Contents

Experiment One

Mathematical Model Response

Object

1-To learn how to derive the transfer function of a linear electric system.

2- To represent the system response on personal computer by using Matlab and Simulink with the transfer function of the system .

Theory

The transfer function of single input single output dynamic system is defined as:

$$G(s) = Y(s) \ / \ X(s) \qquad\qquad (1)$$

Where

$$Y(s) = laplace \ transform \ of \ the \ output \ signal \ \ y(t)$$

$$X(s) = laplace \ transform \ of \ the \ input \ signal \ \ x(t)$$

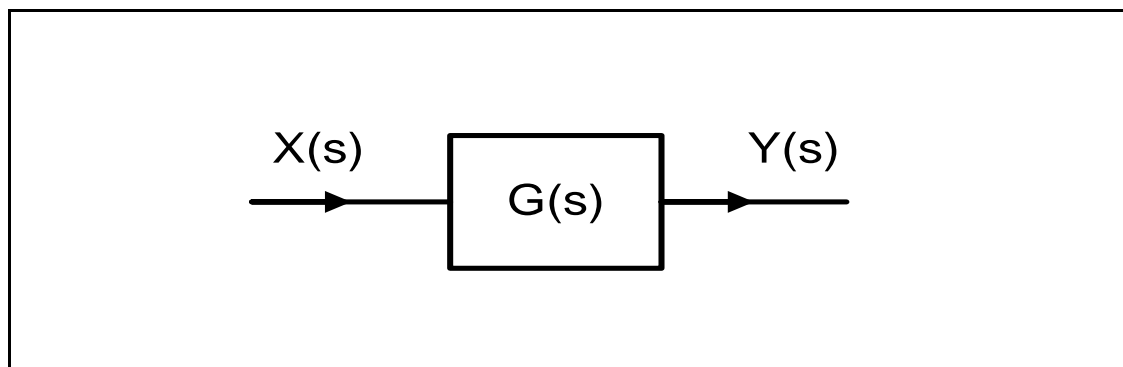The block diagram which represents equation (1) is shown in figure (1-1).

Fig (1-1) Transfer Function block diagram.

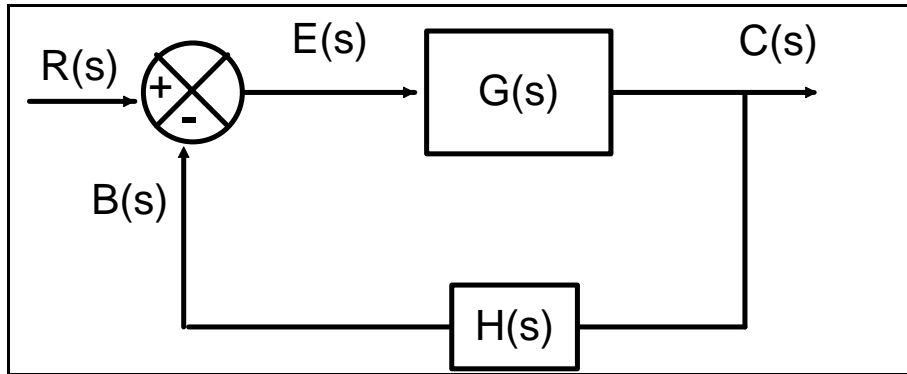Figure (1-2) shows the block diagram for closed loop control system.

Fig. (1-2) Block diagram of closed loop control system.

Where

G(s) is the process (plant) of the system.

H(s) is the feedback measuring unit (transducer).

If the closed loop control system output signal C(s) does not follow the changes of input reference R(s) the error signal E(s) is indicated, which may be positive or negative.

$$E(s) = R(s) - B(s) \qquad \qquad .... \qquad \qquad (2)$$

$$B(s) = C(s) \ H(s) \qquad \qquad ..... \qquad \qquad (3)$$

Substituting for B(s) in (2) from (3) then

$$E(s) = R(s) - C(s) \ H(s) \qquad \qquad ..... \qquad \qquad (4)$$

Where

$$C(s) = E(s) \ G(s) \qquad \qquad ..... \qquad \qquad (5)$$

From (4) and (5) the transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \ H(s)} \qquad \qquad ..... \qquad \qquad (6)$$

Note: if we have positive feedback the transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\ H(s)} \qquad ..... \qquad (7)$$

We can find the transfer function of any passive electric circuit by applying kirckoff's laws, for example to find the transfer function of the (R-L-C) circuit shown in figure (1-3).
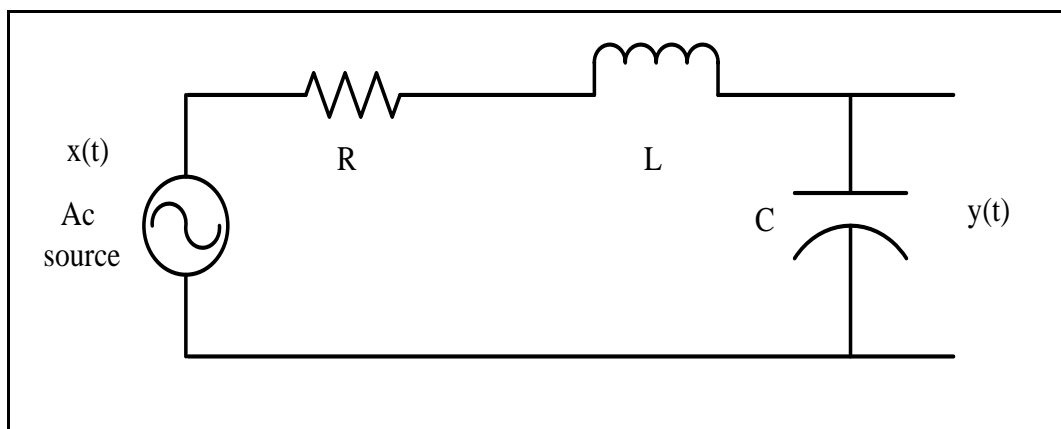

Fig. (1-3): (R-L-C) circuit.

The series connection equation is written as:

$$e_i(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt + R\ i(t) \qquad ..... \qquad (8)$$

Where $i(t)$ is the current in the loop.

By taking Laplace transform and assuming all initial conditions are zero we get:

$$E_i(s) = Ls\ I(s) + \frac{1}{Cs}I(s) + R\ I(s) \qquad ..... \qquad (9)$$

$$E_0(s) = \frac{1}{Cs}\ I(s) \qquad ..... \qquad (10)$$

From (9) and (10) we get :

$$\frac{E_0(s)}{E_i(s)} = \frac{(1/Cs)\, I(s)}{(Ls + 1/Cs + R)\, I(s)} \qquad \ldots\ldots \qquad (11)$$

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{(LCs^2 + RCs + 1)} \qquad \ldots\ldots \qquad (12)$$

By using Matlab and Simulink the transfer function can performed to represent the electric circuit and to obtain the system response as shown in figure (1-4).
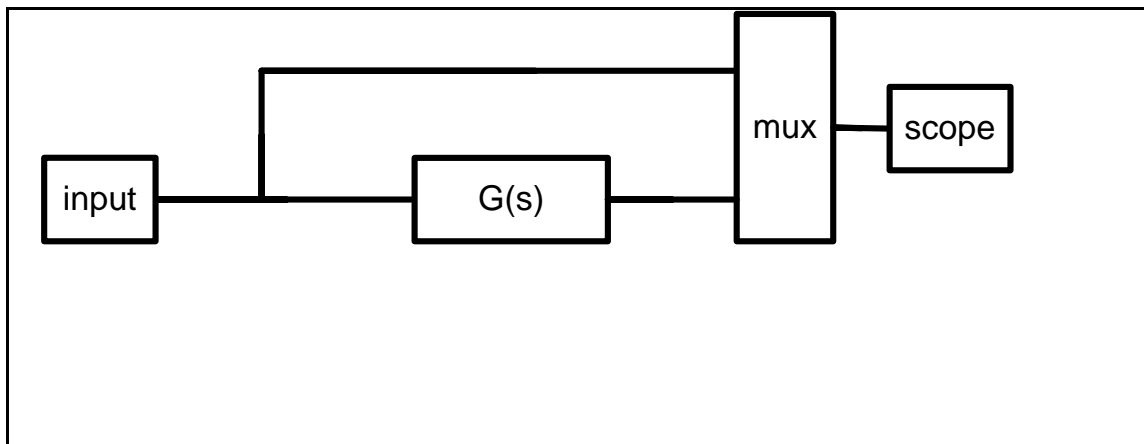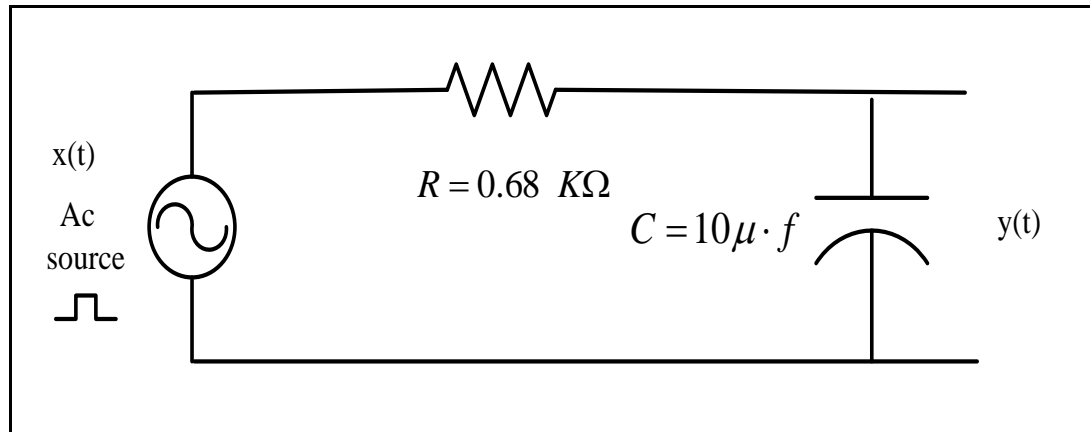

Fig. (1-4): Matlab and Simulink representation.

Electric circuits

$$T(s) = \frac{1}{RCs + 1}$$

Fig. (1-5): (R-C) circuit.
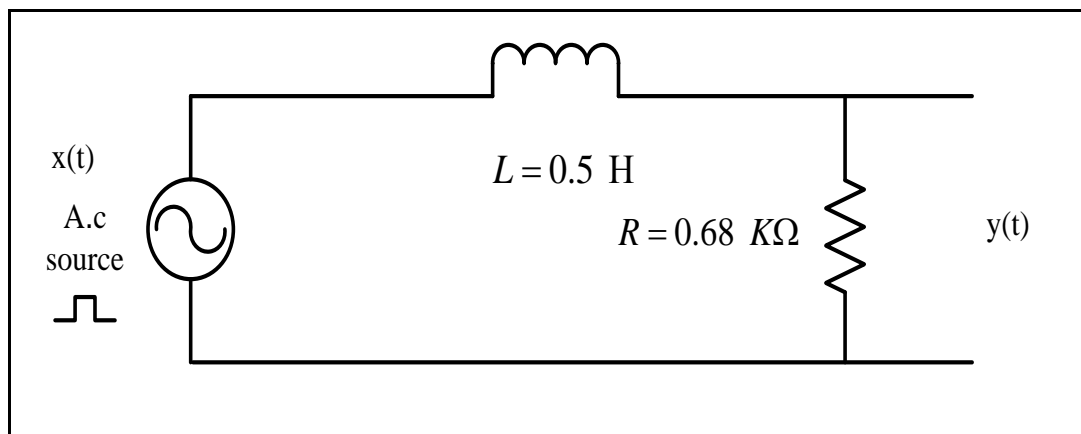
$$T(s) = \frac{1}{(L/R)\ s + 1}$$

Fig. (1-6): (L-R) circuit.
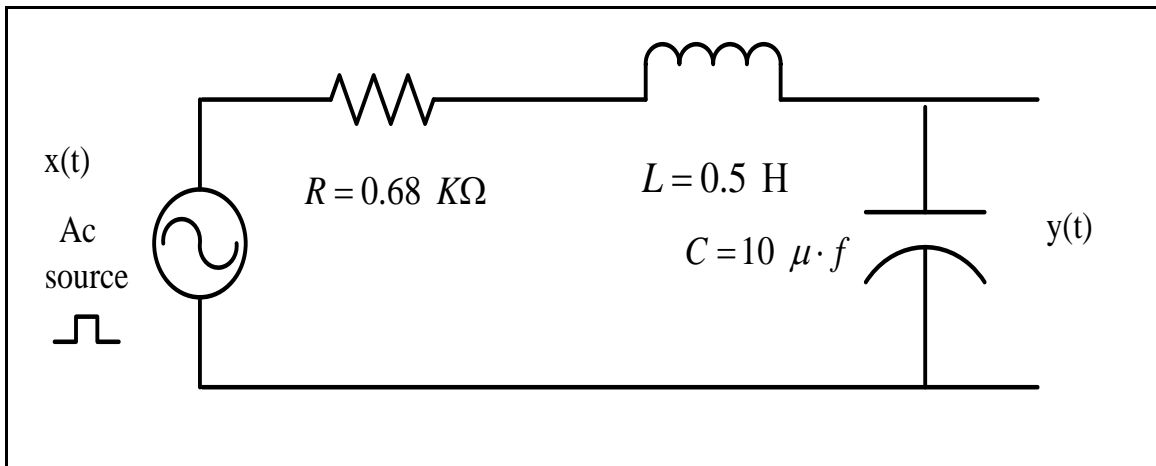
$$T(s) = \frac{1}{\left(LCs^2 + RCs + 1\right)}$$

Fig. (1-7): (R-L-C) circuit.
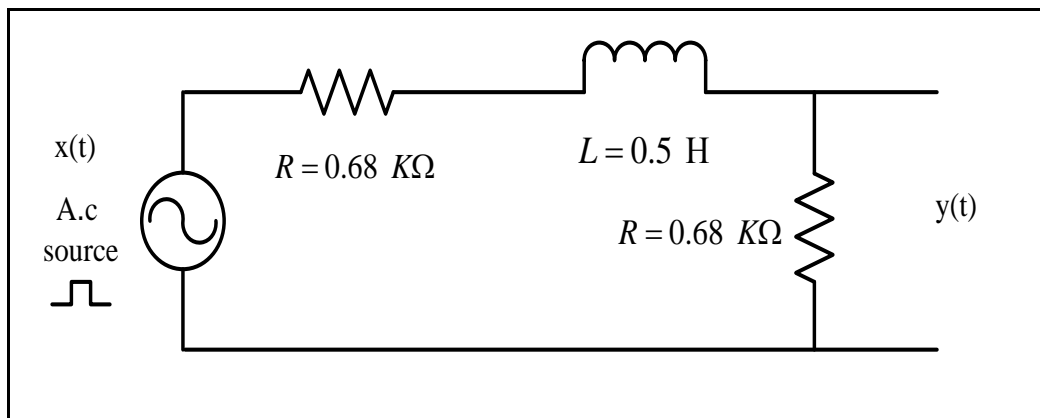
$$T(s) = \frac{1}{2\left((L/2R)\ s + 1\right)}$$

Fig. (1-8): (R-L-R) circuit.

Procedure

1- For the transfer functions of figures (1-5), (1-6), (1-7) and (1-8), apply unit step function x(t) and show the output response y(t) on personal computer using Matlab and Simulink.

2- Plot the output response y (t).

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Discussion

    1- Derive the transfer function of the electric circuits.

