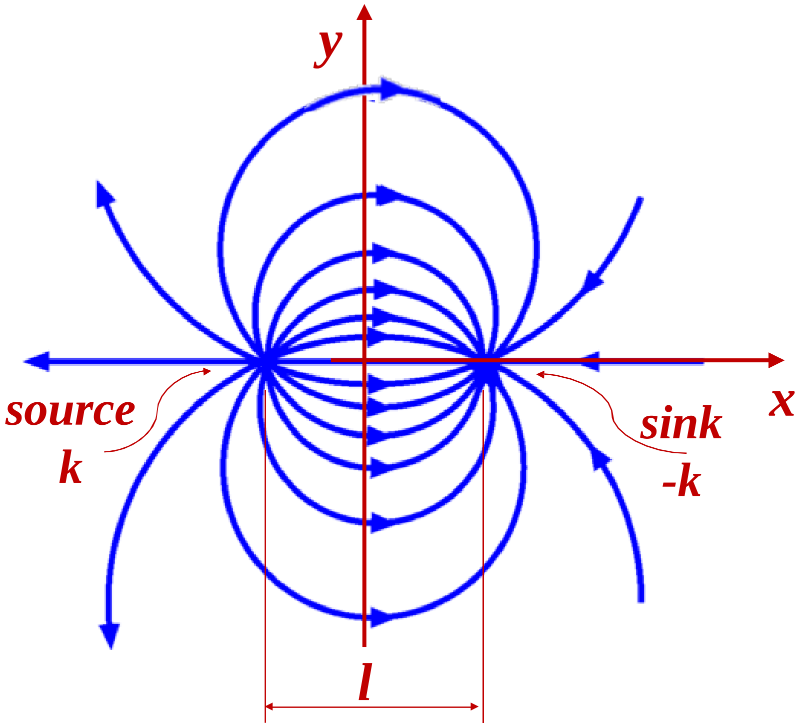
As shown in figure:

$$a = l\,.\sin\theta = \delta\theta\,.\,r$$

$$\delta\theta = \frac{l\,.\sin\theta}{r}$$

$$\therefore\ \psi = -\frac{K.\,l\,.\sin\theta}{r}$$





$$\therefore \ \psi = -\frac{K.l.\sin\theta}{r}$$

$$\therefore \ \psi = -\frac{Q.l.\sin\theta}{2\pi.r}$$

but $Q.l = \mu$

$$\therefore \ \psi = \frac{-\mu}{2\pi}\frac{\sin\theta}{r}$$

and

$$\emptyset = \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$

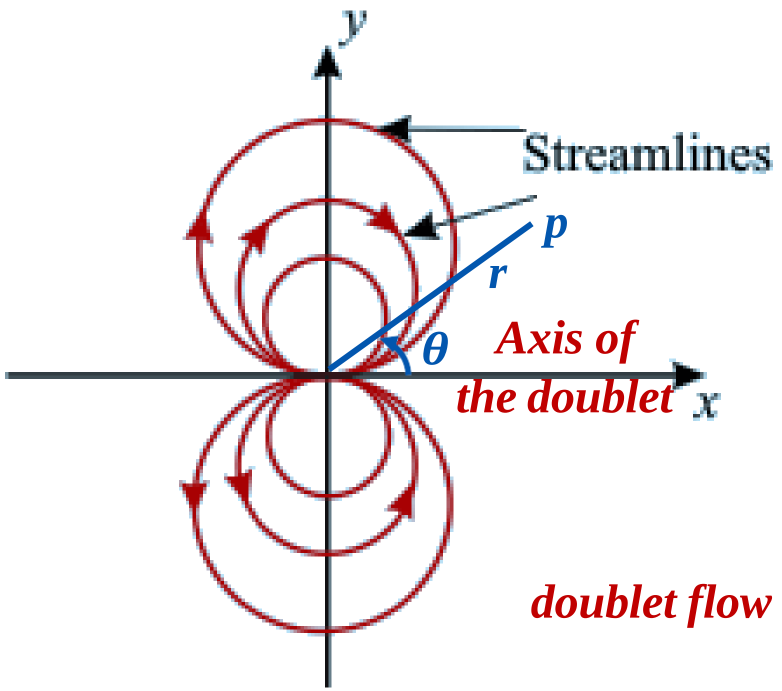
Hint:

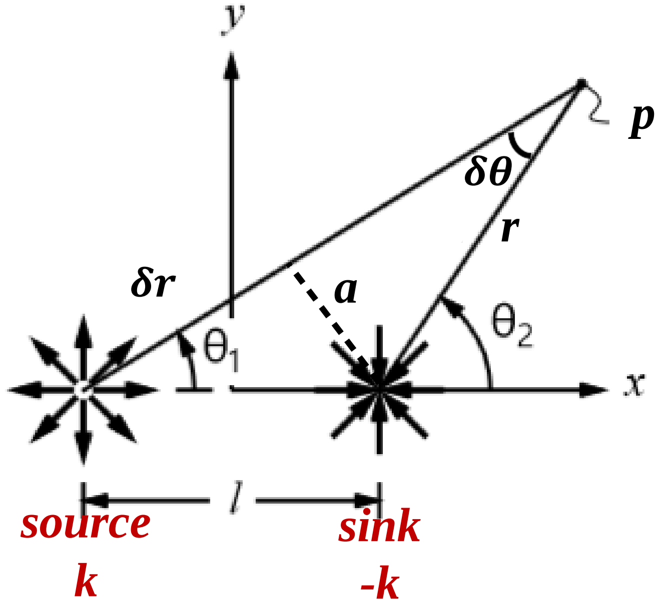
$$\emptyset = K\ \ln(r+\delta r) - K\ \ln r$$

$$\emptyset = K\ \ln\left(1+\frac{\delta r}{r}\right) = K\left(\frac{\delta r}{r} - \frac{(\delta r)^2}{2\,r^2} + \frac{(\delta r)^3}{3\,r^3} + \ \ldots..\right)$$

$$\emptyset \approx K\frac{\delta r}{r}$$

$$\therefore \emptyset = K\frac{\delta r}{r} = K\frac{l.\cos\theta}{r} = \frac{Q\,.\,l.\cos\theta}{2\pi\,r} = \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$



