Dept. of Communication Tech. Engineering

Fourth Stage

Control System

Al- Farahidi University

2023-2024

Lec.3

Mechanical Control System

Assistant lecturer

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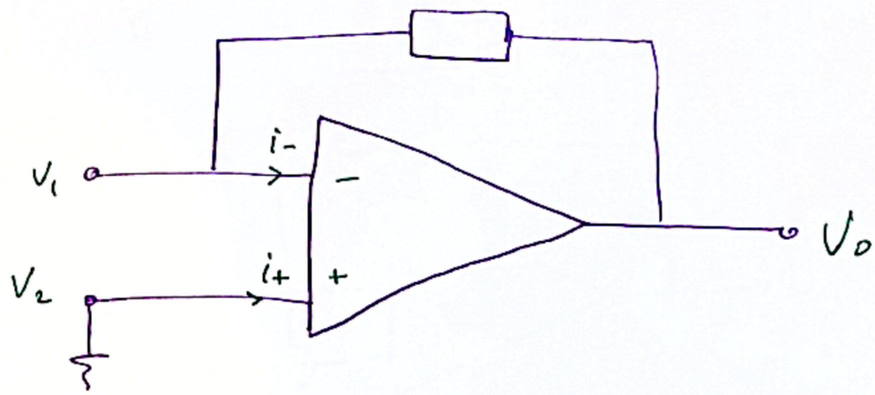
Operational Amplifier [OP-AmP] :~



$V_o = A \times V_d$

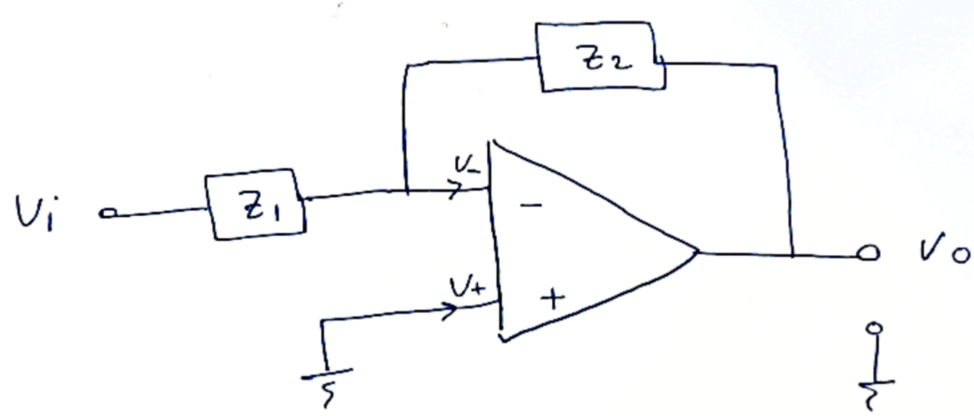
$V_d = V_1 - V_2$

A = open loop Voltage gain [very high, about 10^6]

* zero current in to the OP-Amp $\rightarrow i_- = i_+ = 0$

---

ex: Find the transfer function for the following cct.