    2- Derive the Laplace transform of a test signal :

$$f(t) = 3\sin wt$$

    3- For the (R-C) circuit shown in figure (1-5) , plot the output response
       y(t) for unit step input e(t) mathematically .

Experiment Two

First Order Systems Analysis

Object

To study the characteristics of time response of first order control system.

Theory

The simplest first order control system can be represented by the passive filter (R-C) circuit shown in figure (2-1).
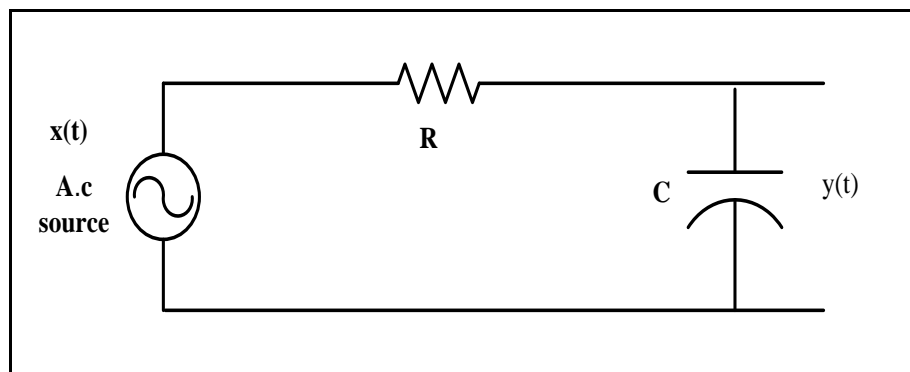
Fig. (2-1): electric (R-C) circuit.

The transfer function can be obtained as follows:

Step 1:

$$e_i(t) = R\ i\ (t) + \frac{1}{C}\int i\ (t)\ dt \qquad ..... \qquad (1)$$

Where $i(t)$ is the current in the loop.

Step 2: Applying Laplace transform assuming that the initial conditions equal to zero, the equation becomes:

$$E_i(s) = R\ I(s) + \frac{1}{Cs}I(s) \qquad ..... \qquad (2)$$

$$E_0(s) = \frac{1}{Cs}I(s) \qquad .... \qquad (3)$$

Step 3: From (2) and (3) the transfer function becomes:

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{(RCs+1)} \qquad \cdots\cdots \qquad (4)$$

The analog solution of the first order system can be obtained from the op-amp active filter, figure (2-2)
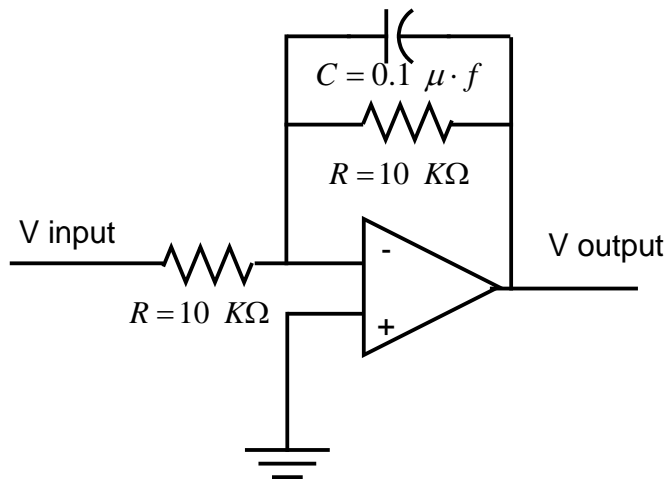
Fig. (2-2): op-amp representation.

The system response to a unit step input for a first order control system can be represented in figure (2-3).
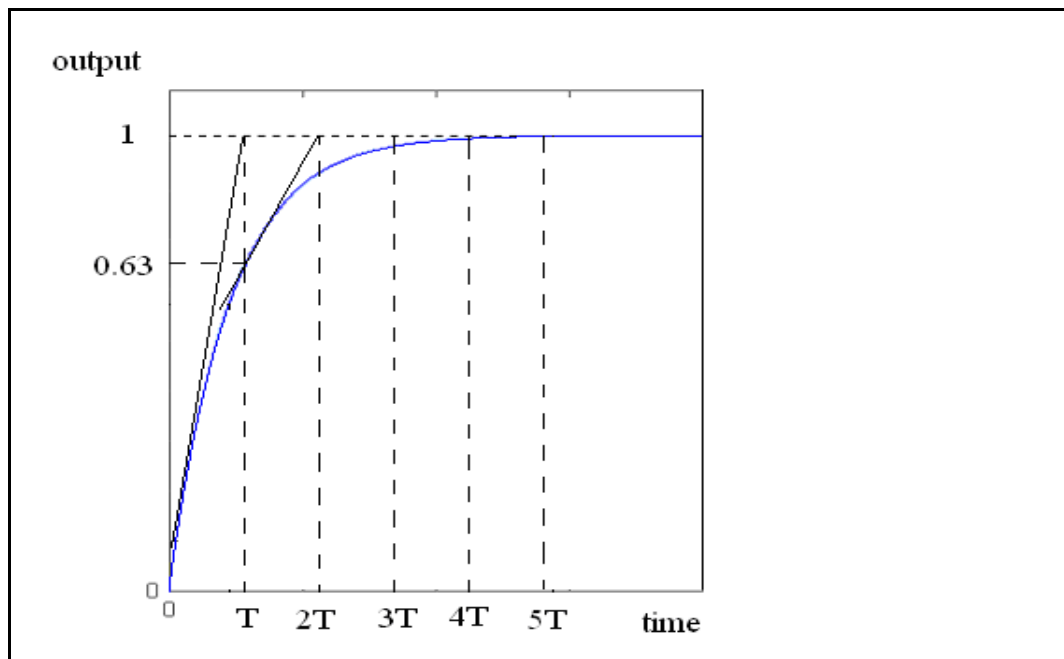
Fig. (2-3): The system response to a unit step input.

The block diagram representation for figure (2-2) is as shown in figure (2-4).
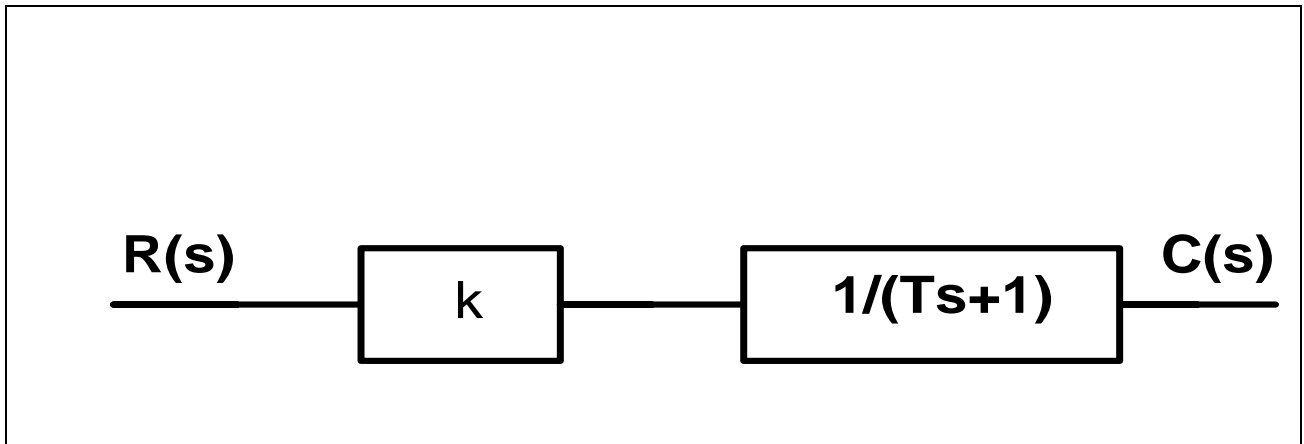
Fig (2-4): Block diagram representation.

Where T =RC   time constant of the system, k = the gain of op- amp.

By using Matlab and Simulink the transfer function can performed to represent it's

response as shown in figure (2-5).
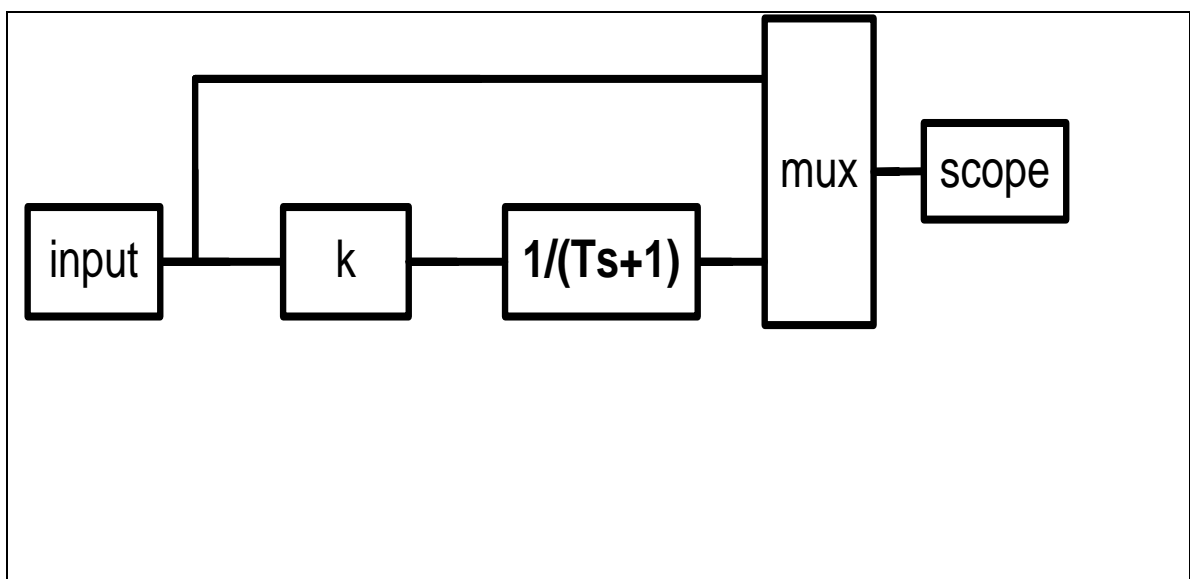
Fig. (2-5): Block Diagram in Matlab.

Procedure

1- Connect the circuit as shown in figure (2-5), apply unit step function assuming that the time constant T=1 and the values of k = 0.5, 1, 2 then sketch the system response y (t).

2- Repeat step 1 for an impulse and ramp input.

Discussion

1- From equation (4) find the transient response using Laplace transforms.

2- Why the output for 3 times constant (3T) is not the same for different cases.

3- Do you observe any relation between the output at one time constant (T) and the value of gain (k).

Experiment Three

Second Order Systems Analysis

Object

To study the characteristics of time response of second order control system.

Theory

The transient response to unit step for a second order control system can be represented in figure (3-1).
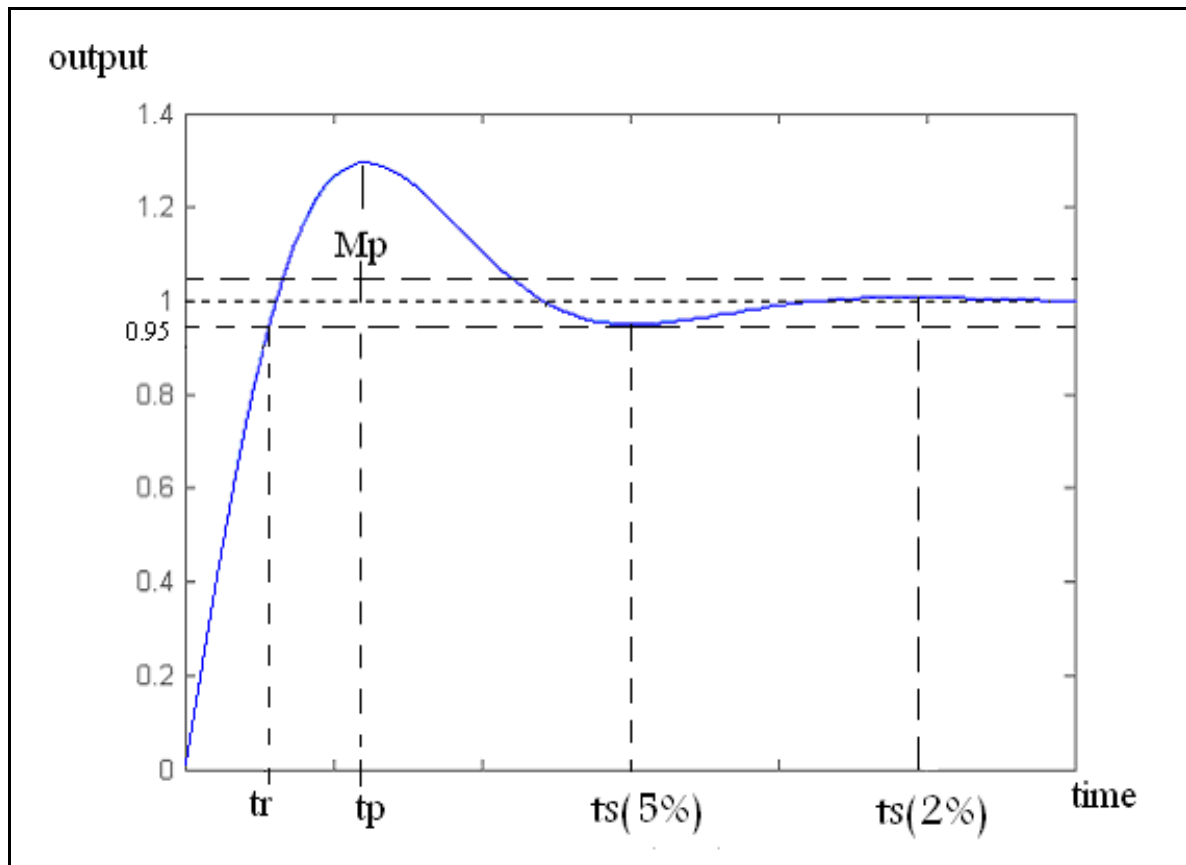
Fig. (3-1): second order transient response.

Transient response terms are as follows:

tr = rise time (the time to reach 0-100 %, 5-95 % and 10-90 % of the input signal).

tp = peak time (the time to reach the maximum overshoot).

Mp = maximum overshoot.

ts = settling time (the time to reach 5% or 2% of steady state error).

We can find the system parameters by the equations:

$$Mp = e^{-\xi\ \Pi/\sqrt{1-\xi^2}} \text{,} \quad tp = \Pi/\omega_d$$

$$ts = \frac{3}{\xi\ \omega_n} \qquad for\ 5\%\ E_{s-s}$$

$$ts = \frac{4}{\xi\omega_n} \qquad for\ 2\%\ E_{s-s}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad , \quad t_{r = \Pi - \beta/\omega d}, \quad \beta = \cos^{-1}\xi$$

$$Mp = \max imum\ overshoot\cdot$$

$$\omega_d = damping\ of\ natural\ frequency\cdot$$

$$\omega_n = natural\ frequency\cdot$$

$$\xi = damping\ ratio\cdot$$

Figure (3-2) shows the block diagram of closed loop second order system
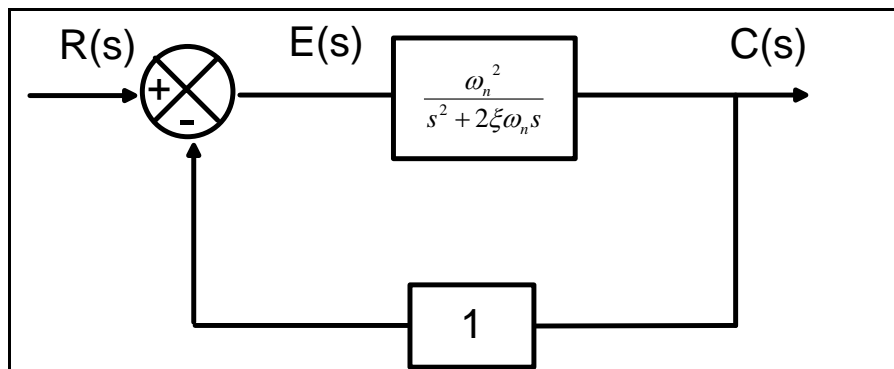
Fig. (3-2): block diagram of second order control system.