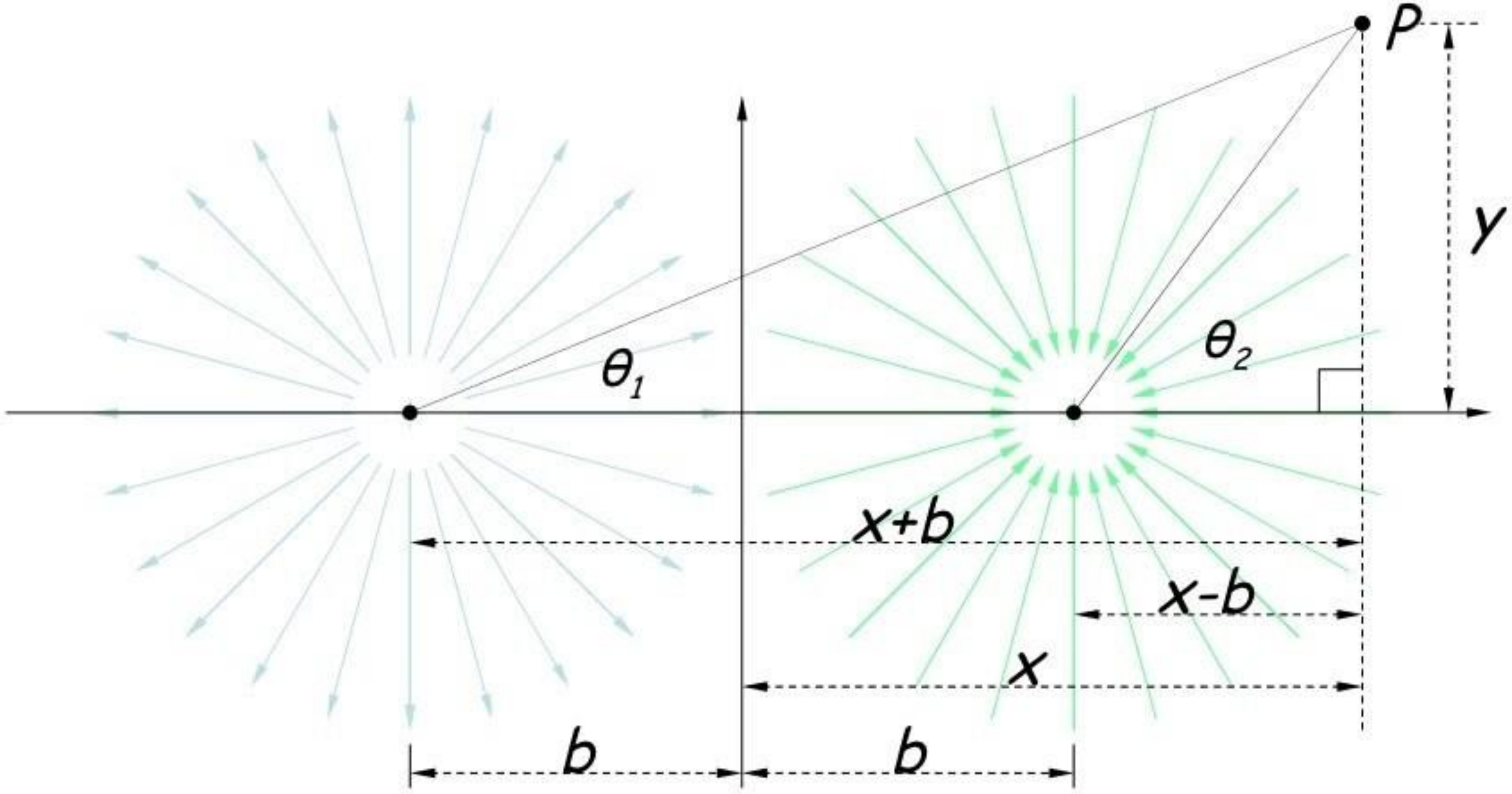


4.7 Source and Sink (of equal strength)

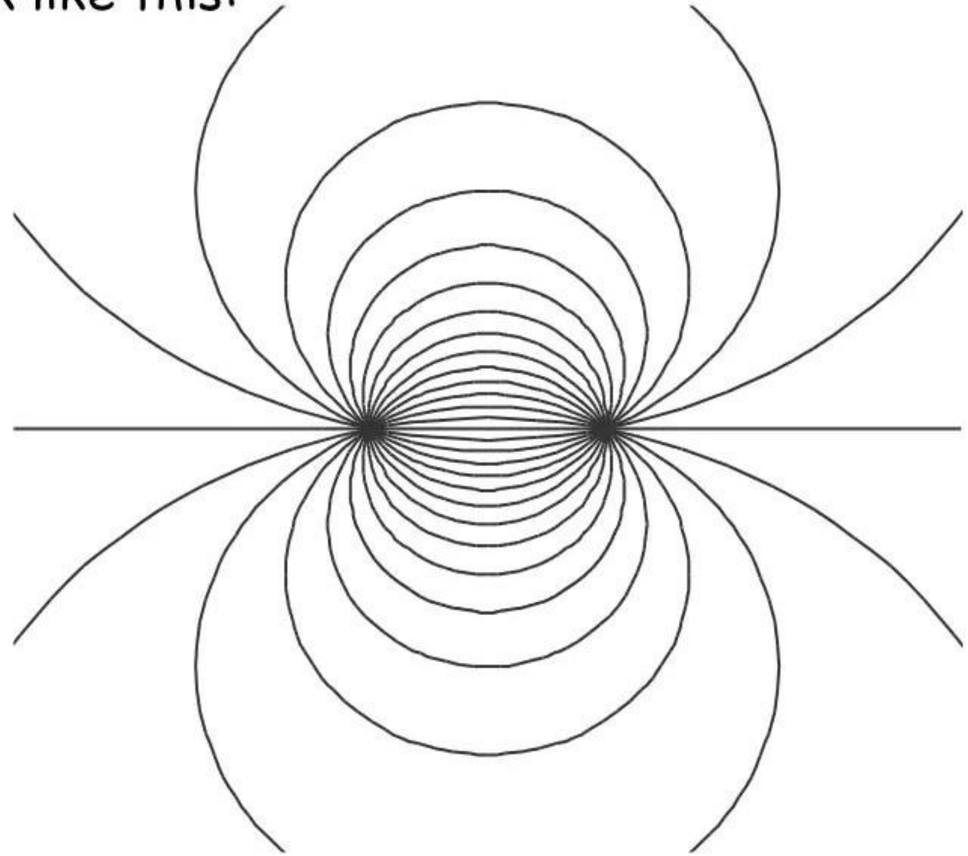
The diagram shows a source and a sink of equal strength m, placed on the x-axis, a distance $2b$ apart.

At point P, $\psi_P = m\,\theta_1/(2\pi) - m\,\theta_2/(2\pi)$

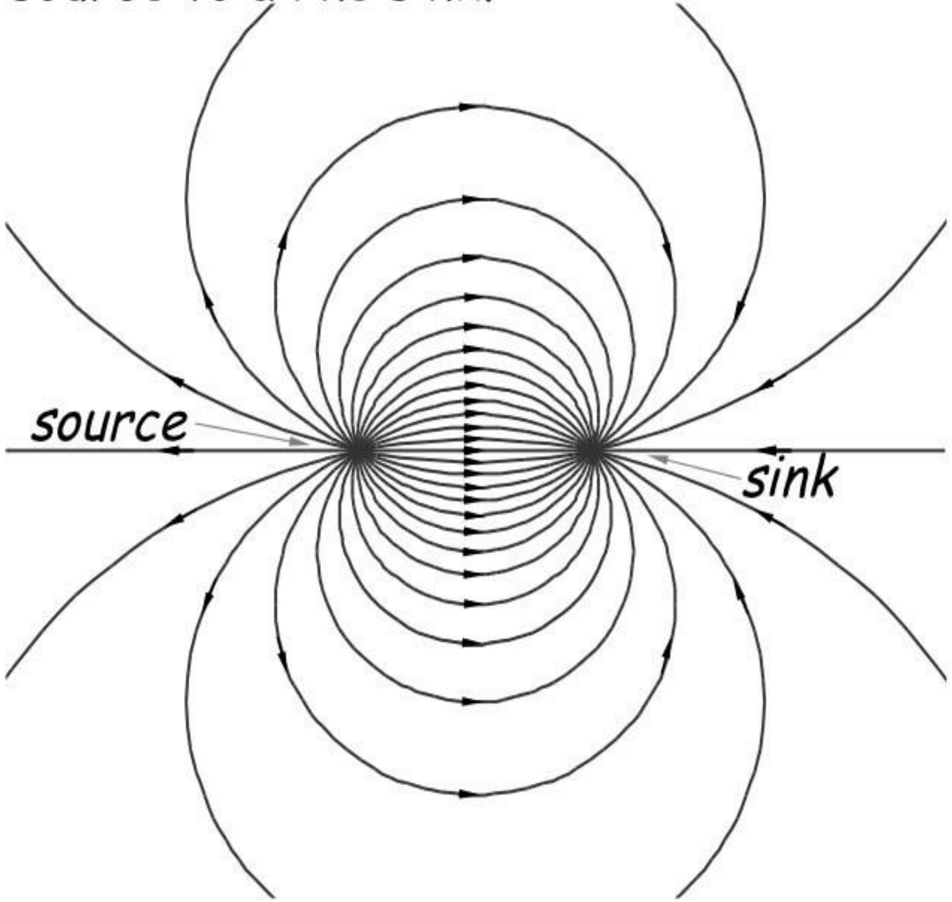
where $\theta_1 = \tan^{-1}(y/(x+b))$ and $\theta_2 = \tan^{-1}(y/(x-b))$



Lines of constant $\psi = m\,\theta_1/(2\pi) - m\,\theta_2/(2\pi)$ look like this:



… so these streamlines represent flow from a line source to a line sink.



11- In an infinite two-dimensional flow filed, a sink of strength $3/2\pi$ $m^3/s.m$ is located at the origin, and another of strength $4/2\pi$ $m^3/s.m$ at (2 , 0). What is the magnitude and direction of the velocity at point (0 , 2). [0.429 m/s ; -68.22°]

$$\phi = -K_1 \ln r_1 - K_2 \ln r_2$$

$$\phi = -K_1 \ln\sqrt{x^2+y^2} - K_2 \ln\sqrt{(x-2)^2+y^2}$$

$$u = \frac{\partial\phi}{\partial x} = -K_1 \frac{2x}{2(x^2+y^2)} - K_2 \frac{2(x-2)}{2((x-2)^2+y^2)}$$

$$v = \frac{\partial\phi}{\partial y} = -K_1 \frac{2y}{2(x^2+y^2)} - K_2 \frac{2y}{2((x-2)^2+y^2)}$$