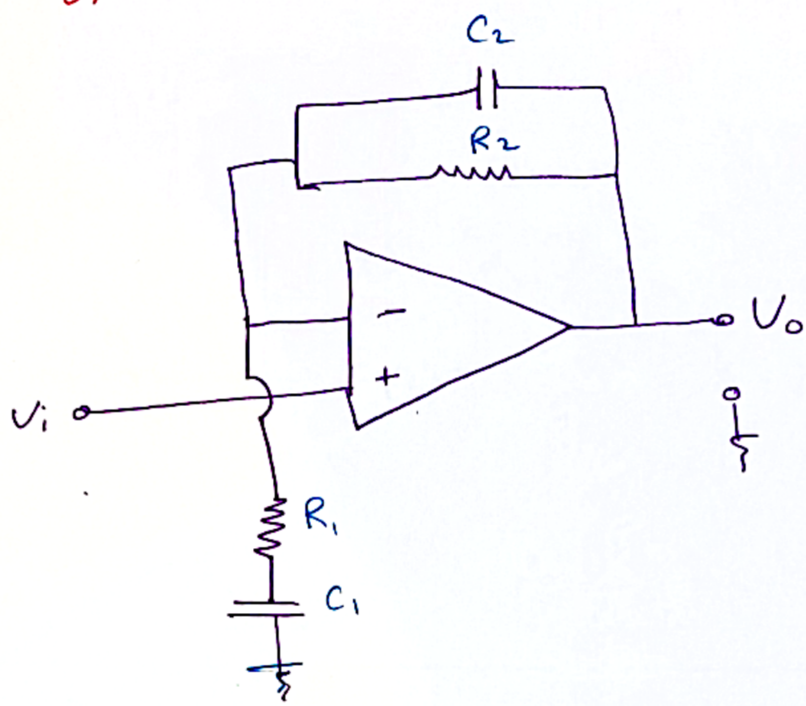


Sol) $V_- = V_+ = 0$

$\therefore$ The current through Z_1 is the same as the current in Z_2.

$$\frac{V_i}{Z_1} = \frac{-V_o}{Z_1} \Rightarrow \frac{V_o}{V_i} = \frac{-Z_2}{Z_1}$$

ex: Find $\frac{V_o}{V_i}$ for the circuit :



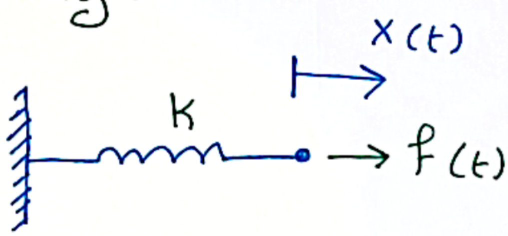
H.W

Ans: $\frac{V_o}{V_i} = \frac{R_1 C_1 S + [1 + R_1 C_1 S][1 + C_2 R_2 S]}{[1 + R_1 C_1 S] * [1 + R_2 C_2 S]}$

Translational Mechanical System :~

There are three basic elements in a translational mechanical system :