The equation becomes:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\ H(s)}$$

$$G(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\xi\omega_n s}$$

$$\mathrm{H}(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^{\ 2}}{s^2 + 2\ \xi\ \omega_n s + \omega_n^{\ 2}}$$

By using Matlab and Simulink the block diagram representation can take the form:
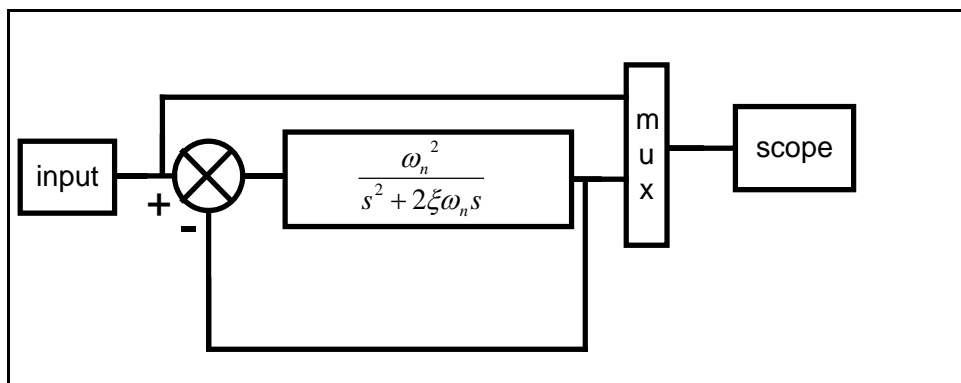
Fig. (3-3): Block diagram in Matlab.

The practical circuit of the second order control system can represented using two integrators and summing circuit with a feedback with a unity gain connected to the summing circuit throw a variable resistance as shown in figure (3-4).

$C = 0.1\ \mu \cdot f$

$R = 100\ K\Omega$

Vout

$R = 15\ K\Omega$ $R = 100\ K\Omega$ $C = 0.1\ \mu \cdot f$

Vin $R = 100\ K\Omega$

$R = 100\ K\Omega$ $R = 100K\Omega$

$R = 33\ K\Omega$

$R = 15\ K\Omega$
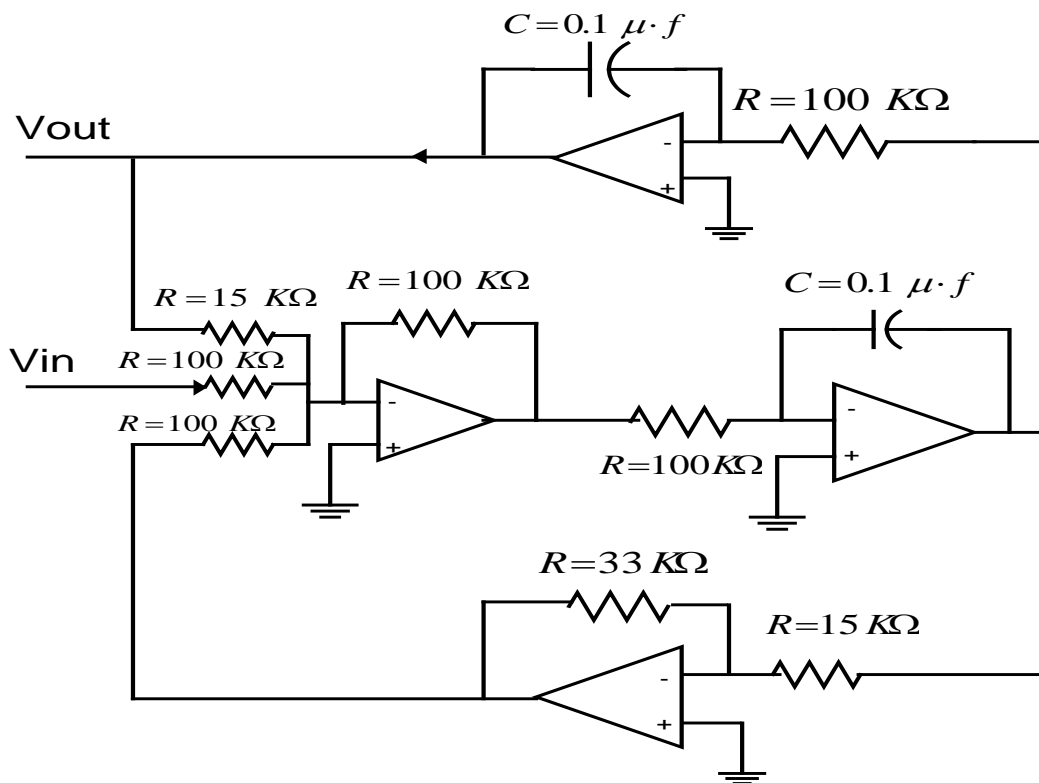
Fig. (3-4): practical circuit of the second order control system.

Procedure

1- Using Matlab and Simulink connect the circuit shown in figure (3-3), show the unit step response y (t) on personal computer for the values:

$$\omega_n = 2 \quad and \quad \xi = 0,\ 0.3\ ,\quad 0.7\ ,\ 1$$

Then sketch the system response.

2- Repeat step 1 for the values: $\xi = 0.7 \quad and \quad \omega_n = 0.5\ ,\ 1\ ,\ 4\ ,\ 10$

Discussion

1- For the values $\xi = 0.7 \quad and \quad \omega_n = 2$ find the rise time, peak time, settling time, and the maximum peak values.

2- Do you observe any relation between the system damping ratio and the system settling time.

3- According to your measurements suggest the relationship between the peak overshoot and the damping ratio.

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Experiment Four

State Space Representation

Object

To represent a modern complex systems using first order differential equation of state space analysis.

Theory

The modern trend in engineering control systems is toward greater complexity due to the requirements of complex tasks and good accuracy. Modern complex systems may have many inputs and many outputs linear or nonlinear systems, to analyze such systems it is essential to reduce the complexity of the mathematical expressions

State-space representation of differential equation used to analysis and design of multi-input multi-output, linear or non-linear, time-variant or time-invariant systems. This new approach is based on the concept of state; so we must define the state variable, state vector and state space.

State Variable: the state variable of a dynamic system are the smallest set of variables which determines the state of the dynamic system, if (n-variables) are needed to completely describe the behavior of the dynamic system then the variables $x_1(t)$, $x_2(t)$, ...$x_n(t)$ are a set of state variables.

State Vector: if (n-state variables) are needed to completely describe the behavior of a given system, then the (n-state variables) can be considered to be a component of a state vector X (t).

State Space: if the (n-dimensional space) whose coordinates axis consist of the axis x_1 $axis$, x_2 $axis$,.......x_n $axis$ is called a state space.

System design in modern control theory can:

•Enables the engineer to design an optimal control system.

•Design multiple-input and multiple-output systems, which may be linear or non-linear system.

•Enables the engineer to include initial conditions in the design.

For the ordinary differential equation of n-order we can find the state space representation by obtaining the algebraic equation and the output equation, which is a set of the first-order differential equation.

Consider the system defined by

$$y^n + a_1 y^{n-1} + ... + a_{n-1} y' + a_n y = u \qquad (1)$$

We can find the state space representation by assuming that

$$x_1 = y$$
$$x_2 = y'$$
$$. \quad . \quad .$$
$$x_n = y^{n-1}$$

Then the equation can be written as

$$x_1' = x_2$$
$$x_2' = x_3$$
$$. \quad . \quad .$$
$$x_{n-1}' = x_n$$
$$x_n' = -a_n x_1 - . \quad . \quad . - a_1 x_n + u$$

Then the algebraic equation of the first-order differential equation can be written as:

$$X^{'} = A\ x + B\ u \quad ..... \quad (2)$$

Where

$$X^{'} = \begin{bmatrix} x_1{'} \\ x_2{'} \\ . \\ . \\ . \\ x_n{'} \end{bmatrix} \quad and \quad A = \begin{bmatrix} 0 & 1 & 0. & . & .0 \\ 0 & 0 & 1. & . & .0 \\ . & . & . & & \\ 0 & 0 & 0. & . & .1 \\ -a_n & -a_{n-1} & -a_{n-2} & ...- & a_1 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix}$$

The output equation for the first order differential equation can be written as:

$$y = C\ x \quad ..... \quad (3)$$

Where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix}$$

$$and$$

$$C = \begin{bmatrix} 1 & 0. & . & .0 \end{bmatrix}$$

Consider the R-L-C circuit
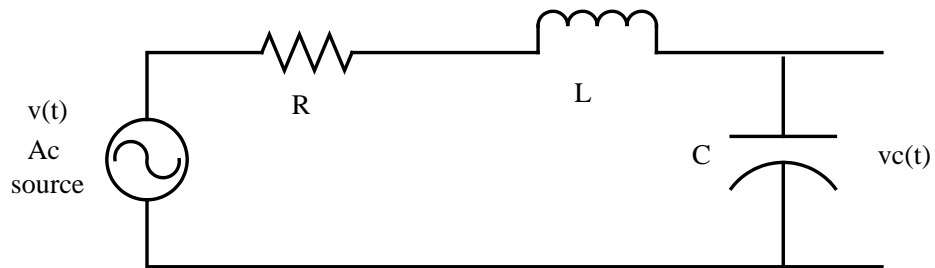
Fig. (4-1): R-L-C circuit.

In vector-matrix notation, we have:

$$C\frac{dvc}{dt} = i$$

$$L\frac{di}{dt} + R\ i + vc = v$$

$$\begin{bmatrix} i^\cdot \\ v^\cdot \end{bmatrix} = \begin{bmatrix} \dfrac{-R}{L} & \dfrac{-1}{L} \\ \\ \dfrac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i \\ vc \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ \\ 0 \end{bmatrix} \begin{bmatrix} v \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \\ vc \end{bmatrix}$$

Example: Consider the system defined by

$$\dddot{y} + 6\,\ddot{y} + 11\,\dot{y} + 6\,y = 6u$$

Where (y) is the output and (u) is the input of the system

Solution:

let

$x_1 = y$

$x_2 = y'$

$x_3 = y''$

$we\ can\ obtain$

$\dot{x_1} = x_2$

$\dot{x_2} = x_3$

$\dot{x_3} = -6\ x_1 - 11\ x_2 - 6\ x_3 + 6\ u$

The output equation and the state equation can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} [u]$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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By using Matlab and Simulink the transfer function of the system can perform to represent the system response on personal computer as shown in figure (4-2).

$$x^{'} = Ax + Bu$$
$$y = Cx$$

$$\frac{6}{s^{3} + 6 s^{2} + 11 \ s + 6}$$

input

m
u
x

scope
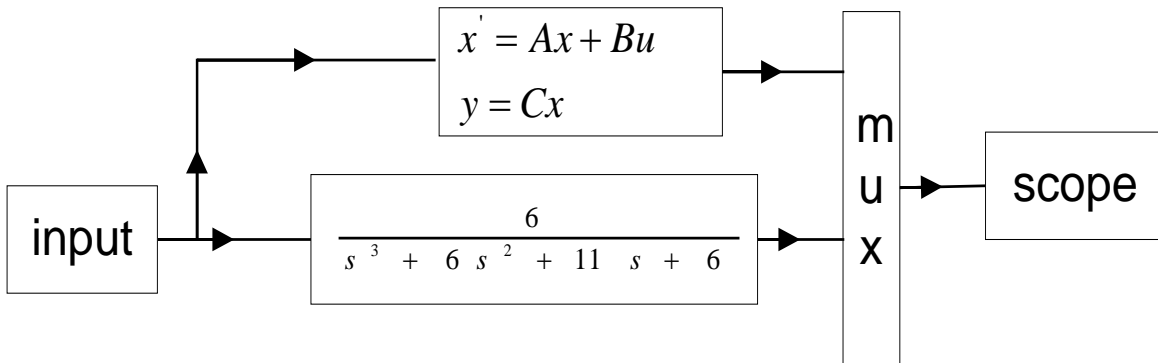
Fig. (4-2): Matlab and Simulink representation.

Procedure

    1. Connect the circuit of figure (4-2) and apply a unit step input signal.

    2. Plot the output system response $y_{(t)}$.

Discussion

    1. Derive the transfer function of the (R-L-R) circuit, and then find the state space representation of the circuit.

    2. Find the output $y_{(t)}$ for the system represented by transfer function shown in figure (4-2) analytically and compare with the simulated result. (Assuming all initial conditions is zero)

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Experiment Five
Steady State Error Analysis

Object

1-To show how to classify the steady state error according to the type of the system.

2-To show the ability of control system to follow step, ramp and parabolic inputs.

Theory

The Steady-State error in controlled system indicates the goodness of the controller Figure (5-1) shows the block diagram for closed loop control system.
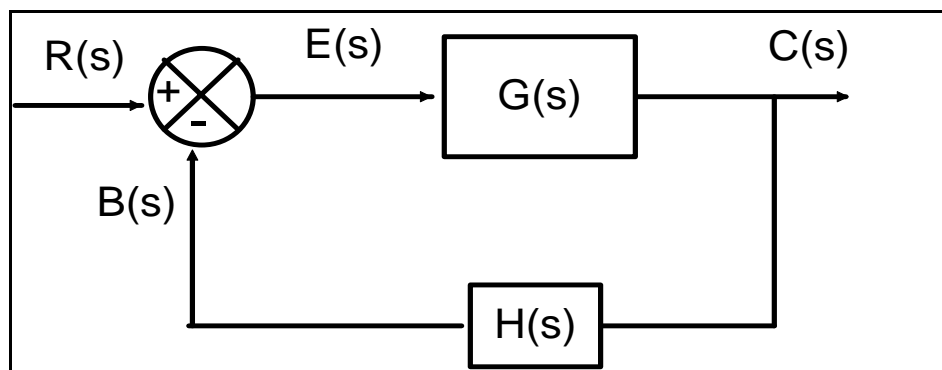
Fig. (5-1): Block Diagram for closed loop control system.

If the closed loop control system output signal C(s) does not follow the changes of input reference R(s) the error signal E(s) is indicated.

$$E(s) = R(s) - B(s) \qquad ... \qquad (1)$$

$$B(s) = C(s) \ H(s) \qquad ... \qquad (2)$$

Substituting for B(s) in (1) from (2) then

$$E(s) = R(s) - C(s) \ H(s) \qquad .... \qquad (3)$$

Where

$$C(s) = E(s) \ G(s) \qquad .... \qquad (4)$$

From (3) and (4) the transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\ H(s)} \qquad \dots \qquad (5)$$

Then we can find that

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)\ H(s)} \qquad \dots \qquad (6)$$

Table (5-1): shows the steady state error in terms of gain K.

	Step input R (t)=1	Ramp input R (t) =t	Acceleration R (t) =t^2
Type 0 system	1/ (1+k)	Infinity	Infinity
Type 1 system	0	1/Kv	Infinity
Type 2 system	0	0	1/Ka

The error coefficient Kp, Kv, Ka describes the ability of the system to reduce or eliminate the Steady-State error. Therefore they are indicative of steady state performance.