at point (0, 2) $x = 0$, $y = 2$

we obtain:-

$u = 0.159$ m/s

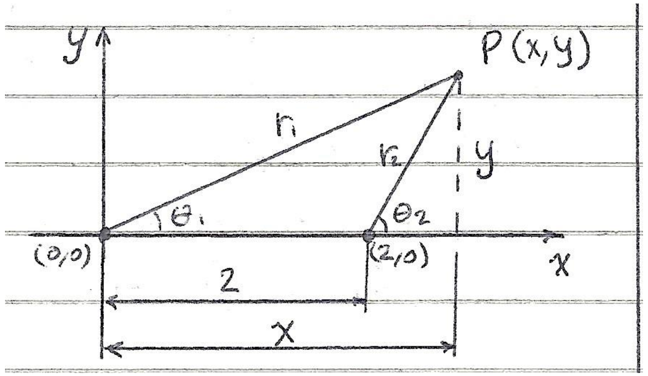
$v = -0.398$ m/s

$q = \sqrt{u^2 + v^2}$

$q = 0.429$ m/s

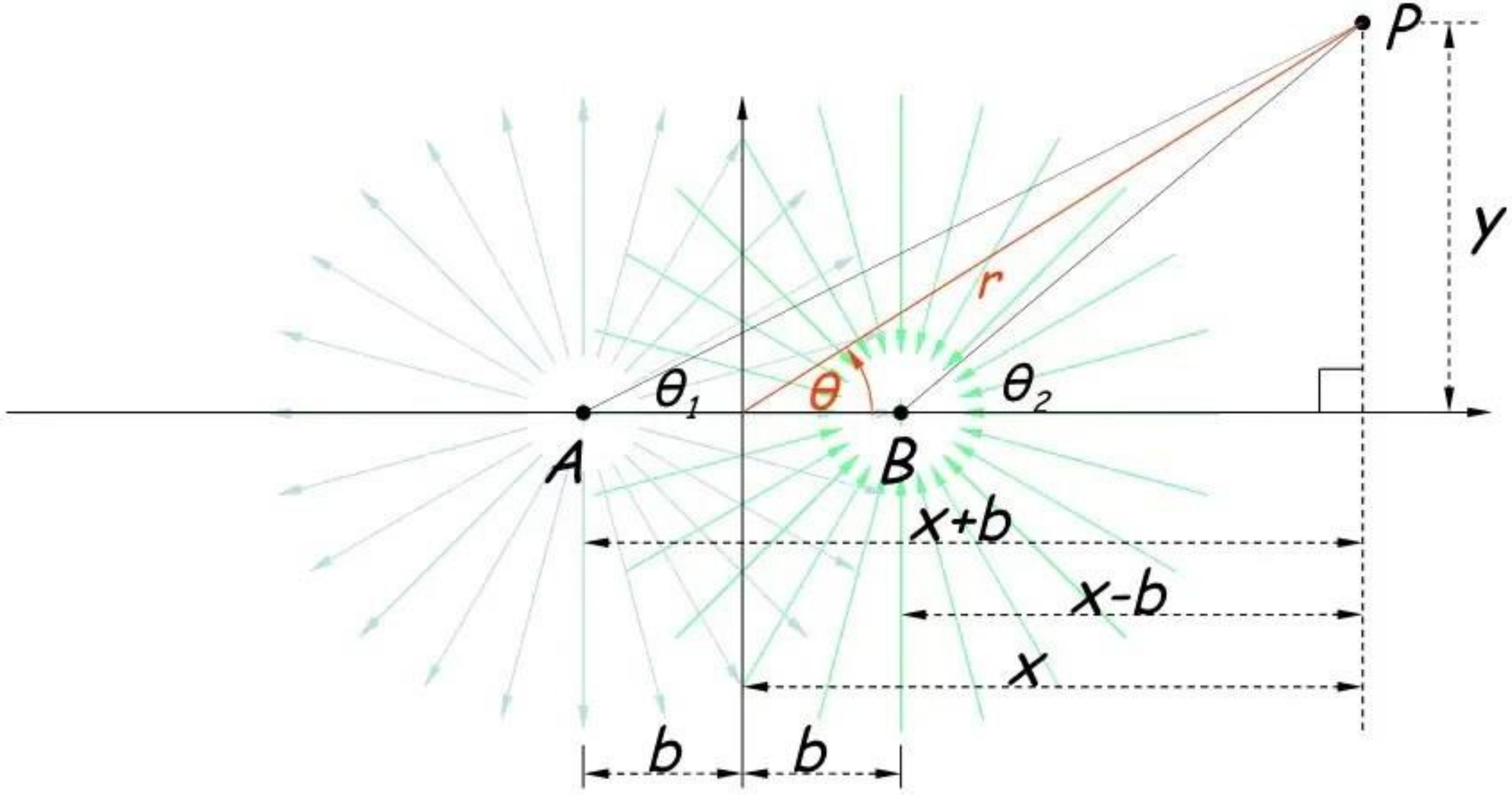
(Ans)

$$\theta = \tan^{-1}\frac{v}{u} = -68.22°$$



4.7 Doublet

A source (A) and a sink (B) of equal strength m are moved progressively closer together, at the same time increasing the strength, so that $k = mb$ = constant.



As $b \to 0$, both θ_1 and $\theta_2 \to \theta$; and both PA and $PB \to r$.
By the sine rule, $\sin(\theta_2 - \theta_1) = \sin\theta_2 \times 2b/(PA)$,
so as $b \to 0$, $\sin(\theta_2 - \theta_1) \to \sin\theta \times 2b/(PA) = (y/r) \times 2b/(r)$.
Since a small angle (in radians) is equal to its sine, this can be written: $(\theta_2 - \theta_1) = 2by/r^2$.

Now the stream function for source and sink is given by:

$\psi = m\,\theta_1/(2\pi) - m\,\theta_2/(2\pi)$

or $\psi = (m/(2\pi)) \times (\theta_1 - \theta_2)$

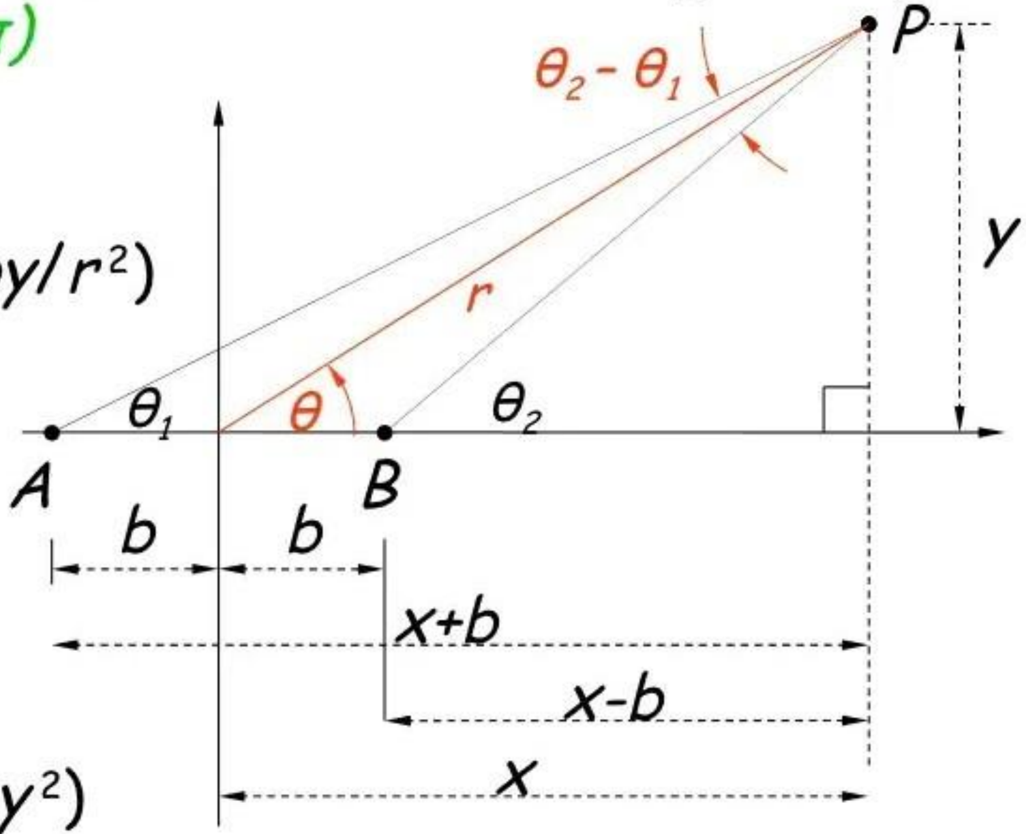
Hence $\psi = (m/(2\pi)) \times (-2by/r^2)$

But $b \times m = k$, so:

$\psi = -(k/(2\pi)) \times 2y/r^2$,

or: $\psi = -(k/\pi) \times y/r^2$,

or: $\psi = -(k/\pi) \times y/(x^2 + y^2)$



A system consisting of a source and sink placed very close together is called a "**Doublet**". The equations for stream function for a doublet are summarised below:

$\psi = -(k/\pi) \times y/r^2$, or:

$\psi = -ky/(\pi r^2)$,

or, since $y = r\sin\theta$ and $x^2 + y^2 = r^2$,

$\psi = -k\sin\theta/(\pi r)$

$\psi = -ky/(\pi(x^2 + y^2))$

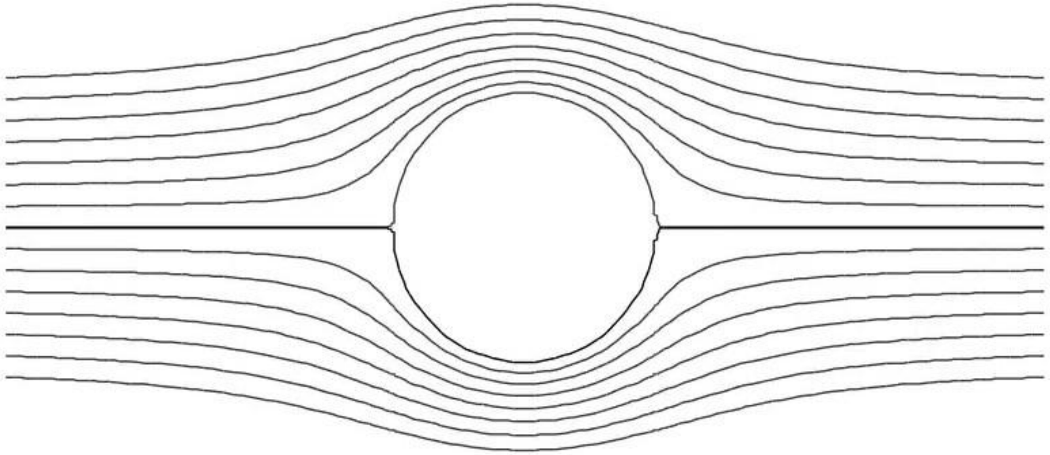
(Remember: the source and the sink are $2b$ apart. Their strengths are m and $-m$ respectively, and $k = b \times m$.)

4.8 Uniform Flow + Doublet

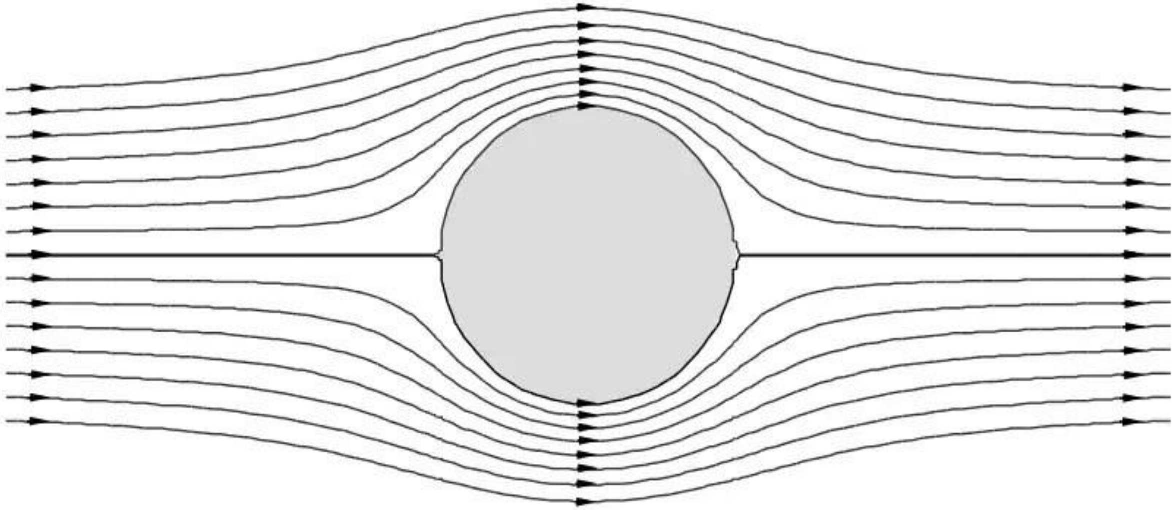
The stream function for this combination is given by:

$\psi = u\,y - k\,y/(\pi\,(x^2 + y^2))$

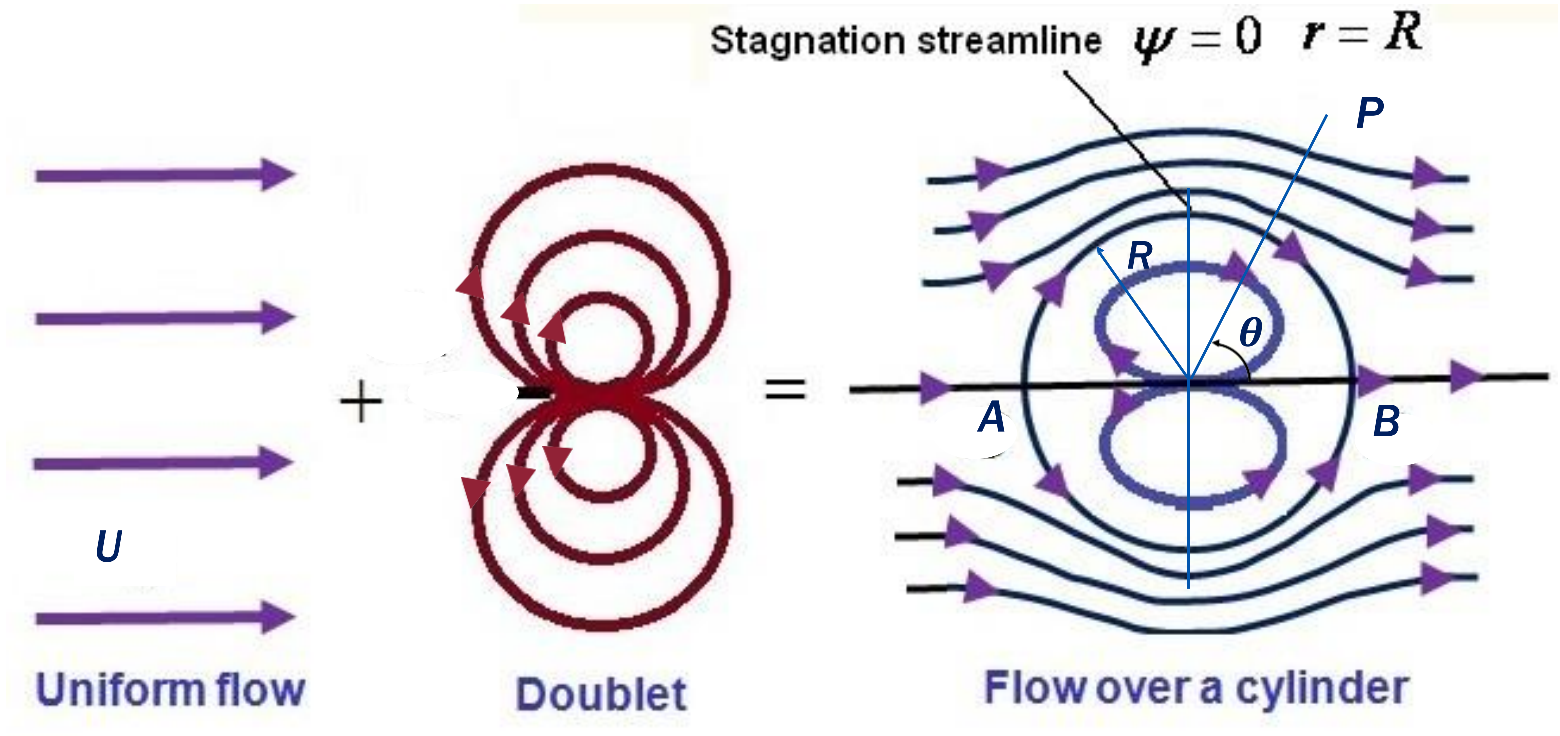
and, outside the dividing streamlines, lines of constant ψ look like this:



This time the "dividing streamlines" form a circle", and the streamlines outside represent flow of an ideal fluid round a cylinder.



1.6.3 Uniform Flow and a Doublet (Non Lifting flow over a Cylinder)



$$\psi = U\,y$$

$$\phi = U\,x$$

$$\psi = -\frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

$$\phi = \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$

$$\psi = U\,y - \frac{\mu}{2\pi}\frac{\sin\theta}{r}$$

$$\phi = U\,x + \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$

The resulting ψ is:

$$\psi = \psi_{uniform\ flow} + \psi_{doublet}$$

$$\psi = U\,y - \frac{\mu}{2\pi}\frac{\sin\theta}{r} = U\,r\sin\theta - \frac{\mu}{2\pi}\frac{\sin\theta}{r} = const$$

$$\psi = U\,r\sin\theta\left(1 - \frac{\mu}{2\pi\,U\,r^2}\right)$$

$$let;\quad R^2 \equiv \frac{\mu}{2\pi\,U}$$

$$then;\ \psi = U\,r\sin\theta\left(1 - \frac{R^2}{r^2}\right)$$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta\left(1 - \frac{R^2}{r^2}\right)$$

$$u_\theta = -\frac{\partial\psi}{\partial r} = -\,U\sin\theta\left(1 + \frac{R^2}{r^2}\right)$$

Points A and B are stagnation points and can be located by setting the equations for

$\boldsymbol{u_r}$ and $\boldsymbol{u_\theta}$ equal to zero

- Coordinates of $\boldsymbol{A(r,\theta) = (R,\pi)}$
- Coordinates of $\boldsymbol{B(r,\theta) = (R,0)}$

Substitute the coordinates of A and B in the equation of $\boldsymbol{\psi}$ yields:

$$\boldsymbol{\psi = U\ R} \sin\pi \left(\boldsymbol{1 - \frac{R^2}{R^2}}\right)$$

$$\boldsymbol{\psi = 0}$$

∴ the streamline passing through A and B is a dividing streamline.