1- Spring :~



x = distance

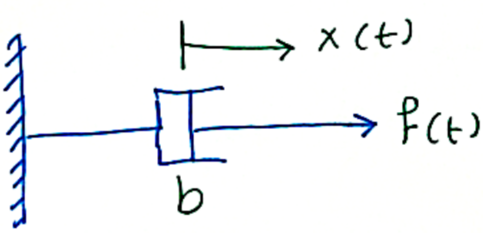
K = spring constant

$$f(t) = k \cdot x(t)$$

$$F(s) = k \cdot X(s)$$

$$\therefore \frac{F(s)}{X(s)} = K$$

2- Viscous Damper :~



b = damping constant = coefficient of friction

x = distance

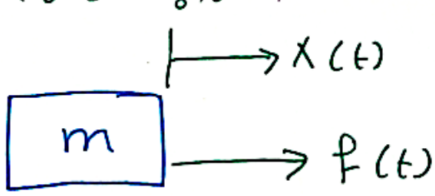
$$f(t) = b \cdot \frac{dx(t)}{dt}$$

$$= b \cdot \dot{x}$$

$$F(s) = S \cdot b \cdot X(s)$$

$$\frac{F(s)}{X(s)} = b \cdot S$$

3- Mass :~

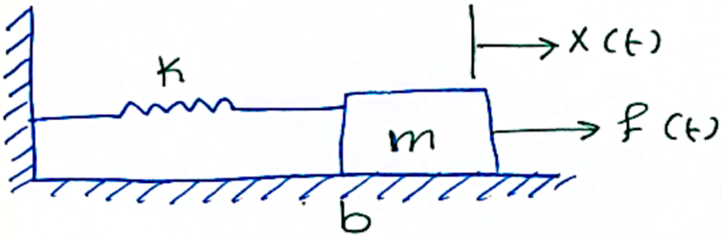


$$f(t) = m \frac{d^2x}{dt^2} = m\ddot{x}$$

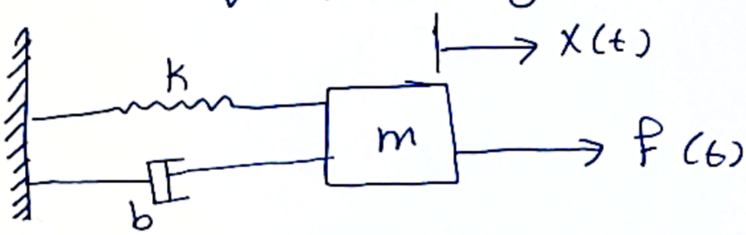
$$F(s) = m \cdot S^2 \cdot X(s)$$

$$\frac{F(s)}{X(s)} = m S^2$$

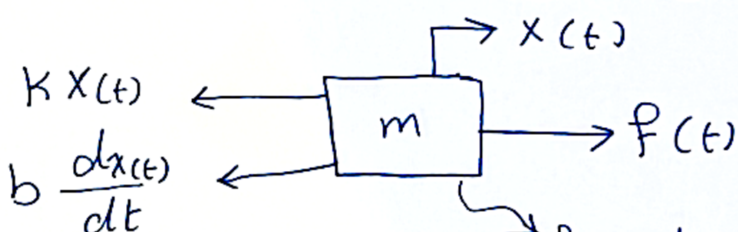
ex :~ Find The T.F for the following system.



⇓ equivalently :



⇓ free-body diagram



force due to acceleration

« قوة وزنها »

by applying Newton's law:

net force = 0

$$f(t) = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx(t)$$

⇓

$$F(s) = ms^2X(s) + bsX(s) + kX(s)$$

$$\therefore F(s) = X(s)\left[ms^2 + bs + k\right]$$

$$\boxed{\therefore \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}}$$

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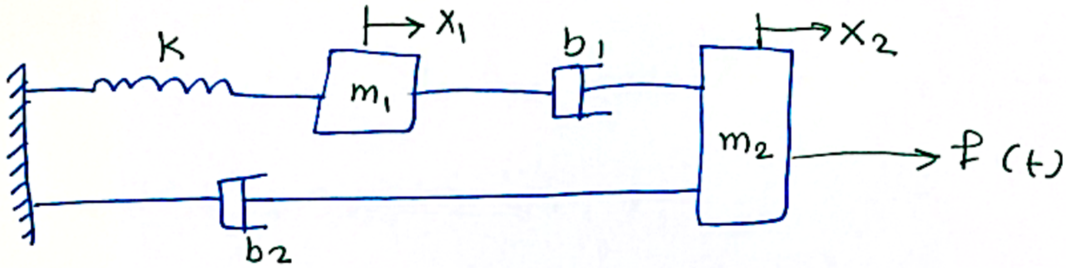
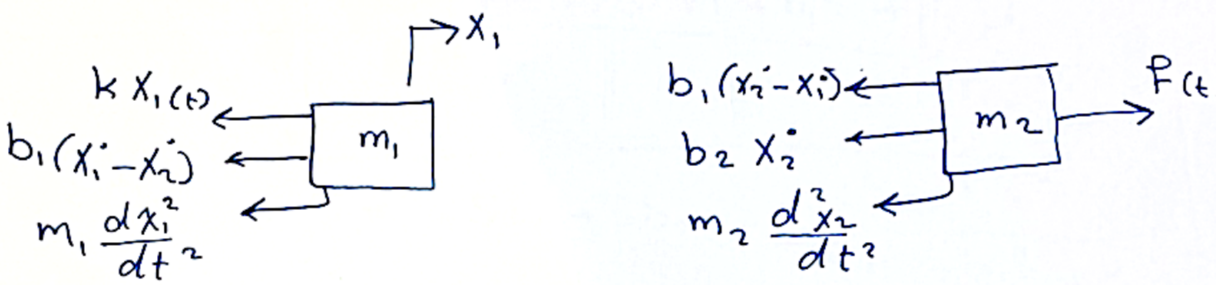
ex:

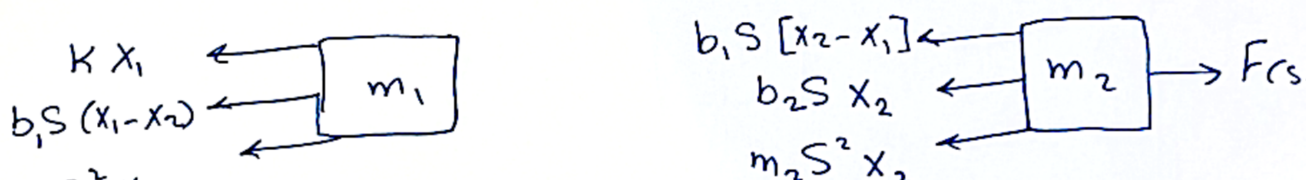


Find $\dfrac{X_2(s)}{F(s)}$?

Sol) First: draw the free body diagram.



⇓



⇓

now: write the balance eq.

$$s^2 m_1 X_1 + b_1 s (X_1 - X_2) + K X_1 = 0 \quad \text{---} \; (1)$$

$$s^2 m_2 X_2 + b_2 s X_2 + b_1 s [X_2 - X_1] = F(s) \quad \text{---} \; (2)$$

$$s^2 m_1 X_1 + b_1 s X_1 - b_1 s X_2 + K X_1 = 0 \quad \text{---} \; (1)$$

$$X_1 [s^2 m_1 + b_1 s + K] = b_1 s X_2$$

$$\therefore \boxed{X_1 = \frac{b_1 s}{[s^2 m_1 + b_1 s + K]} \times X_2}$$

sub in (2)

$$S^2 m_2 X_2 + S b_1 X_2 - S b_1 X_1 + S b_2 X_2 = F(s) \quad \text{---- } ②$$

$$X_2 [S^2 m_2 + S b_1 + S b_2] - S b_1 X_1 = F(s)$$

$$X_2 [S^2 m_2 + S b_1 + S b_2] - S b_1 * \frac{S b_1}{[S^2 m_1 + b_1 S + K]} * X_2 = F(s)$$

$$X_2 \left[S^2 m_2 + S b_1 + S b_2 - \frac{S^2 b_1^2}{S^2 m_1 + b_1 S + K} \right] = F(s)$$

$$\boxed{\therefore \frac{X_2}{F(s)} = \frac{1}{S^2 m_2 + S b_1 + S b_2 - \dfrac{S^2 b_1^2}{S^2 m_1 + b_1 S + K}}}$$