By using Matlab and Simulink the transfer function can performed to represent the system response to step input in personal computer as shown in figure (5-2) for type zero, figure (5-3) for type one and figure (5-4) for type two.
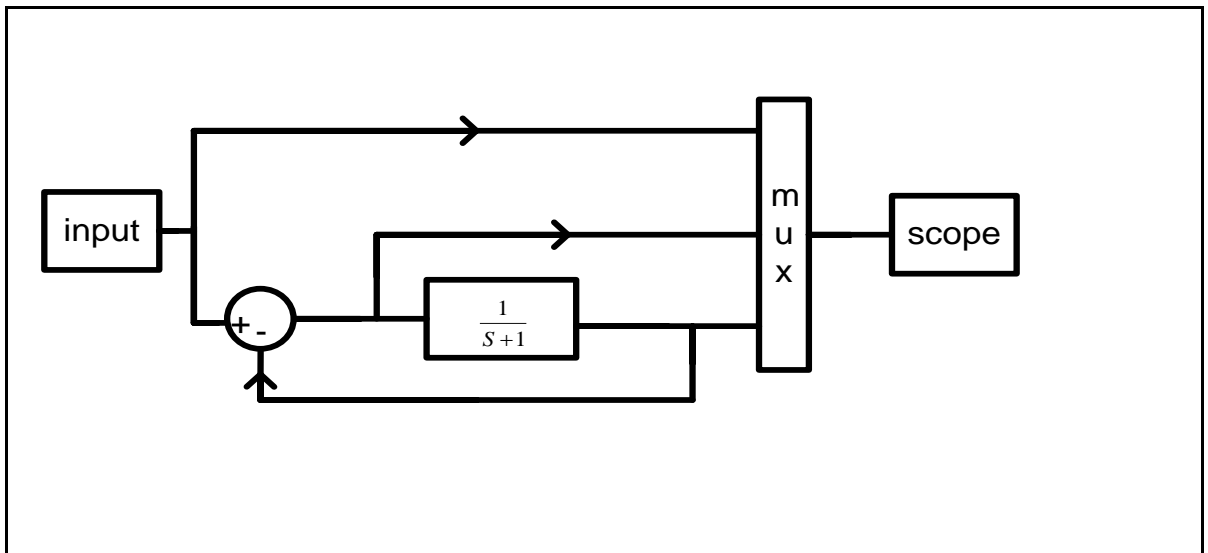
Fig. (5-2): Matlab and Simulink representation.
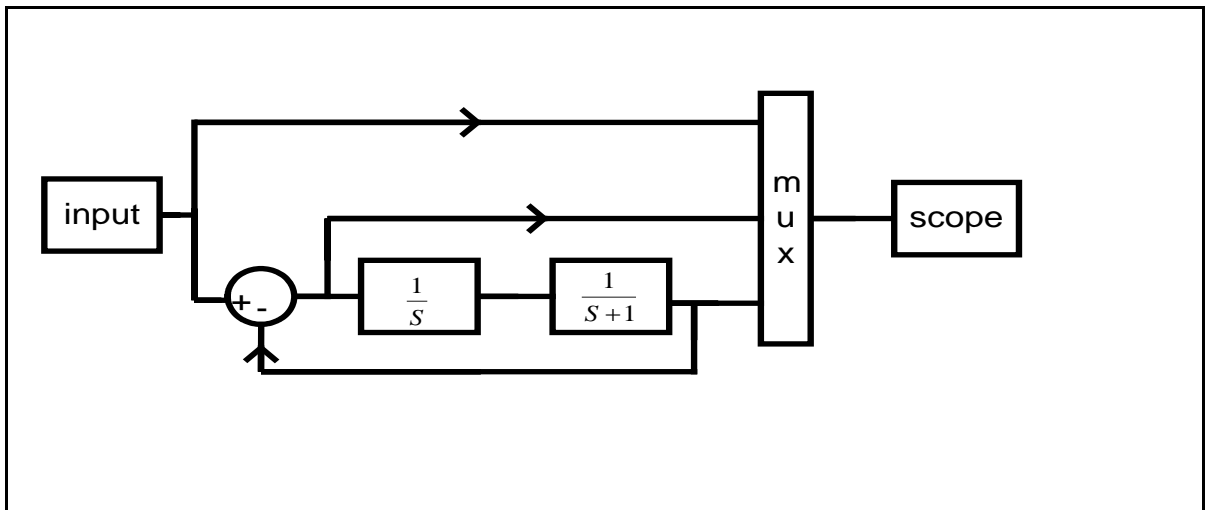
Fig. (5-3): Matlab and Simulink representation.
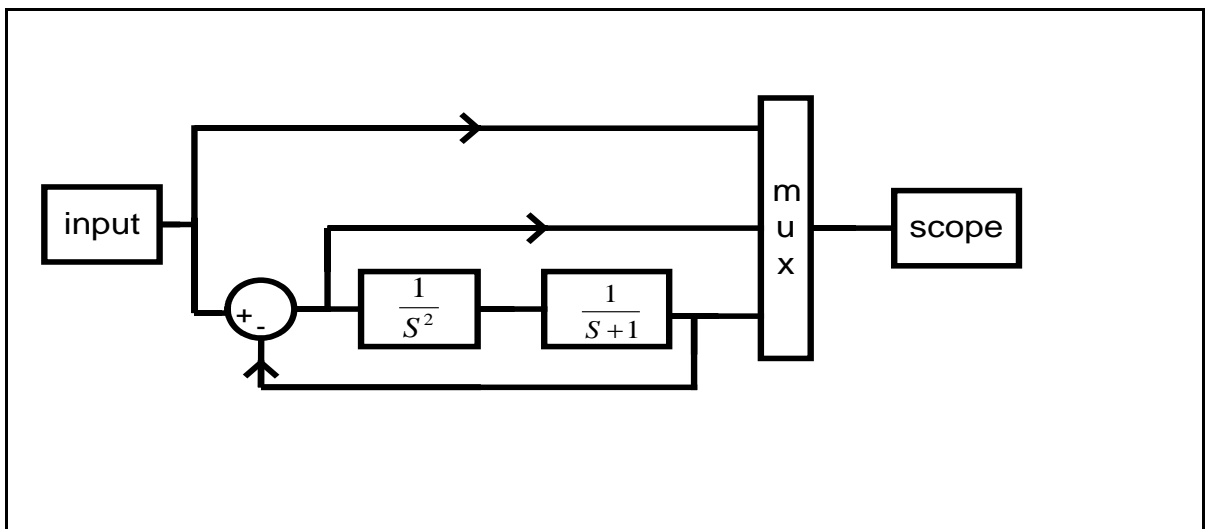
Fig. (5-4): Matlab and Simulink representation.

Procedure

1- For the block diagram of figures (5-2), (5-3) and (5-4) show the output response y (t) for a unit step input on personal computer using Matlab and Simulink.

2- Repeat step 1 for a ramp input.

Discussion

1. Derive the transfer function of figures (5-2), (5-3) and (5-4).

2. Find the steady state error using Laplace transform for a unit step, and ramp input.

3. Compare the results obtained with the simulated results.

Experiment Six

Root Locus plot

Object

1-To locate the closed loop poles in s-plane.

2-To investigate the closed loop system stability.

Theory

A root loci plot is simply a plot of the zeros and the poles values on a graph with real and imaginary coordinates. The root locus is a curve of the location of the poles of a transfer function as some parameter is varied.

Such a plot shows clearly the contribution of each open loop pole or zero to the locations of the closed loop poles. This method is very powerful graphical technique for investigating the effects of the variation of a system parameter on the locations of the closed loop poles. The closed loop poles are the roots of the characteristic equation of the system while the locus of the roots as the gain varies from zero to infinity.

From the design viewpoint, in some systems simple gain adjustment can move the closed loop poles to the desired locations. Root loci are completed to select the best parameter value for stability. A normal interpretation of improving stability is when the real part of a pole is further left of the imaginary axis.

A control system is often developed into an equation as shown below

$$\frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)} = F(s) \qquad (1)$$

$$F(s) = \frac{K \cdot (s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2)(s - p_3) \cdots (s - p_n)} \qquad (2)$$

$$N(s) = (s - z_1)(s - z_2) \cdots (s - z_m) \qquad (3)$$
$$D(s) = (s - p_1)(s - p_2)(s - p_3) \cdots (s - p_n) \qquad (4)$$

where $\qquad n > m$

$F(s) = 0 \qquad$ *when* $\qquad s = z_1, z_2 \cdots z_m$

$F(s) = \infty \qquad$ *when* $\qquad s = p_1, p_2, p_3 \cdots p_n$

A typical feedback system is shown in figure (6-1).

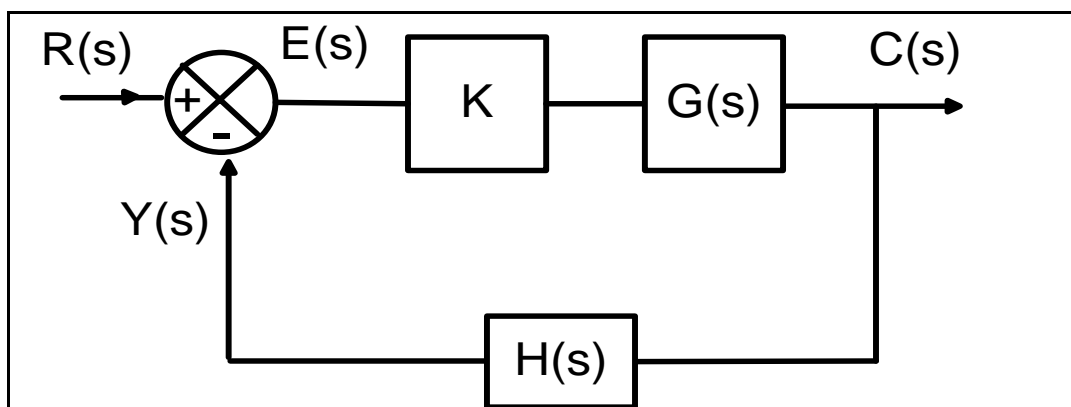Fig. (6-1): block diagram of feedback control system.

The open-loop transfer function between the input R(s) and the measured output Y(s) is:

$$T(s) = K \cdot G(s)H(s) \qquad \ldots \qquad (5)$$

The closed-loop transfer function is:

$$\frac{C(s)}{R(s)} = \frac{K \cdot G(s)}{1 + K \cdot G(s)H(s)} \qquad \ldots \qquad (6)$$

if $\qquad \dfrac{C(s)}{R(s)} = \infty \qquad$ *then* $\qquad K \cdot G(S)\ H(S) = -1$

It can be written as:

$$|K| = \frac{1}{|G(s)H(s)|} \qquad \dots \qquad (7)$$

$$\theta = \angle KG(s)H(s) = odd \ multiples \ of \ 180°\dots \quad (8)$$

The complex numbers in polar form have the following properties:

$$|Z_1 \cdot Z_2| = |Z_1| \cdot |Z_2| \qquad and \qquad \left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$$

$$\angle(Z_1 \cdot Z_2) = \angle Z_1 + \angle Z_2 \qquad and \qquad \angle \left[\frac{Z_1}{Z_2}\right] = \angle Z_1 - \angle Z_2$$

Example 1

For a closed loop system

$$G(s) = \frac{1}{s(s+2)} \qquad , \qquad H(s) = 1$$

The root locus plot can be obtained from the closed loop characteristic equation:

$$s^2 + 2s + K = 0$$

The root of the characteristic equation:

$$s = -1 \pm \sqrt{1-K}$$

$$when \ K = 0 \quad the \ poles \ at \quad s = 0 \ , \ s = -2$$

$$when \ K = 1 \quad the \ poles \ at \quad s = -1$$

$$when \ K = \infty \qquad s = -1 \pm j\infty$$

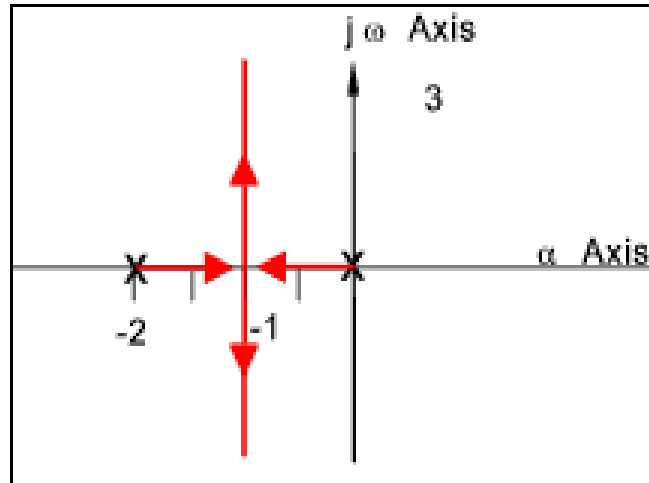The relevant root locus is as shown in figure (6-2).

Fig.(6-2): The relevant root locus.

The system is stable for all values of $K \geq 0$.

Example 2

For a closed loop system

$$G(s) = \frac{2s}{s^2 + 1} \quad , \quad H(s) = 1$$

The root locus plot can be obtained from the closed loop characteristic equation:

$$s^2 + 2K\ s + 1 = 0$$

The root of the characteristic equation:

$$s = -K \pm \sqrt{K^2 - 1}$$

$$when \ K = 0 \quad the \ poles \ at \quad s = +j \ , \ s = -j$$

$$when \ K = 1 \quad the \ poles \ at \quad s = -1$$

$$when \ K = \infty \quad s = -\infty \quad s = 0$$

The relevant root locus is as shown in figure (6-3)
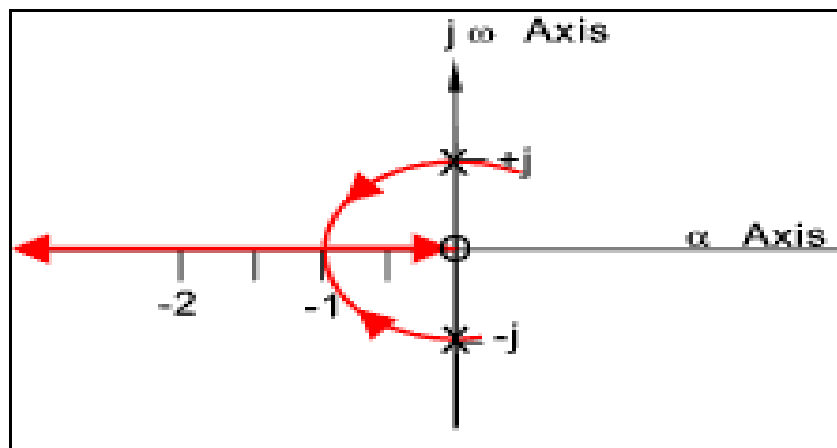

Fig.(6-3): The relevant root locus.

The system has the best stability point at K = 1, at values below this root loci moves towards the instability boundary.

Procedure

Using MATLAB and SIMULINK to draw the root locus diagram for the following transfer function:

$$G(s) = \frac{s + 2}{(3s + 1)(2s + 1)}$$

Use the instruction:

$$num = [1 \quad 2] \; ;$$

$$den = conv \; (\; [3 \quad 1], [2 \quad 1] \;) \; ;$$

$$rlocus(num, den);$$

Discussion

1. What is the type and order of the system?

2. From the simulated result obtained is the system stable.

3. Draw the theoretical (asymptotic only) polar plot of the open loop system and compare it with the simulated result.

4- What are the advantages of root locus method?