The streamline could be replaced by a solid surface of the same shape forming a circular cylinder with radius:

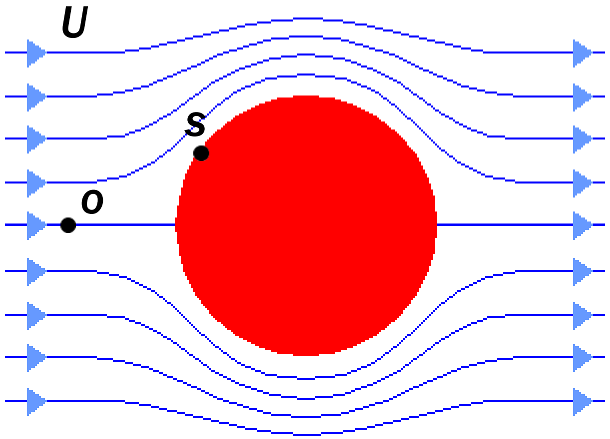
$$\boldsymbol{R = \sqrt{\frac{\mu}{2\pi\ U}}}$$

•The pressure distribution on the cylinder surface is obtained as follows:

<u>Boundary Conditions:</u>

Velocity component normal to the surface = 0

- at r = R , $\boldsymbol{u_{r_s}} = \boldsymbol{0}$
- and $\boldsymbol{u_{\theta_s}} = -\boldsymbol{2}\ \boldsymbol{U}\ \sin\boldsymbol{\theta}$



- Bernoulli equation between (o) and (s); assume $Z_o = Z_s$

$$\boldsymbol{P_o + \frac{1}{2}\rho U^2 = P_s + \frac{1}{2}\rho u_{\theta_s}^2}$$

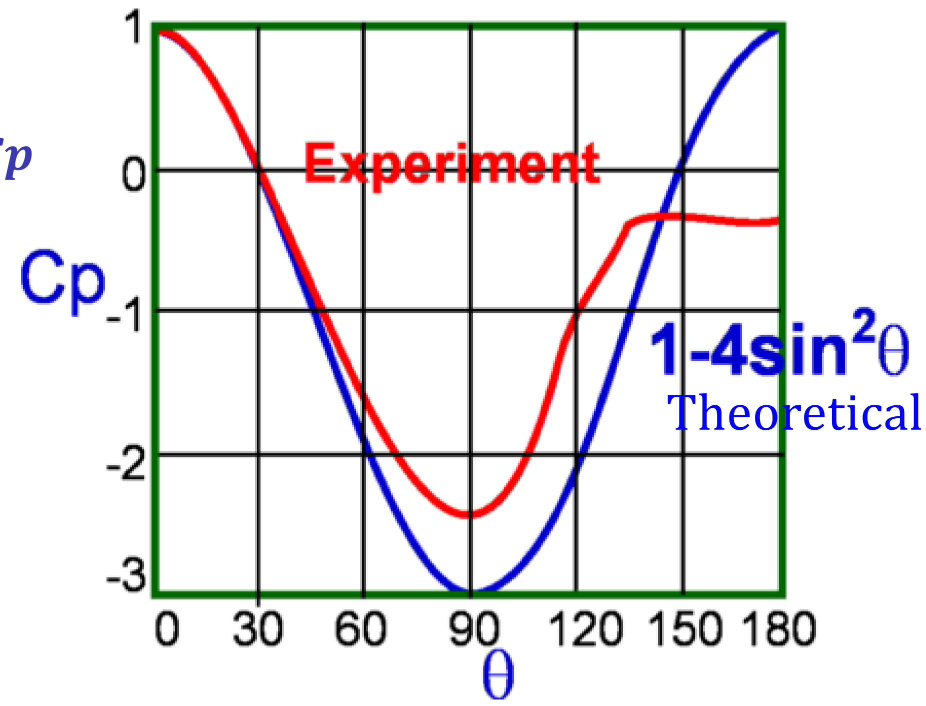
$$\Longrightarrow \boldsymbol{P_s = P_o + \frac{1}{2}\rho U^2\left(1 - 4\ \sin^2\theta\right)} \qquad \dots\dots\dots\dots(*)$$

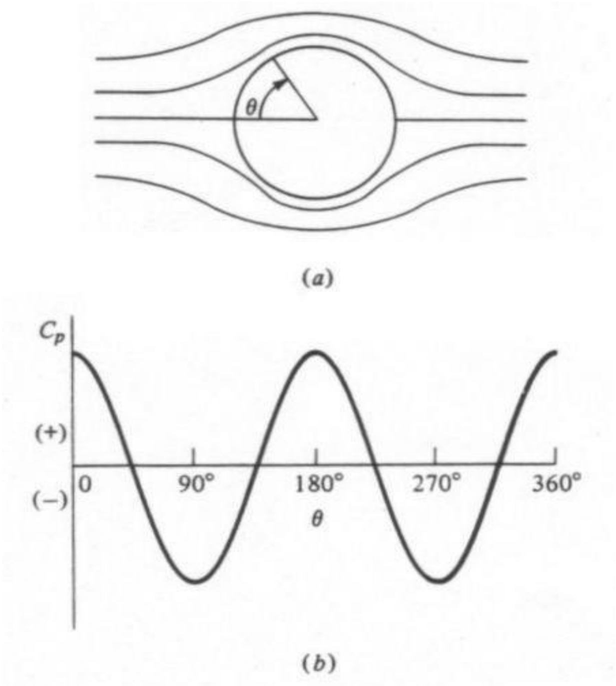
- The surface pressure as obtained by equ (*) is the theoretical (non-viscous) pressure distribution.

- The figure shows a comparison of theoretical with experimental distribution.