Suspensio System :~ النظام المعلق

[quarter Car model] :~

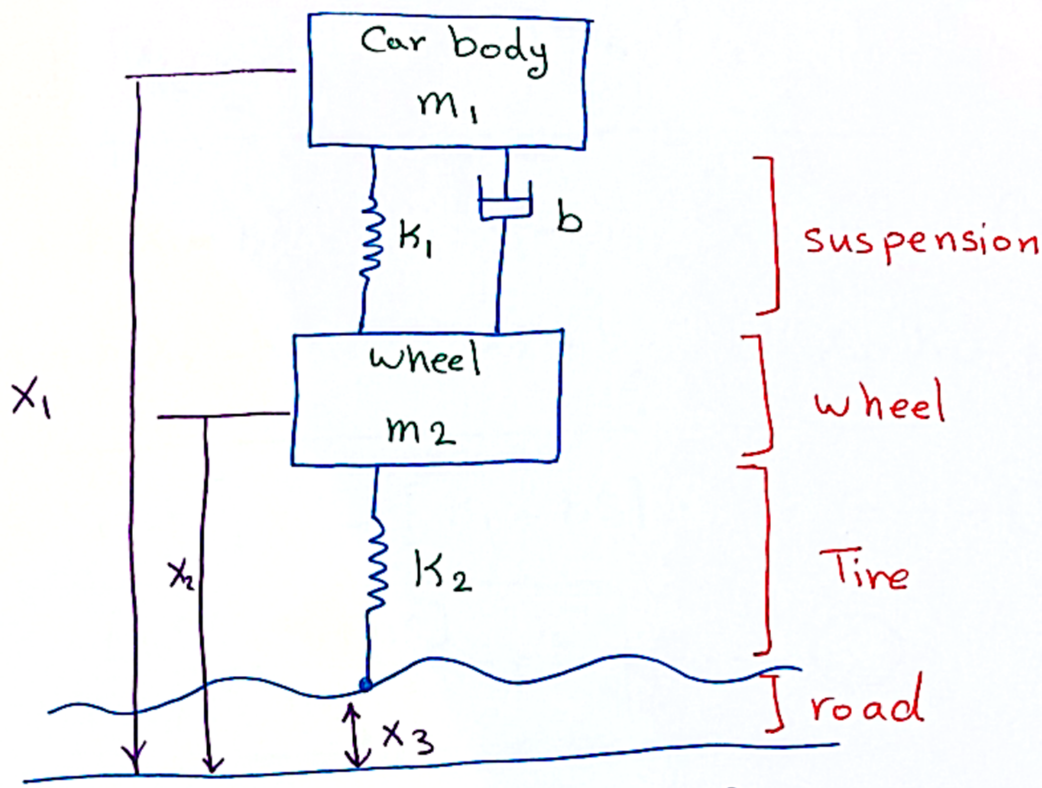
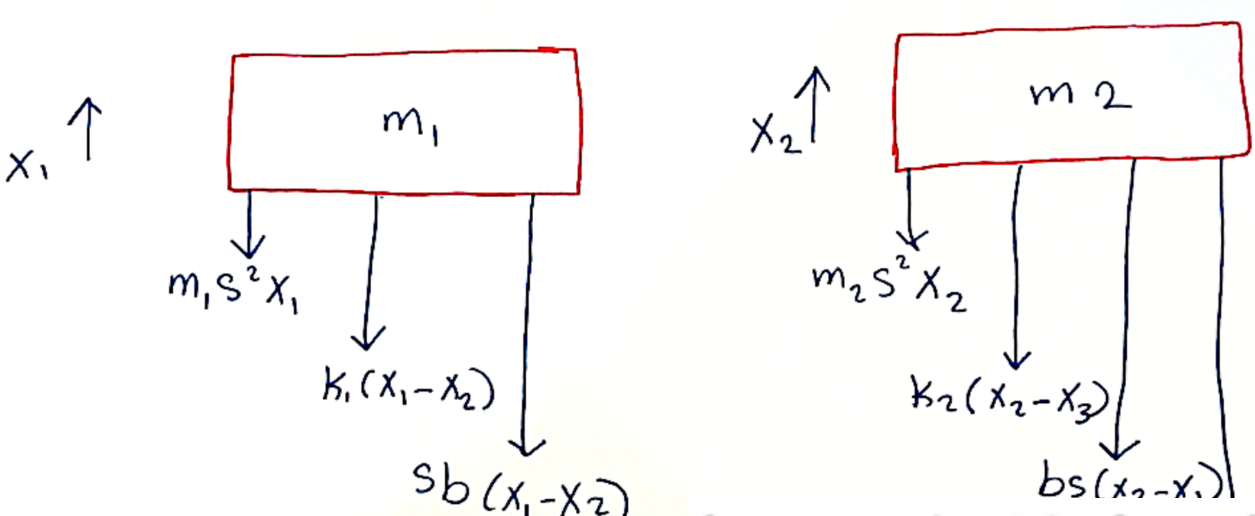


Figure out the transfer function from X_3 to X_1, So that, we can analyze how we can minimize the impact of X_3 on X_1.