Experiment Seven

Bode Plot

Object

1-To study the frequency response analysis by using bode plot algorithm.

2-To investigate the closed loop system stability using open loop transfer function.

Theory

The frequency response is the steady state response of a system to a sinusoidal input; the category is the plot of the magnitude of the output in db versus frequency using logarithmic scale, and the phase shift versus frequency as in figure (7-1).
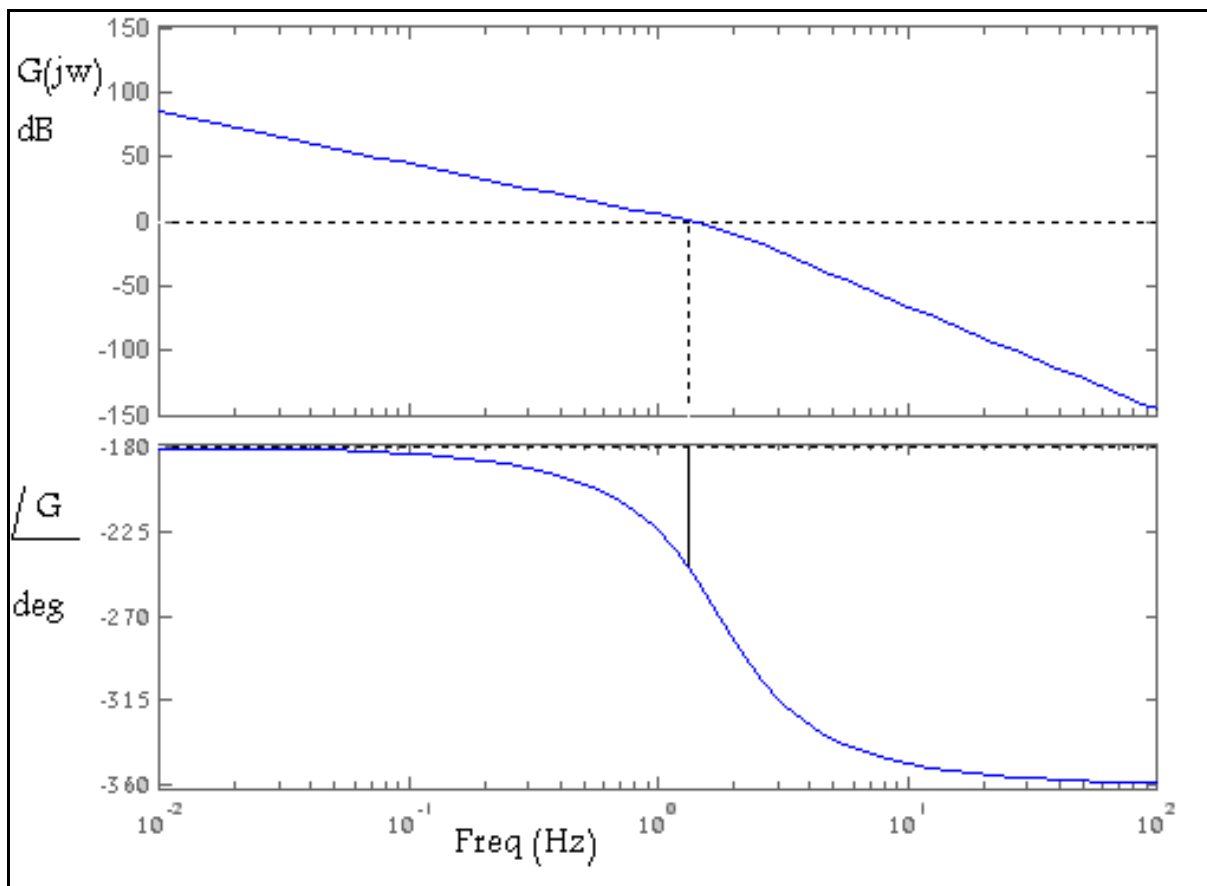

Fiq. (7-1): Bode plot

The frequency response characteristics of the system can be obtained directly from the sinusoidal transfer function in which "s" is replaced by "jw", where "w" is the frequency.

Consider the linear time invariant system shown in figure (7-2) where the transfer function is G(s) and the input is a sinusoidal and it is given by x (t) and the output is y (t) :
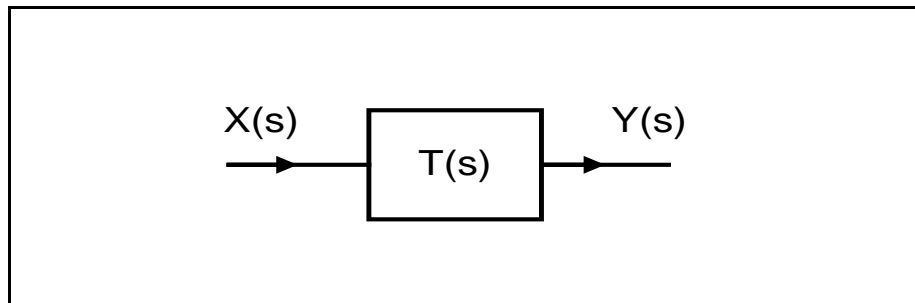

Fig. (7-2): linear time invariant system.

$$i / p = X \ \sin(\omega t)$$

$$o / p = y \ \sin(\omega t + \phi)$$

Shows G (jw) is complex quantity and it can be written as:

$$G(j\omega) = |G(j\omega)| \ e^{j\phi}$$

$$| \ G(jw) \ | = 20\log \sqrt{(\text{Re})^2 + (\text{Im})^2}$$

$$\phi = \angle G(j\omega) = \tan^{-1}[\frac{imaginary \ part \ of \ G(j\omega)}{real \ part \ of \ G(j\omega)}]$$

$$y(t) = X \ |G(j\omega)| \ \frac{e^{j(wt+\phi)} - e^{-j(wt+\phi)}}{2j}$$

$$y(t) = X \ |G(j\omega)| \ \sin \ (\omega t + \phi)$$

The amplitude and phase shift ratio of the output to the input is given by:

$$|G(j\omega)| = \left|\frac{y(j\omega)}{X(j\omega)}\right|$$

$$\angle G(j\omega) = \angle \frac{y(j\omega)}{X(j\omega)}$$

To completely characterize a linear system in the frequency domain we must specify both the amplitude ratio and the phase angle as function of frequency.

Example

$$G(s) = \frac{10(s+4)}{(s)(s+2)(s+5)(s+10)}$$

For the open loop transfer function:

Find the closed loop frequency response.

Solution:

Replace (s) by (jw)

$$G(jw) = \frac{10(jw+4)}{(jw)(jw+2)(jw+5)(jw+10)}$$

Find the overall gain

$$G(jw) = \frac{0.4\left(\dfrac{jw}{4}+1\right)}{(jw)\left(\dfrac{jw}{2}+1\right)\left(\dfrac{jw}{5}+1\right)\left(\dfrac{jw}{10}+1\right)}$$

Then plot of the magnitude of the output versus frequency and the phase shift versus frequency using semi-log paper.

$$G(jw) = 20\log 0.4 + 20\log\left|\frac{jw}{4} + 1\right| - 20\log|jw| - 20\log\left|\frac{jw}{2} + 1\right| - 20\log\left|\frac{jw}{5} + 1\right| - 20\log\left|\frac{jw}{10} + 1\right|$$

$$\angle G(jw) = 0 + \tan^{-1}\frac{w}{4} - 90 - \tan^{-1}\frac{w}{2} - \tan^{-1}\frac{w}{5} - \tan^{-1}\frac{w}{10}$$

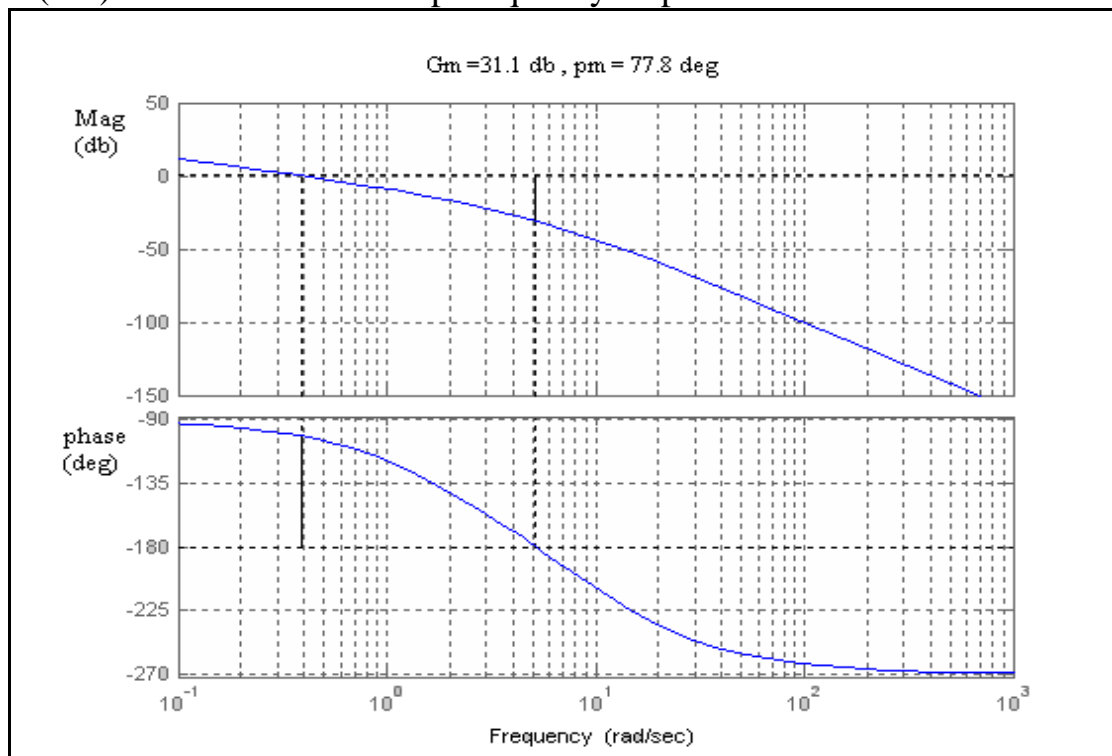Figure (7-3) shows the closed loop frequency response.

Fig. (7-3): closed loop frequency response.

Example

Find the steady state response for the system where the open loop transfer function is given by:

$$G(s) = \frac{64}{(s^2 + 8s + 64)}$$

$$\frac{64}{(s^2 + 8s + 64)} = \frac{1}{1 + 2\zeta\left(j\frac{w}{w_n}\right) + \left(j\frac{w}{w_n}\right)^2}$$

$$M_r = \frac{1}{\sqrt{\left(1 - \frac{w^2}{w_n^2}\right)^2 + \left(2\zeta\frac{w}{w_n}\right)^2}} \qquad ; w_r = w_n\sqrt{1 - 2\zeta^2}$$

$$\phi = -\tan^{-1}\frac{2\,\xi\,\dfrac{\omega}{\omega_n}}{1 - \dfrac{\omega^2}{\omega_n^2}}$$

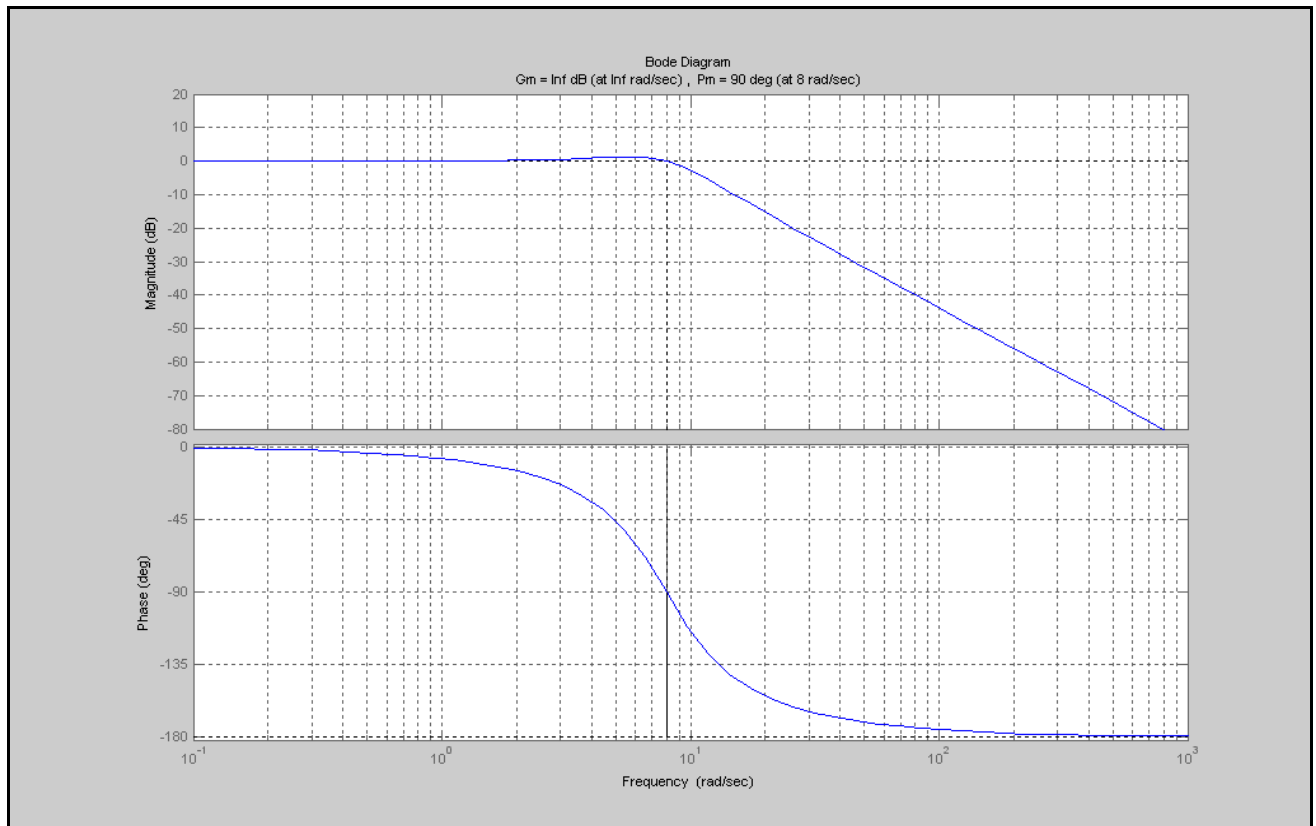Figure (7-4) shows the closed loop frequency response.

Fig. (7-4) closed loop frequency response.

Procedure

Using Mat lab and Semolina to draw the bode diagram for the following transfer function:

$$G(s)H(s) = \frac{s+2}{s(s+1)(s^2 + 6s + 8)}$$

Use the instruction:

$num = [1 \quad 2] \ ;$

$den = conv \ (\ [1 \quad 1 \quad 0], [1 \quad 6 \quad 8] \ ) \ ;$

$bode(num, den);$

Discussion

1. From the values of G.M. and the P.M. Is the system stable?
2. Draw the theoretical (asymptotic only) bode plot of the open loop system and compare it with the simulated result.
3. What are the advantages of frequency response method?

Experiment Eight
Effects of the Addition of Poles and Zeros
on the Root-Locus plot

Object :

To study the adding effects of the poles and zeros on the resultant output root-locus shaping .and to study their effects on the time response.