$$\frac{P_s - P_o}{\frac{1}{2}\rho U^2} = (1 - 4\sin^2\theta) = Cp$$

Cp =pressure coefficient

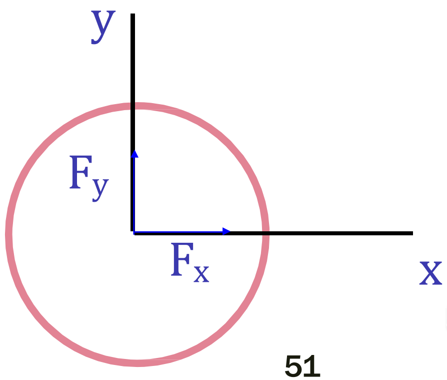




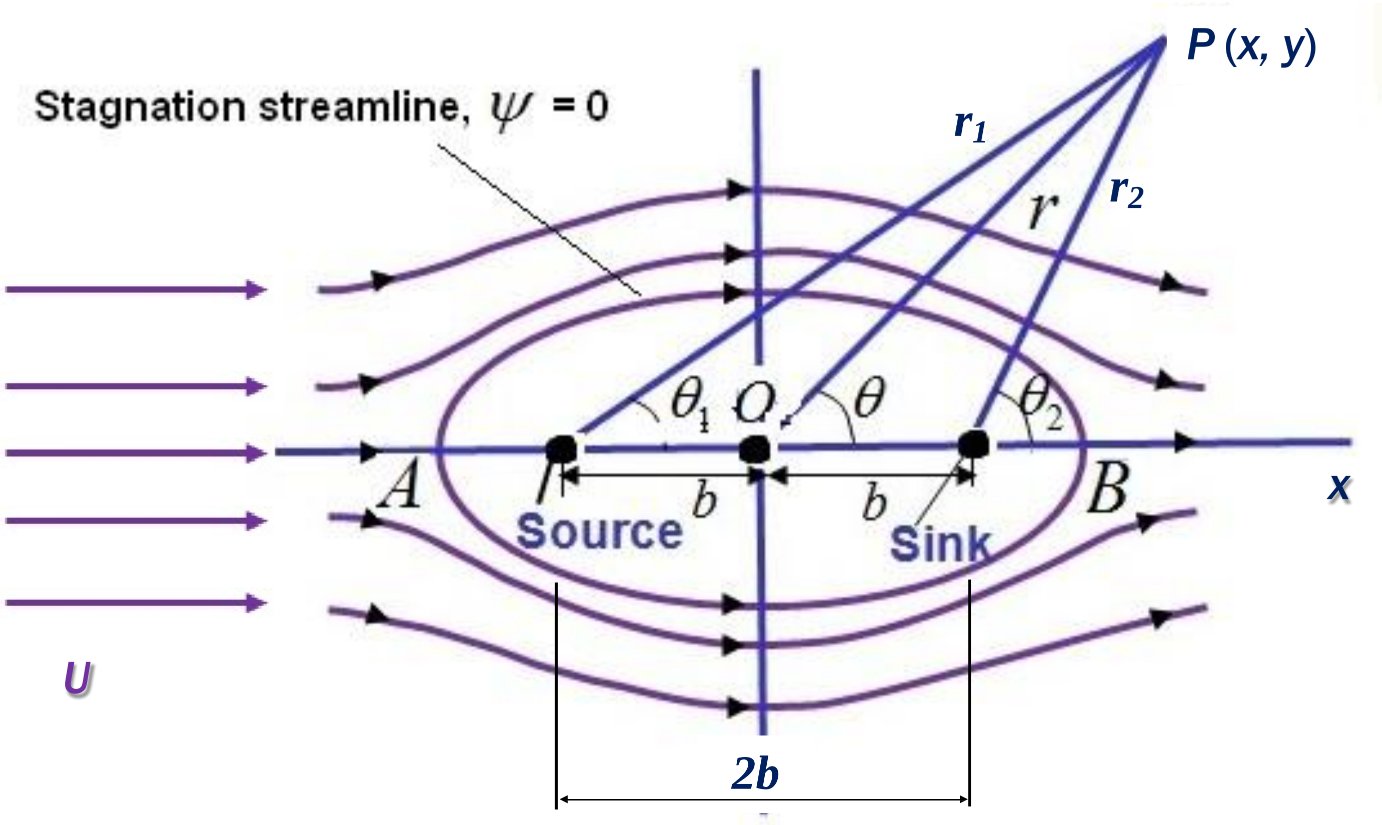
- The pressure distribution is symmetrical around the cylinder and the resultant force developed on the cylinder = zero

$\therefore F_x = 0$ (drag force)

and $F_y = 0$ (lift force)



1.6.2 Uniform Flow and Source – Sink pair



$\boldsymbol{The\ strength\ of\ the\ source\ and\ sink\ are\ K\ and\ -K\ respectively\ (equal\ and\ opposite)}$.

The resulting $\boldsymbol{\psi\ is}$:

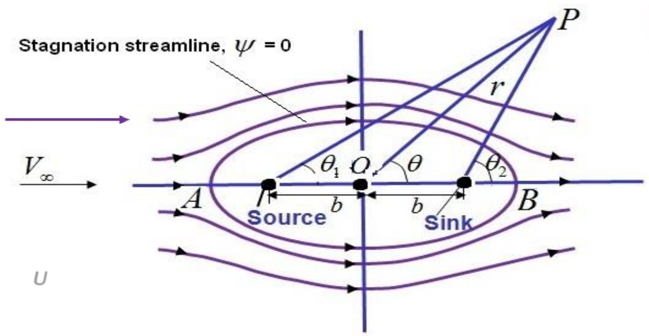
$$\psi = \psi_{uniform\ flow} + \psi_{source} + \psi_{sink}$$

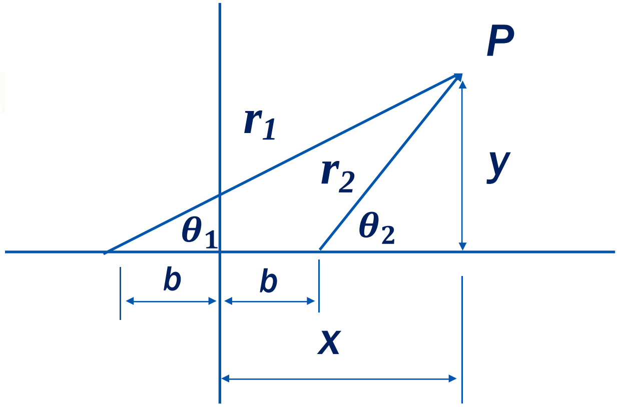
$$\psi = U\,y + K\,\theta_1 - K\,\theta_2 = constant$$

or

$$\psi = U\,y + K\,\tan^{-1}\frac{y}{x+b} - K\tan^{-1}\frac{y}{x-b}$$

$$u = \frac{\partial\psi}{\partial y}$$





$$u = U + K\frac{1}{(x+b)\left[1+\left(\frac{y}{x+b}\right)^2\right]} - K\frac{1}{(x-b)\left[1+\left(\frac{y}{x-b}\right)^2\right]}$$

Points A and B are stagnation points and can be located by setting the equation for u equal to zero, with $y = 0, x = OB$ or OA

$$0 = U + K\left[\frac{1}{(x+b)} - \frac{1}{(x-b)}\right] = U + K\left[\frac{-2b}{x^2-b^2}\right] \Rightarrow x = OB = OA = \pm\sqrt{b^2+\frac{2Kb}{U}}$$

at the stagnation point A; $\theta = \pi, y = 0$

at the stagnation point B; $\theta = 0, y = 0$

$$0 = U + K\left[\frac{1}{(x+b)} - \frac{1}{(x-b)}\right] = U + K\left[\frac{-2b}{x^2-b^2}\right] \Rightarrow x = OB = OA = \pm\sqrt{b^2 + \frac{2Kb}{U}}$$

$$0 = U + K\left[\frac{-2b}{x^2-b^2}\right]$$

نستخرج الـ (X)

$$U = K\frac{2b}{x^2-b^2}$$

$$U = \frac{2Kb}{x^2-b^2} \Rightarrow 2Kb = U\left[x^2-b^2\right]$$

$$2Kb = Ux^2 - Ub^2 \Rightarrow \boxed{2Kb + Ub^2 = Ux^2}$$

$$x^2 = \frac{2Kb}{U} + \frac{\cancel{U}b^2}{\cancel{U}} \Rightarrow x = \mp\sqrt{\frac{2Kb}{U} + b^2}$$

Set these values in the equation of ψ *we obtain* $\psi = 0$

$\therefore$ *The streamline passing through A and B is a dividing streamline.*

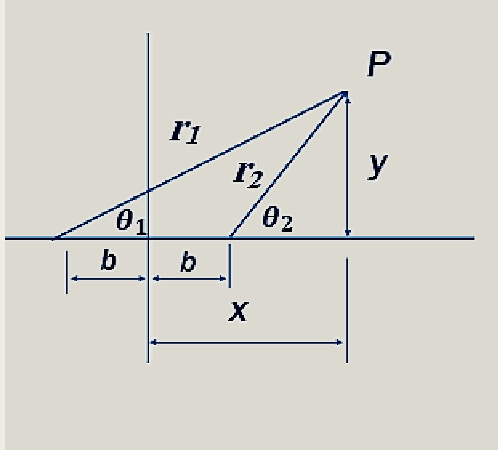
This streamline could be replaced by a solid surface of the same shape,

*forming an oval called a **Rankine oval**.*

The velosity potential is:

$$\emptyset = \boldsymbol{U}\,\boldsymbol{x} + \boldsymbol{K}\,\ln r_1 - K \ln r_2$$

$$\therefore \emptyset = \boldsymbol{U}\,\boldsymbol{x} + \boldsymbol{K}\,\ln \frac{r_1}{r_2}$$



where:

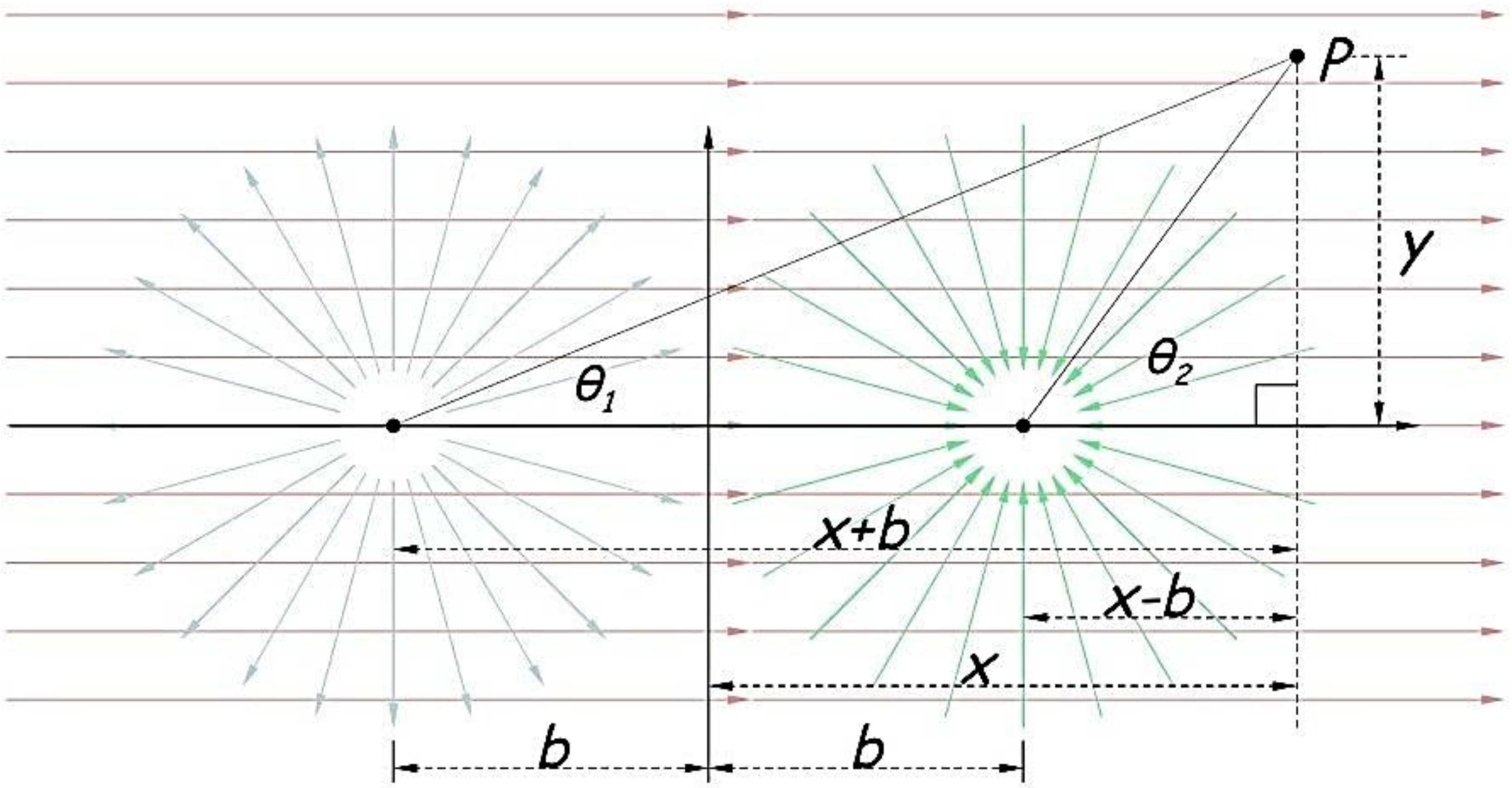
$$\boldsymbol{r}_1 = \sqrt{\boldsymbol{y}^2 + (\boldsymbol{x} + \boldsymbol{b})^2}$$

$$\boldsymbol{r}_2 = \sqrt{\boldsymbol{y}^2 + (\boldsymbol{x} - \boldsymbol{b})^2}$$

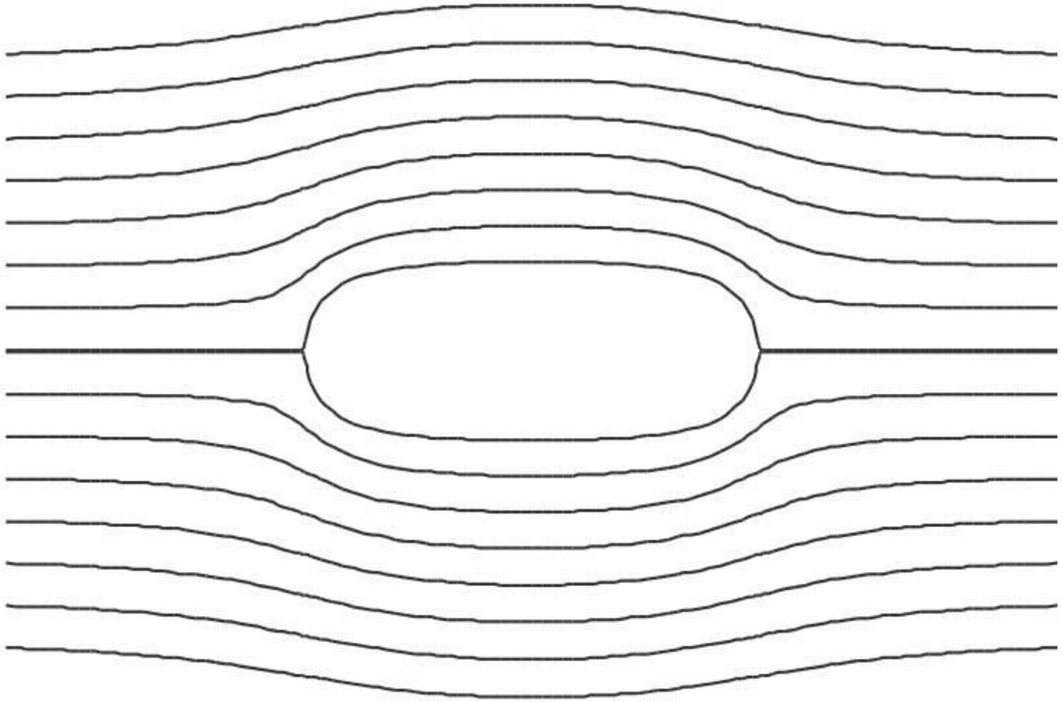
4.8 Source and Sink (of equal strength) combined with Uniform Flow

At any point, $\psi_P = u\,y + m\,\theta_1/(2\pi) - m\,\theta_2/(2\pi)$

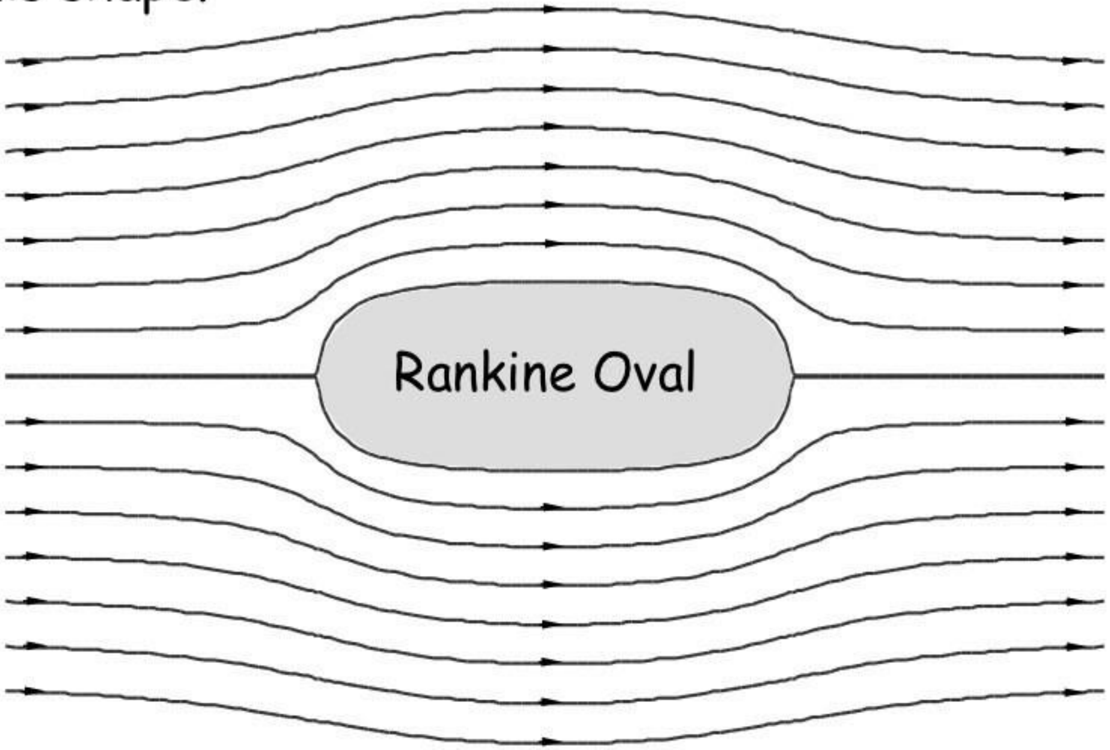
where $\theta_1 = \tan^{-1}(y/(x+b))$ and $\theta_2 = \tan^{-1}(y/(x-b))$



Outside the dividing streamlines, lines of constant $\psi = u\,y + m\,\theta_1/(2\pi) - m\,\theta_2/(2\pi)$ look like this:



The "dividing streamlines" represent a shape called a "Rankine Oval", and the streamlines outside represent flow of an ideal fluid round a solid of this shape.