Free-body Diagram :~



Two Balance equations :~

$$m_1 S^2 X_1 + K_1 [X_1 - X_2] + bS [X_1 - X_2] = 0 \quad ---- (1)$$

$$m_2 S^2 X_2 + K_1 [X_2 - X_1] + bS [X_2 - X_1] + K_2 [X_2 - X_3] = 0 \quad --- (2)$$

$\Downarrow$

$$m_1 S^2 X_1 + K_1 X_1 - K_1 X_2 + bS X_1 - bS X_2 = 0 \quad ---- (1)$$

$$m_2 S^2 X_2 + K_1 X_2 - K_1 X_1 + bS X_2 - bS X_1 + K_2 X_2 - K_2 X_3 = 0 \quad --- (2)$$

$\Downarrow$

$$[m_1 S^2 + K_1 + bS] X_1 - [K_1 + bS] X_2 = 0 \quad ---- (1)$$

$$\boxed{\therefore \; X_2 = \frac{[m_1 S^2 + K_1 + bS]}{[K_1 + bS]} * X_1} \quad ------ (*)$$

put (*) in (2)

eq(2) :

$$- [K_1 + bS] X_1 + [m_2 S^2 + K_1 + bS + K_2] * \frac{[m_1 S^2 + K_1 + bS]}{[K_1 + bS]} * X_1 = K_2 X_3$$

$$\therefore \; \frac{X_1(S)}{X_3(S)} = \frac{K_2 [K_1 + bS]}{- [bS + K_1] + [m_2 S^2 + K_1 + bS + K_2][m S^2 + K_1 + bS]}$$

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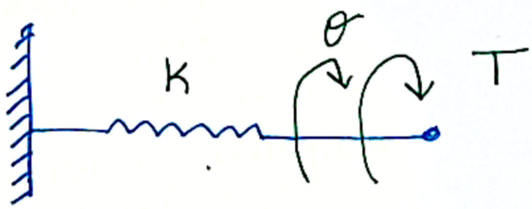
Rotational Mechanical System :~

There are three basic elements in a Rotational mechanical system :

1- Spring [Torsional Spring]

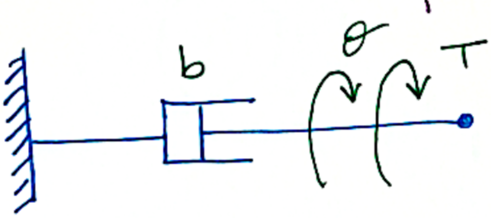


T = Torque
θ = Tortional distance.