Theory :

The root-locus method is a graphical method for determining the locations of all closed-loop poles from knowledge of the locations of the open-loop poles and zeros as some parameter (usually the gain) is varied from zero to infinity. The method yields a clear indication of the effects of parameter adjustment.

In practice, the root-locus plot of a system may indicate that the desired performance cannot be achieved just by the adjustment of gain. In fact, in some cases, the system may not be stable for all values of gain. Then it is necessary to reshape the root loci to meet the performance specifications.

In designing a control system, if other than a gain adjustment is required, we must modify the original root loci by inserting a suitable compensator. Once the effects on the root locus of the addition of poles and/or zeros are fully understood, we can readily determine the locations of the pole(s) and zero(s) of the compensator that will reshape the root locus as desired. In essence, in the design by the root-locus method, the root loci of the system are reshaped through the use of a compensator so that a pair of dominant closed-loop poles can be placed at the desired location. (Often, the damping ratio and un - damped natural frequency of a pair of dominant closed-loop poles are specified).

Effects of the Addition of Poles:

The addition of a pole to the open-loop transfer function has the effect *of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response* Figure (8-1) shows examples of root loci illustrating the effects of the addition of a pole to a single-pole system and the addition of two poles to a single-pole system.
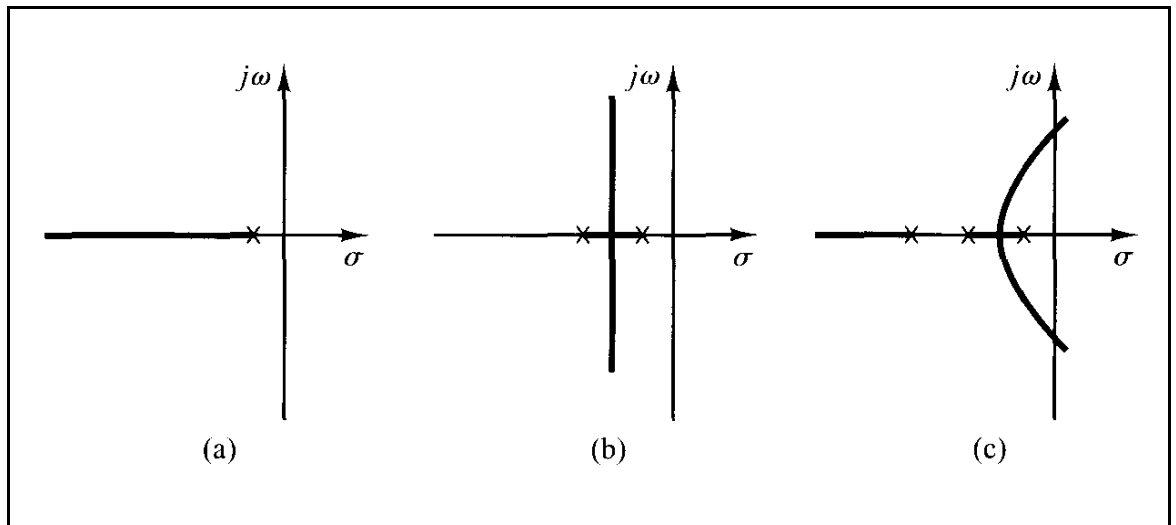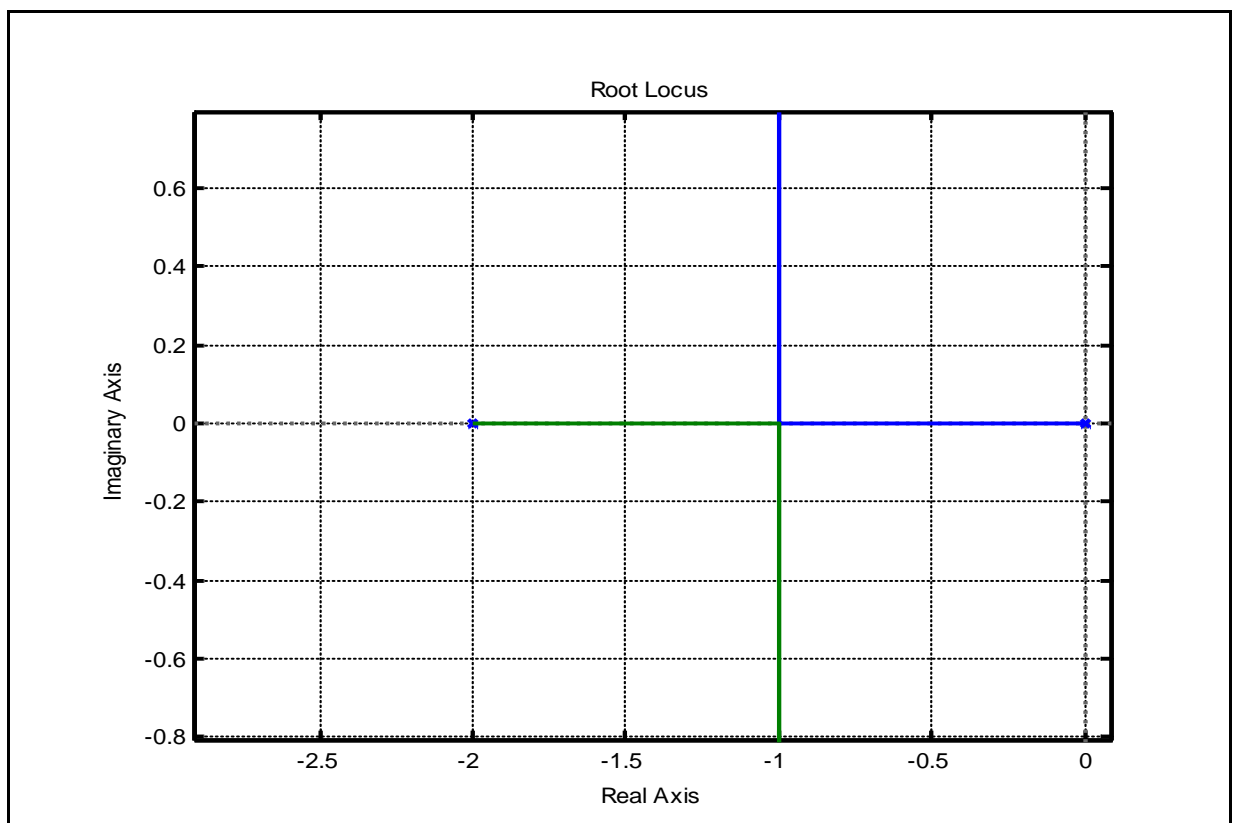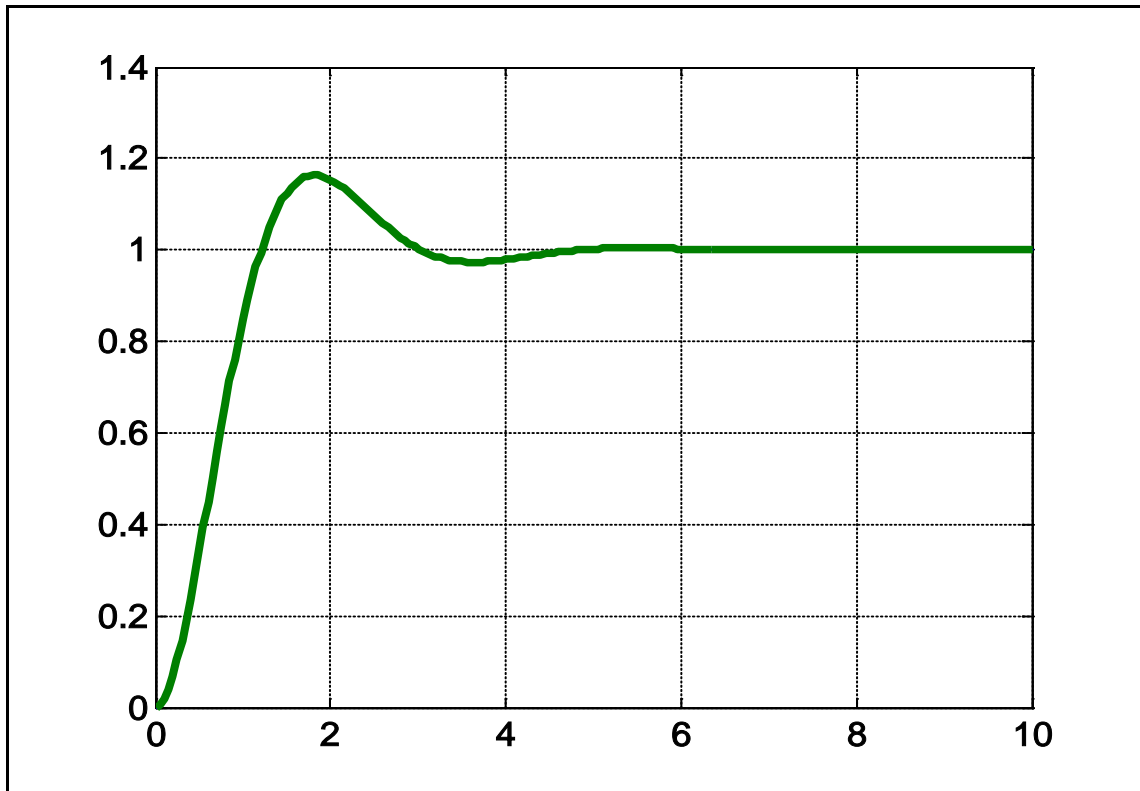
Figure (8-1)

Procedure:

1. For the open loop transfer function $G(s) = 4 / s (s + 2)$, plot the root locus by using the following program:

```
clear
num=[4];
den=[1 2 0];
rlocus(num,den);
```

2. And then, we have to see output response (closed loop system with unity feedback) for the unit step with using the following program :

```
clear
num=[4];
den=[1 2 4];
t=0:0.05:20;
c=step(num,den,t);
plot(t,c);
grid on;
```


3. Repeat steps (1 and 2) with adding a single real pole s = - 10 .

4. Repeat steps (1 and 2) with adding a single real pole s = - 6

5. Repeat steps (1 and 2) with adding a single real pole s = - 1

6. Compare the resultant shapes of the root locus with the actual system

   on the same graphical paper.

Discussion:

- Discuss the effect of adding a pole on the root locus shape, through the relative stability.

- Discuss the effect of adding a pole on time response, through the speed

- Response, overshoot ….etc**.**

Effects of the Addition of Zeros:

      The addition of a zero to the open-loop transfer function has the effect *of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response.* (Physically, the addition of a zero in the feed-forward transfer function means the addition of derivative control to the system. The effect of such control is to introduce a degree of anticipation into the system and speed up the transient response.)

      Figure (8-2)(a) shows the root loci for a system that is stable for small gain but unstable for large gain .

      Figures (8-2)(b), (c), and (d) show root-locus plots for the system when a zero is added to the open-loop transfer function. Notice that when a zero is added to the system of Figure (8-2)(a), it becomes stable for all values of gain.
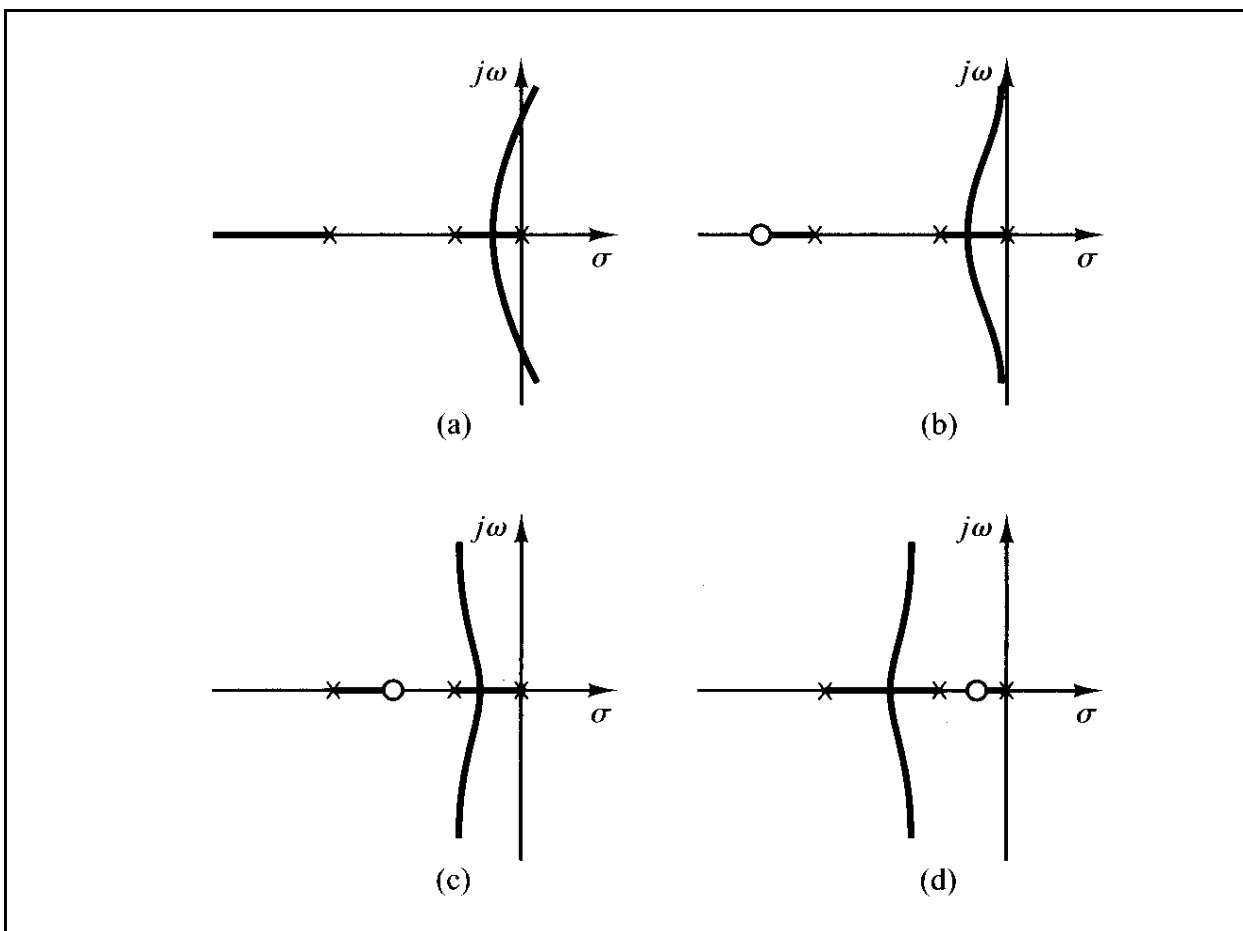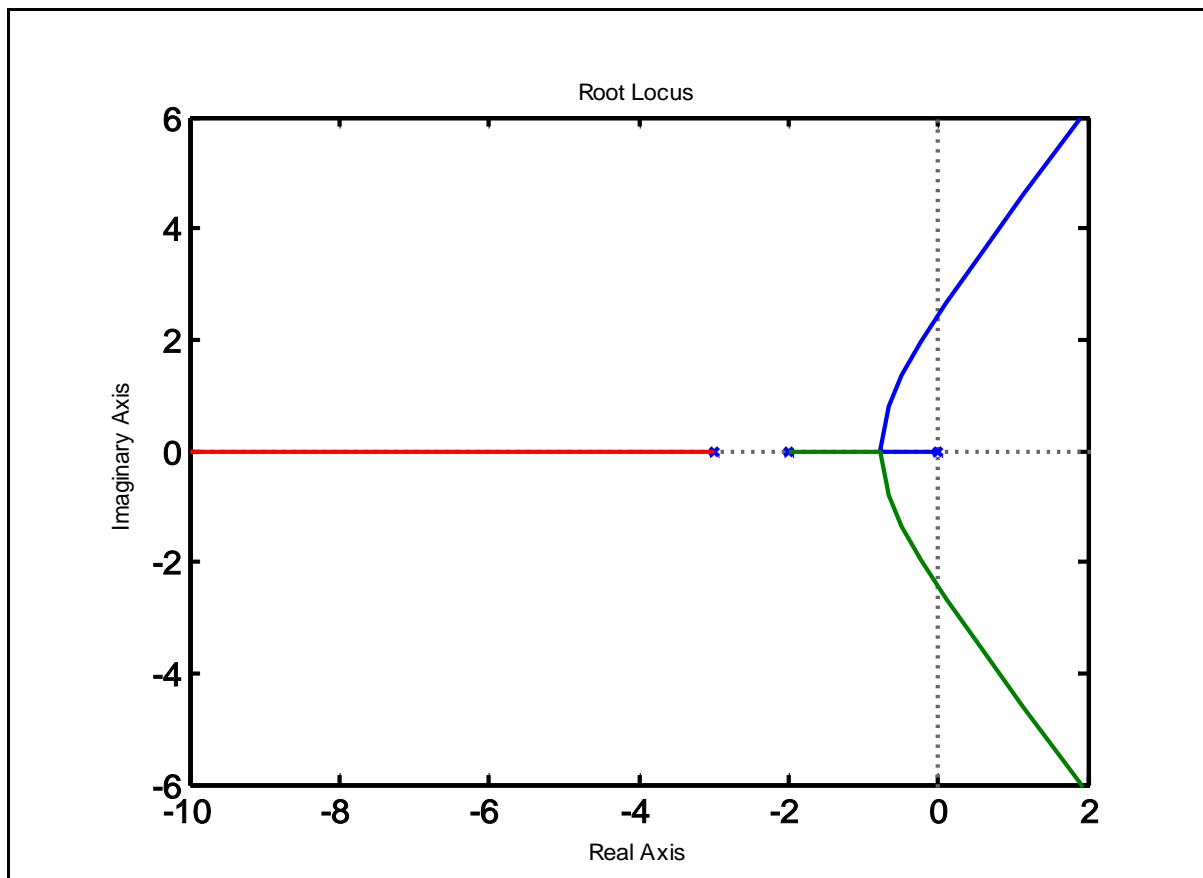