Summary

$Type\ of\ flow$	ψ	$\emptyset$
$Uniform flow\ in\ x-direction$ $Uniform flow\ in\ y-direction$ $General\ uniform flow$	$\psi = u\, y$ $\psi = -v\, x$ $\psi = u\, y\ - v\, x$	$\emptyset = u\, x$ $\emptyset = v\, y$ $\emptyset = u\, x + v\, y$
$Source$	$\psi = K\, \theta$	$\emptyset = K\, ln\, r$
$Sink$	$\psi = -K\, \theta$	$\emptyset = -K\, ln\, r$
$Doublet$	$\psi = \frac{-\mu}{2\pi} \frac{\sin\theta}{r}$	$\emptyset = \frac{\mu}{2\pi} \frac{\cos\theta}{r}$
$Free\ vortex$	$\psi = \frac{-\Gamma}{2\pi} \ln r$	$\emptyset = \frac{\Gamma}{2\pi}\, \theta$

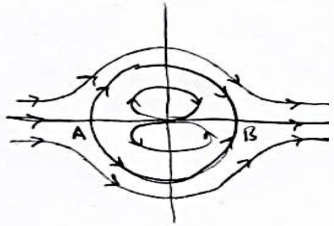
Q1: A doublet is placed at the origin of coordinates (0,0). It is found that the velocity (q) at point (0,5) is 10 m/s. Calculate the required doublet strength. Also calculate the value of ψ for the streamline passing through point (0,5).

1. What are the characteristics of an ideal flow? Describe every characteristic.
2. Show that the two-dimensional flow described by the equation $\psi = x + x^2 - y^2$ is irrotational. Find the velocity potential for this flow.
3. A source strength (0.72 m2/s) is located at (-1,0) and a sink of twice the strength is located at (+2,0) for free stream velocity of (2m/s). Find the velocity at (0,1) and (1,1).

Q1)

$$\psi = U r \sin\theta \left(1 - \frac{\mu}{2\pi U r^2}\right)$$

$$R^2 = \frac{\mu}{2\pi U}$$



$$u_r = U \cos\theta \left(1 - \frac{R^2}{r^2}\right)$$

$$u_\theta = -U \sin\theta \left(1 + \frac{R^2}{r^2}\right)$$

$$q = \sqrt{u_r^2 + u_\theta^2}$$

at $R \Rightarrow u_r = 0$ (normal component velocity is zero)

u_θ = the surface velocity componanet at point (0, 5)

⇒ means that the max velocity which at max thickness (5)

$\Rightarrow \theta = -90°$

$$q = \sqrt{u_r^2 + u_\theta^2} \Rightarrow 10 = \sqrt{0 + u_\theta^2} \Rightarrow u_\theta = 10 \text{ m/s}$$

at $r = R \Rightarrow u_r = 0$

and $u_{\theta_s} = -U \sin\theta \left(1 + \frac{R^2}{r^2}\right) = -2U \sin\theta$

$$10 = -2U \sin(-90)$$

$\therefore U = 5$ m/s

$$R = \frac{\mu}{2\pi U} \Rightarrow \mu = 5 * 2\pi * 5 = 50\pi \ \frac{m^3}{s}$$

$$\psi = U r \sin\theta \left(1 - \frac{R^2}{r^2}\right) = 0$$

Q1: A doublet is placed at the origin of coordinates (0,0). It is found that the velocity (q) at point (0,5) is 10 m/s. Calculate the required doublet strength. Also calculate the value of ψ for the streamline passing through point (0,5).

5 marks

Q2)

1. What are the characteristics of an ideal flow? Describe every characteristic.

(A) 1. non viscous ($\mu = 0$)

2. incompressible (ρ = constant)

$$\frac{\partial \rho}{\partial t} = 0 , \quad \frac{\partial \rho}{\partial x} = 0 , \quad \frac{\partial \rho}{\partial y} = 0 , \quad \frac{\partial \rho}{\partial z} = 0$$

3. the continuity equation

$$\nabla \cdot \vec{q} = 0 \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

4. irrotational flow (vorticity = 0)

$$\nabla \times \vec{q} = 0$$

2. Show that the two-dimensional flow described by the equation $\psi = x + x^2 - y^2$ is irrotational. Find the velocity potential for this flow.

(B) $\psi = x + x^2 - y^2$

$$u = \frac{\partial \psi}{\partial y} = -2y$$

$$v = -\frac{\partial \psi}{\partial x} = -1 - 2x$$

for two-dimensional flow

$$\omega_x = 0 , \quad \omega_y = 0$$

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2 + 2 = 0$$

∴ flow is irrotational

$$\phi = \int u\,dx = \int -2y\,dx = -2yx + f(y)$$

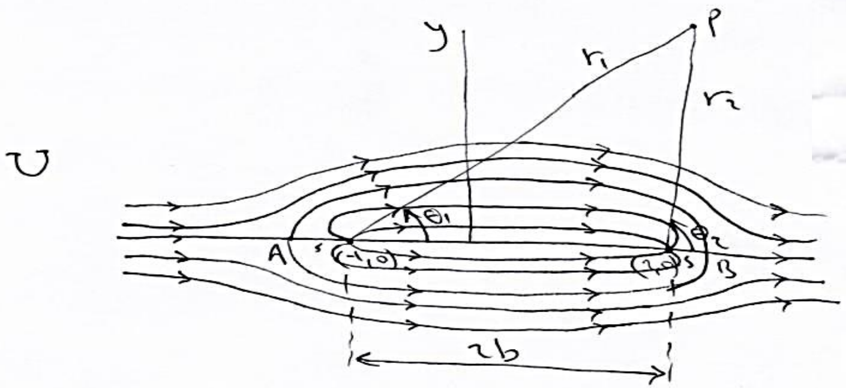
$$v = \frac{\partial \phi}{\partial y} \Rightarrow -1 - 2x = -2x + f'(y)$$

$$f'(y) = -1 \Rightarrow f(y) = -y + c$$

$$\therefore \phi = -y - 2yx + c$$

3. A source strength (0.72 m2/s) is located at (-1,0) and a sink of twice the strength is located at (+2,0) for free stream velocity of (2m/s). Find the velocity at (0,1) and (1,1).

ⓒ



$2b = (1+2)$

$b = \frac{3}{2}$

$$\psi = \psi_{uniform} + \psi_{source} + \psi_{sink}$$

$$\psi = Uy + k\theta_1 - 2k\theta_2 = \text{constant}$$

$$\psi = Uy + k \tan^{-1}\frac{y}{x+b} - 2k\tan^{-1}\frac{y}{x-b}$$

$$u = \frac{\partial \psi}{\partial y}$$

$$\therefore u = U + k\frac{1}{(x+b)\left[1+\left(\frac{y}{x+b}\right)^2\right]} - 2k\frac{1}{(x-b)\left[1+\left(\frac{y}{x-b}\right)^2\right]}$$

at point (0,1)

$$u = 2 + (0.72)\frac{1}{(0+\frac{3}{2})\left[1+\left(\frac{1}{0+\frac{3}{2}}\right)^2\right]} - 2(0.72)\frac{1}{(0-\frac{3}{2})\left[1+\left(\frac{1}{0-\frac{3}{2}}\right)^2\right]}$$

$u = 3$ m/s

at point (1,1)

$$u = 2 + (0.72)\frac{1}{(1+\frac{3}{2})\left[1+\left(\frac{1}{1+\frac{3}{2}}\right)^2\right]} - 2(0.72)\frac{1}{(1-\frac{3}{2})\left[1+\left(\frac{1}{1-\frac{3}{2}}\right)^2\right]}$$

$u = 2.824$ m/s

EXAMPLE 6.2: The velocity components in a two-dimensional velocity field for an incompressible fluid are expressed as

$$u = \frac{y^3}{3} + 2x - x^2 y$$

$$v = xy^2 - 2y - \frac{x^3}{3}$$

Show that these functions represent a possible case of an irrotational flow.

SOLUTION: The functions given satisfy the continuity equation (Equ. 6.3), for their partial derivatives are

$$\frac{\partial u}{\partial x} = 2 - 2xy \qquad \text{and} \qquad \frac{\partial v}{\partial y} = 2xy - 2$$

so that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0$$

EXAMPLE 6.3: A stream function is given by

$$\psi = 3x^2 - y^3$$

Determine the magnitude of velocity components at the point (3,1).

SOLUTION: The x and y components of velocity are given by

x-component: $u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}\left(3x^2 - y^3\right) = -3y^2$

y-component: $v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}\left(3x^2 - y^3\right) = -6x$

At the point (3,1)

u = -3 and v = -18

and the total velocity is the vector sum of the two components.

$$\vec{V} = -3\vec{i} - 18\vec{j}$$

Note that ∂u/∂x=0 and ∂v/∂y=0, so that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

may also be expressed in terms of ψ by substituting Eqs. (6.12) and (6.13) into Equ. (6.14)

$$\zeta = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}$$

However, for irrotational flows, ζ = 0, and the classic Laplace equation,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \nabla^2 \psi = 0$$

results. This means that the stream functions of all irrotational flows must satisfy the Laplace equation and that such flows may be identified in this manner; conversely, flows whose ψ does not satisfy the Laplace equation are rotational ones. Since both rotational and irrotational flow fields are physically possible, the satisfaction of the Laplace equation is no criterion of the physical existence of a flow field.