$$T_{(t)} = K \cdot \theta_{(t)}$$

$$T_{(s)} = K \cdot \theta_{(s)}$$

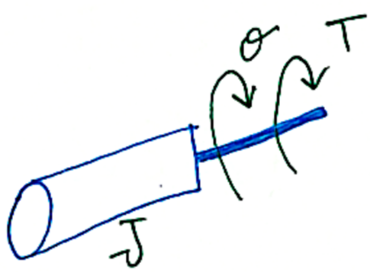
2- Viscous Damper :~



$$T(t) = b \frac{d\theta}{dt}$$

$$T(s) = s \cdot b \cdot \theta(s)$$

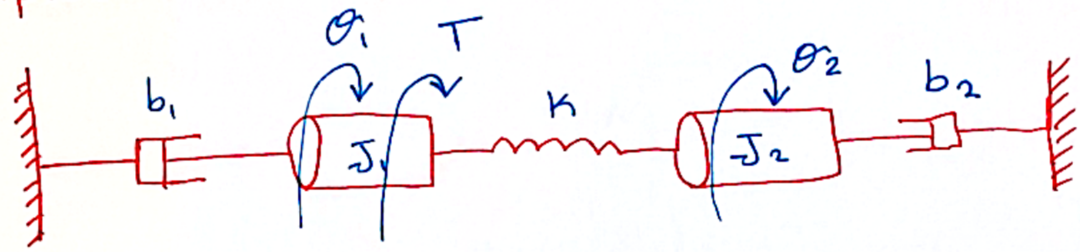
3- Inertia :~



$$T_{(t)} = J \frac{d^2\theta}{dt^2}$$

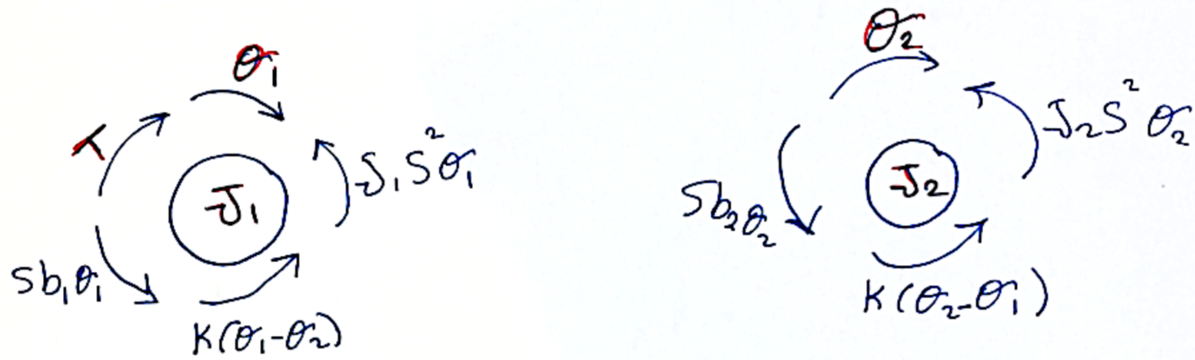
$$T(s) = S^2 \cdot J \cdot \theta(s)$$

Example :-



Find $\dfrac{\theta_2(s)}{T(s)}$?

Free body diagram :-



∴ Torque Balance equations :-

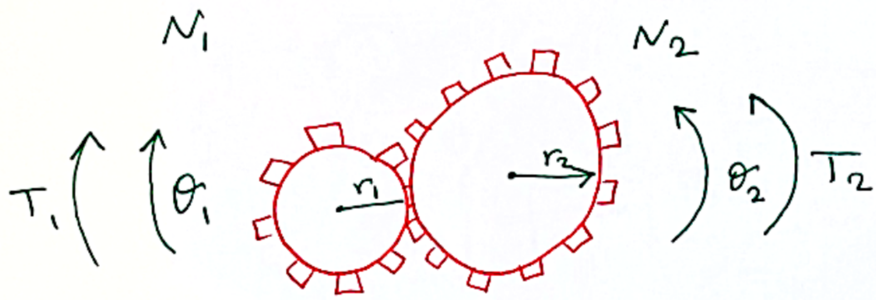
$$T(s) = J_1 S^2 \theta_1 + b_1 S \theta_1 + K(\theta_1 - \theta_2)$$

$$0 = J_2 S^2 \theta_2 + b_2 S \theta_2 + K(\theta_2 - \theta_1)$$

$$\Downarrow$$

$$\frac{\theta_2}{\theta_1} = H \cdot W$$

Gears :~



all teeth have the same size for both gears:

N = no. of teeth

r = radius

$$r_1 \cdot \theta_1 = r_2 \cdot \theta_2$$

$$\Downarrow$$

$$r_1 \dot{\theta}_1 = r_2 \dot{\theta}_2$$

$$\frac{\dot{\theta}_1}{\dot{\theta}_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1}$$

$$\frac{N_1}{N_2} = n$$

$$\frac{N_1}{N_2} = \frac{r_1}{r_2}$$

$$\Downarrow$$

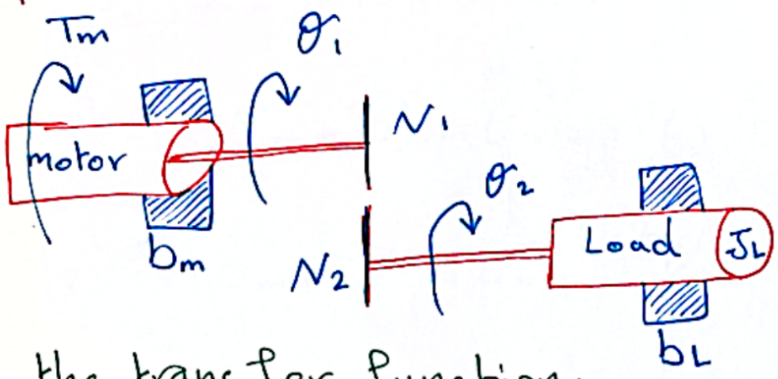
$$\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1}$$