Figure (8-2)

(a) Root-locus plot of a three-pole system; (b), (c), and (d) root-locus plots showing effects of addition of a zero to the three-pole system.

Procedure:

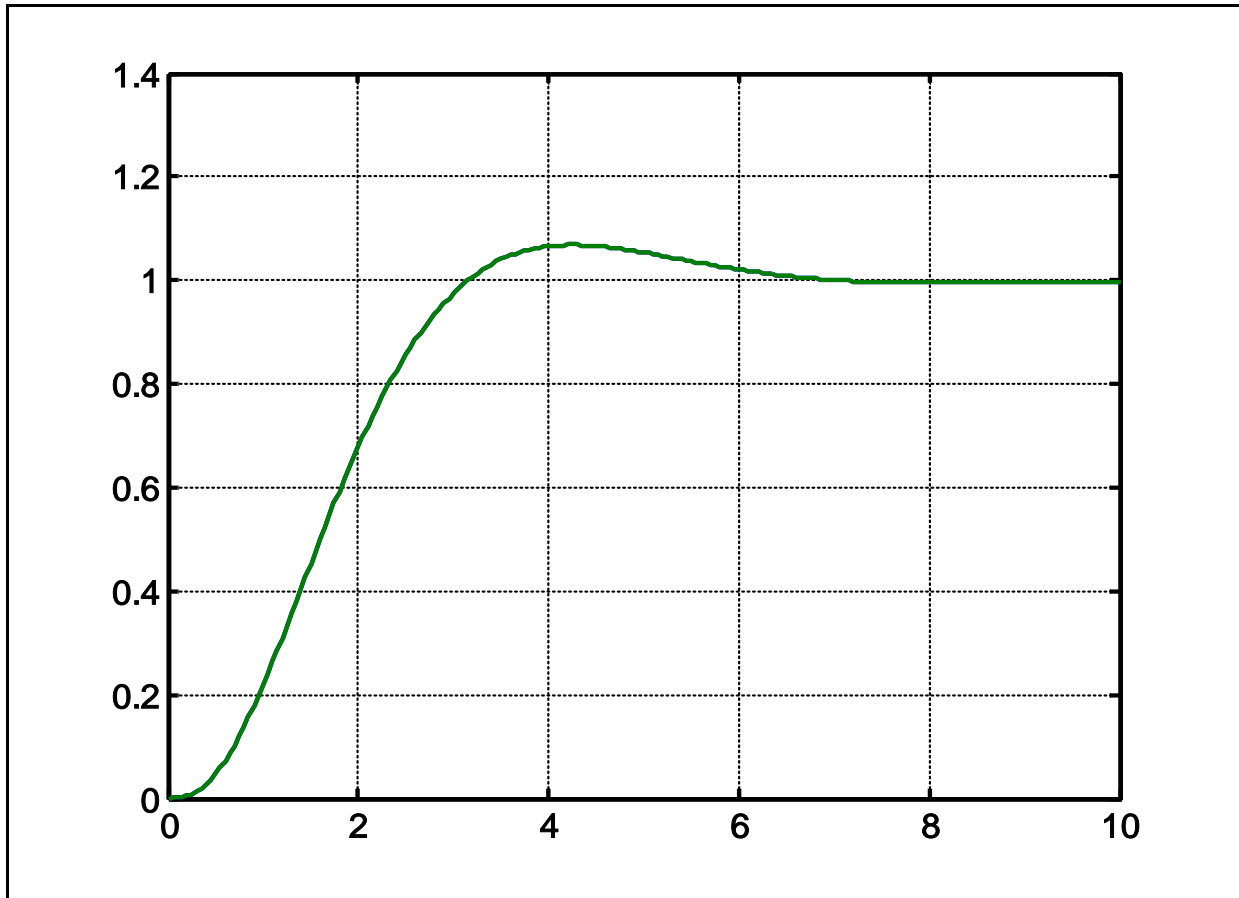1. For the open loop transfer function G(s) = 4 / s (s^2 + 5*s + 6) , plot

   the root locus by using the following program :

```
clear
num=[4];
den=[1 5 6 0];
rlocus(num,den);
```


2. And then, we have to see output response (closed loop system with

   unity feedback) for the unit step with using the following program

```
clear
num=[4];
den=[1 5 6 4];
t=0:0.05:20;
c=step(num,den,t);
plot(t,c);
grid on;
```

3 . Repeat steps (1 and 2) with adding a single real zero s = - 4 .

4 . Repeat steps (1 and 2) with adding a single real zero s = -2.5

5 . Repeat steps (1 and 2) with adding a single real zero s = -1 .

6 . Compare the resultant shapes of the root locus with the actual system on the same graphical paper.

Discussion:

1. Discuss the effect of adding a zero on the root locus shape, through the relative stability.

2. Discuss the effect of adding a zero on time response, through the speed response , overshoot ….etc**.**

Experiment Nine

Lead Compensation Techniques Based on
the Root-Locus Approach

Object:

To study and design the Lead Compensator using Root-Locus.

Theory:

The root-locus approach to design is very powerful when the specifications are given in terms of time-domain quantities, such as the damping ratio and un damped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time, and settling time.

Consider a design problem in which the original system either is unstable for all values of gain or is stable but has undesirable transient-response characteristics. In such a case, the reshaping of the root locus is necessary in the broad neighborhood of the jω axis and the origin in order that the dominant closed-loop poles be at desired locations in the complex plane. This problem may be solved by inserting an appropriate lead compensator in cascade with the feed forward transfer function shown in figure (9-1).
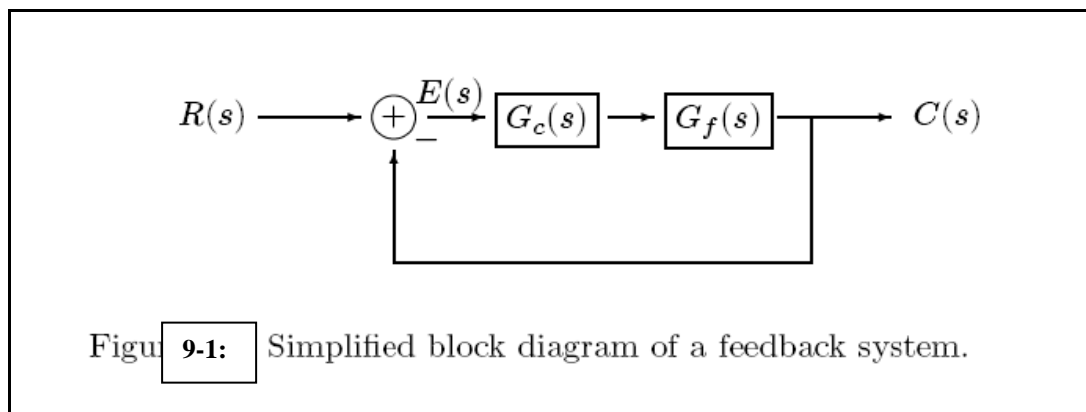

Figure **9-1:** Simplified block diagram of a feedback system.

Once, a compensator has been designed, check to see whether all performance specifications have been met. If the compensated system does not meet the performance specifications, then repeat the design procedure by adjusting the compensator pole and zero until all such specifications are met.

Lead Compensators:

There are many ways to realize continuous-time (or analog) lead compensators, such as electronic networks using operational amplifiers and electrical RC networks; Figure (9-2) shows an electronic circuit using operational amplifiers.
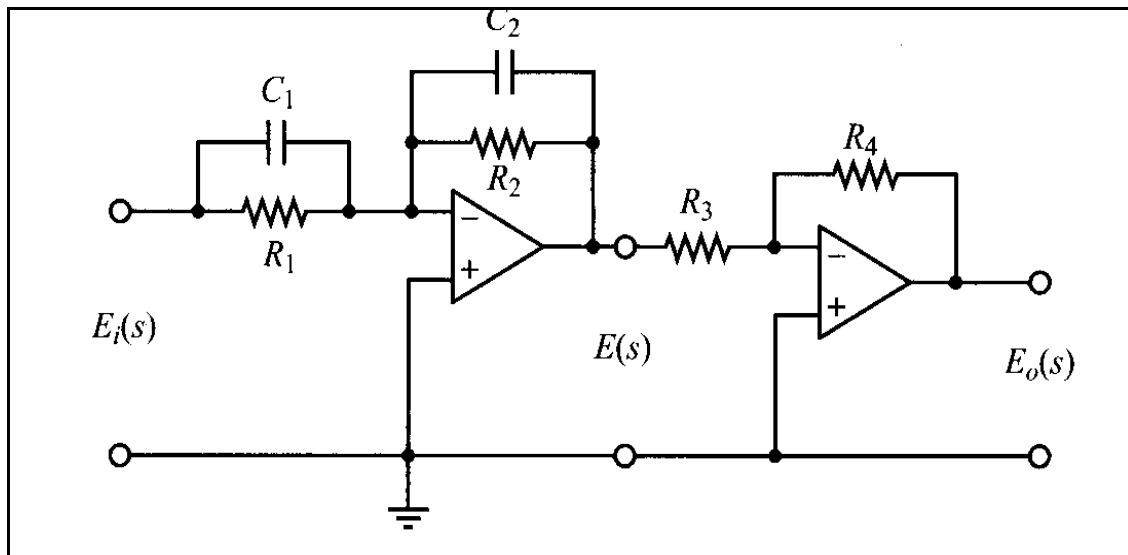

Figure (9-2)

The transfer function for this circuit was obtained as follows:

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4}{R_1 R_3} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = \frac{R_4 C_1}{R_3 C_2} \frac{s + \dfrac{1}{R_1 C_1}}{s + \dfrac{1}{R_2 C_2}}$$

$$= K_c \alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\alpha T}} \qquad (1-1)$$

where

$$T = R_1 C_1, \qquad \alpha T = R_2 C_2, \qquad K_c = \frac{R_4 C_1}{R_3 C_2}$$

Notice that

$$K_c \alpha = \frac{R_4 C_1}{R_3 C_2} \frac{R_2 C_2}{R_1 C_1} = \frac{R_2 R_4}{R_1 R_3}, \qquad \alpha = \frac{R_2 C_2}{R_1 C_1}$$

This network has a gain of $K_c \, \alpha = R_2 \, R_4 \, / \, (\, R_1 \, R_3 \,)$

From Equation (1-1), we see that this network

is a lead network   if $R_1 \, C_1 > R_2 \, C_2$ or $\alpha < 1$

is a lag network   if $R_1 \, C_1 < R_2 \, C_2$

The pole-zero configurations of this net work when $R_1 \, C_1 > R_2 \, C_2$
and $R_1 \, C_1 < R_2 \, C_2$ are shown in Figure (9-3)(a) and (b), respectively.

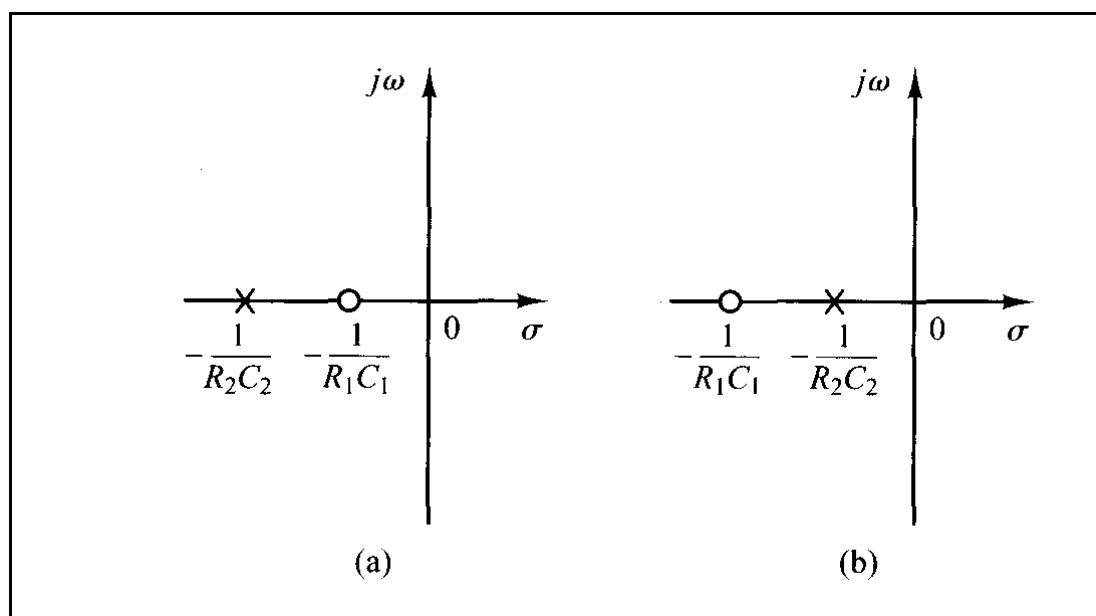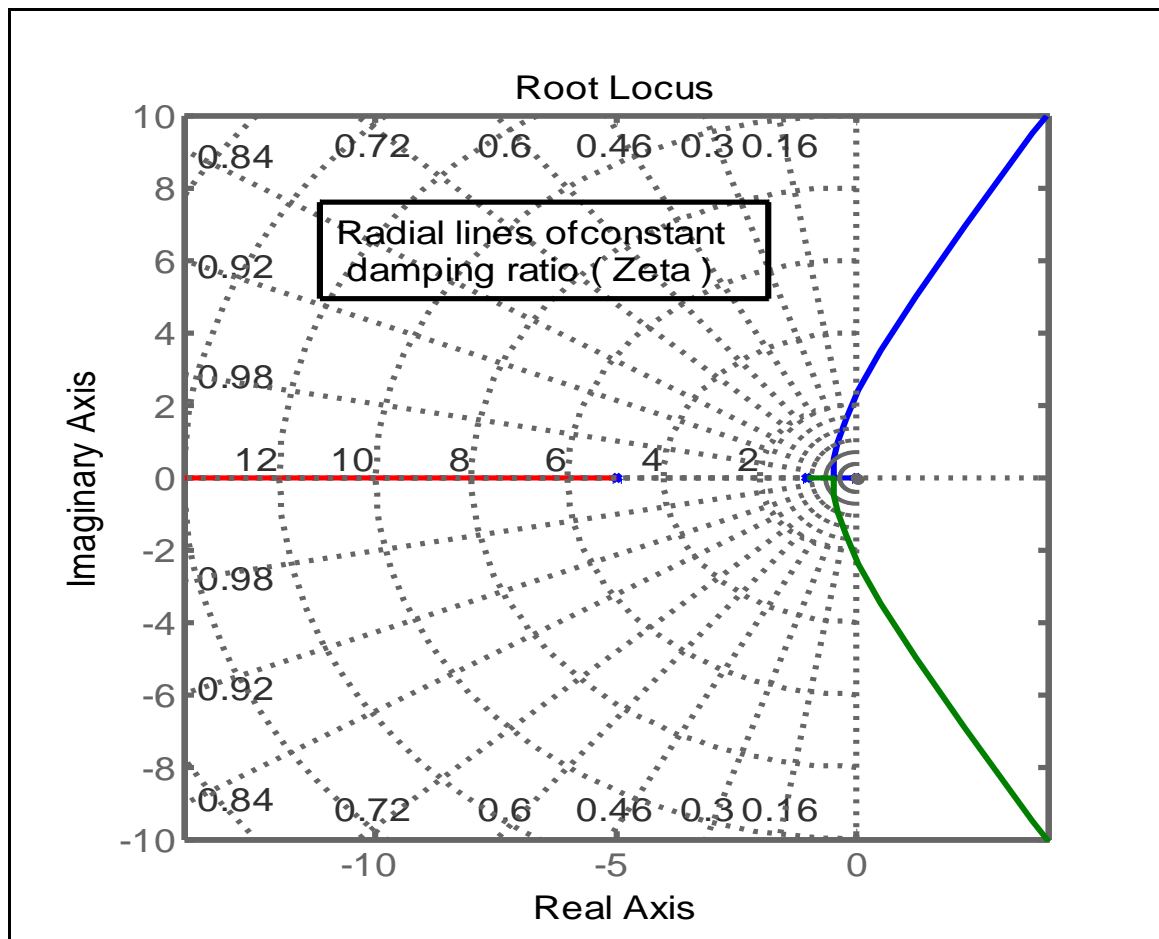(a)   (b)

Figure (9-3)

Procedure:

    1. For the original system below, find the root locus with grid property :

$$G_x(s) = \frac{K_1}{s(1 + s)(1 + 0.2s)} = \frac{K}{s(s + 1)(s + 5)}$$

Root Locus

2. Use the transfer function of the lead compensator

$$G_c(s) \;=\; K_c \alpha \, \frac{Ts \,+\, 1}{\alpha Ts \,+\, 1} \;=\; K_c \frac{s \,+\, \dfrac{1}{T}}{s \,+\, \dfrac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

using a value $\alpha \;=\; 0.1$.

3. For three selected values ($T \;=\; 4\,/\,3$, $T = 1$, $T = 2\,/\,3$) of zero position ($s = -1/T$), a three compensator will be made ,Plot the root locus for each compensated open loop system (using grid property of the root locus) .

4. Let the required damping ratio($\zeta = 0.45$) , find the dominant complex roots, natural frequency, settling time and maximum overshoot by using direct searching on root locus plots with using the following table:

Compensator	Dominant complex roots	Open Loop gain K'	Undamped freq, ω_n (rad/sec)	Mp %	for 2% t_s (sec.)
Uncompensated					
Zero at T=4/3 $\dfrac{s+0.75}{s+7.5}$					
Zero at T=1 $\dfrac{s+1}{s+10}$					
Zero at T=2/3 $\dfrac{s+1.5}{s+15}$					

5. Simulate (using Simulink) the time response (unit step input) for the uncompensated system and the three cases of lead compensation on the one screen of the oscilloscope, using open loop gain for each case.

Discussion:

1. For which purpose the lead compensator is used.
2. Discuss the effect of zeros position on the shapes of the root locus.
3. Discuss the effect of zeros position on the time responses.

Experiment Ten

Lag Compensation Techniques Based on
the Root-Locus Approach

Object:

   To study and design the Lag Compensator using Root-Locus.

Theory:

   Consider the problem of finding a suitable compensation network for the case where the system exhibits satisfactory transient-response characteristics but unsatisfactory steady-state characteristics. Compensation in this case essentially consists of increasing the open- loop gain without appreciably changing the transient-response characteristics. ***This means that the root locus in the neighborhood of the dominant closed-loop poles should not be changed appreciably , but the open-loop gain should be increased as much as needed***. This can be accomplished if a lag compensator is put in cascade with the given feedforward transfer function as shown in Figure (10-1) .
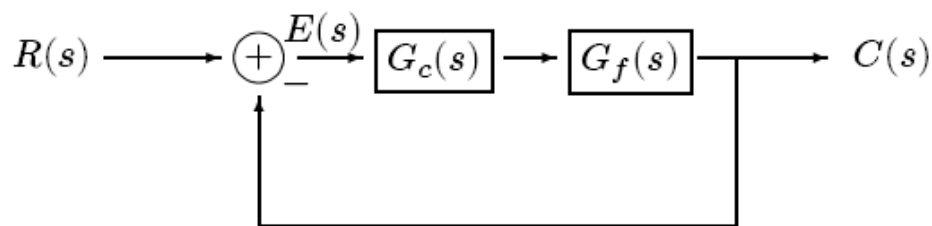

Figure 10-1 Simplified block diagram of a feedback system.

   Electronic Lag Compensator Using Operational Amplifiers. The configuration of the electronic lag compensator using operational amplifiers is the same as that for the lead compensator as shown in Figure (10-2).
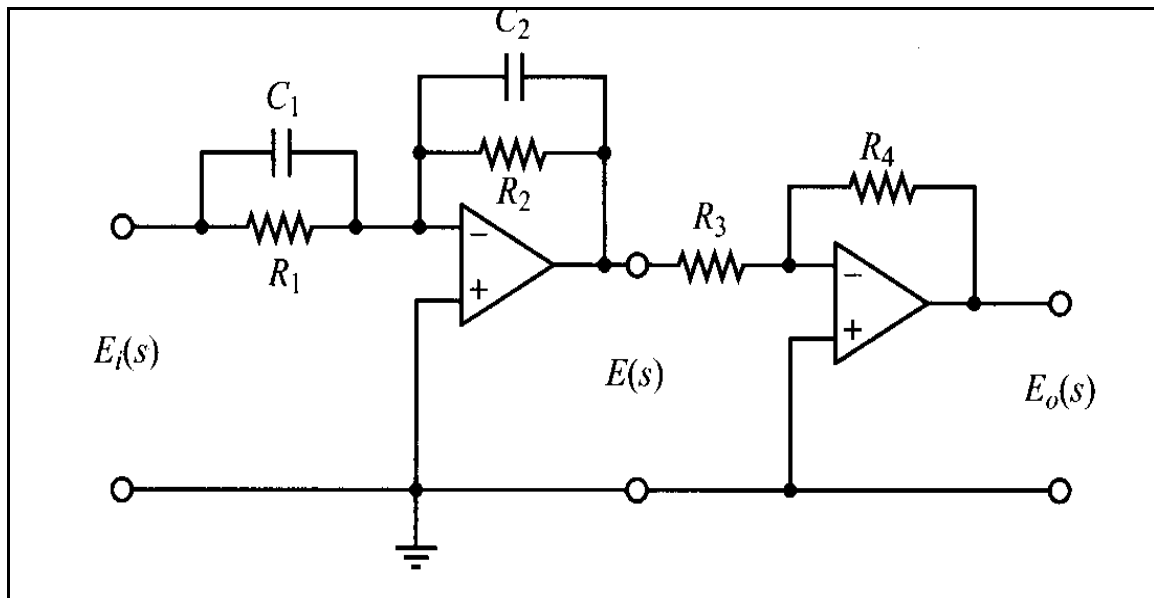
Figure (10-2)

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 R_4}{R_1 R_3} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = \frac{R_4 C_1}{R_3 C_2} \frac{s + \dfrac{1}{R_1 C_1}}{s + \dfrac{1}{R_2 C_2}}$$

$$= K_c \alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\alpha T}}$$

where

$$T = R_1 C_1, \qquad \alpha T = R_2 C_2, \qquad K_c = \frac{R_4 C_1}{R_3 C_2}$$

Notice that

$$K_c \alpha = \frac{R_4 C_1}{R_3 C_2} \frac{R_2 C_2}{R_1 C_1} = \frac{R_2 R_4}{R_1 R_3}, \qquad \alpha = \frac{R_2 C_2}{R_1 C_1}$$

since we talking about lag compensator , therefore   R1 C1 < R2 C2
is essential condition for the design.

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Lag compensator design

To avoid an appreciable change in the root loci, the ***angle contribution of the lag net work should be limited to a small amount***, say 5°. To assure this, we place the pole and zero of the lag network relatively close together and near the origin of the s plane. Then the closed-loop poles of the compensated system will be shifted only slightly from their original locations. Hence, the transient-response characteristics will be changed only slightly.

Procedure of design:

1. For the original system below, find the root locus with grid property:

$$G_x(s) = \frac{K_1}{s(1 + s)(1 + 0.2s)} = \frac{K}{s(s + 1)(s + 5)}$$
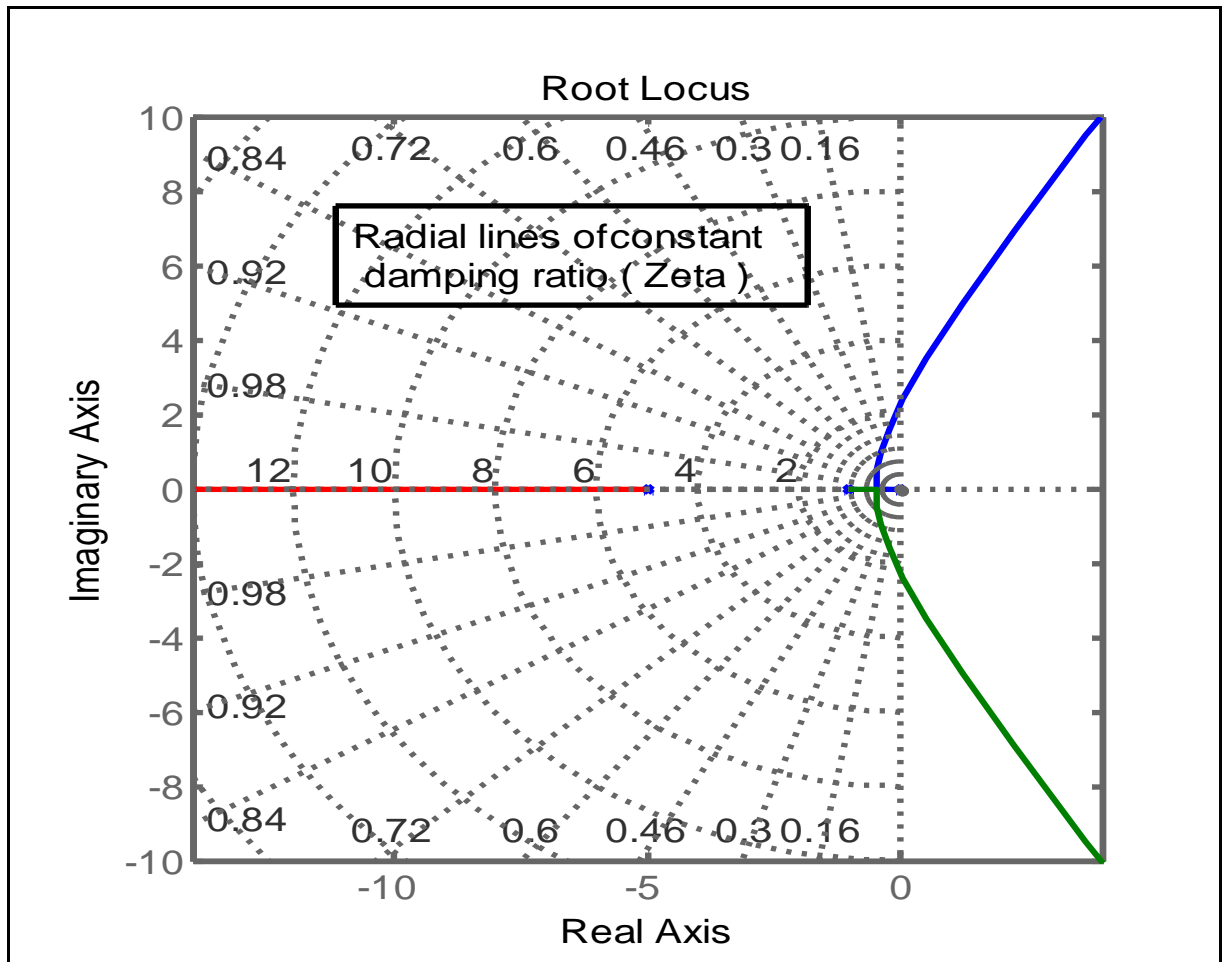
$$G_c(s) \;=\; \hat{K}_c\beta\,\frac{Ts + 1}{\beta Ts + 1} \;=\; \hat{K}_c\,\frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\beta T}}$$

.

2. Use the transfer function of the lag compensator

$$\left(\; Gc(s) = \hat{K}\beta\;\frac{Ts + 1}{\beta Ts + 1} \;=\; \hat{K}_c\,\frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\beta T}} \qquad \beta > 1 \right.$$

using a value $\;\beta = 10\;$.

3. For three selected values ($T = 4$, $T = 5$, $T = 10$) of poles position ($s = -1 / \beta\, T$), a three compensator will be made ,Plot the root locus for each compensated open loop system (using grid property of the root locus) .

4. Let the required damping ratio($\zeta = 0.45$) , find the dominant complex roots, natural frequency, settling time and maximum overshoot by using direct searching on root locus plots with using the following table:

Compensator	Dominant complex roots	Open Loop gain K'	Undamped freq, ω_n (rad/sec)	Mp %	for 2% t_s (sec.)
Uncompensated					
poles at T=4 $\dfrac{s+0.25}{s+0.025}$					
poles at T=5 $\dfrac{s+0.2}{s+0.02}$					
poles at T=10 $\dfrac{s+0.1}{s+0.01}$					

5. Simulate (using Simulink) the time response (ramp input) for the uncompensated system and the three cases of lag compensation on the one screen of the oscilloscope, using open loop gain for each case.

Discussion

1. For which purpose the lag compensator is used.

2. Discuss the effect of poles position on the shapes of the root locus.

3. Discuss the effect of poles position on the time responses.