Assuming no power loss in transferring energy from one gear to the other :~

$$T_1 \dot{\theta}_1 = T_2 \dot{\theta}_2$$

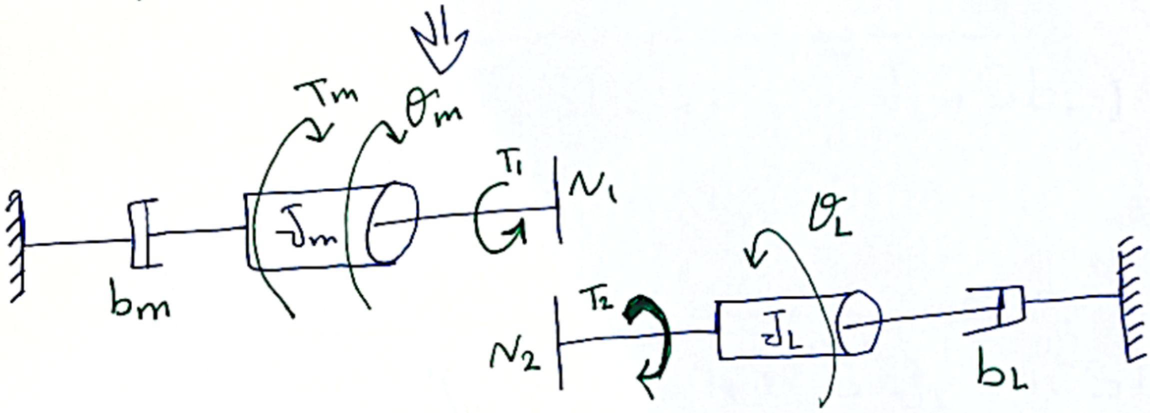
$$\boxed{\frac{T_2}{T_1} = \frac{\dot{\theta}_1}{\dot{\theta}_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1}}$$

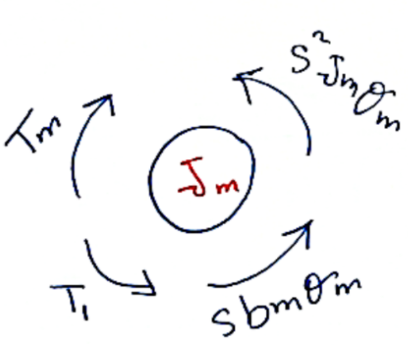
example :~

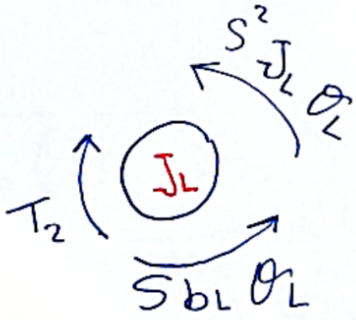


Find the transfer function:

$$\frac{\theta_L}{T_m} = ?$$







∴ The balance eq. are:

$$T_m = S^2 J_m \theta_m + S b_m \theta_m + T_1 \quad ----(1)$$

$$T_2 = S^2 J_L \theta_L + S b\, \theta_L \quad ----(2)$$

$$\frac{T_2}{T_1} = \frac{N_2}{N_1} = \frac{1}{n} \Rightarrow T_1 = n\, T_2$$

$$\frac{\theta_L}{\theta_m} = \frac{N_1}{N_2} = n \Rightarrow \theta_L = n\,\theta_m$$

$$T_m = S^2 J_m \theta_m + S b_m \theta_m + n\left[S^2 J_L \theta_L + S b_L \theta_L\right]$$

$$T_m = S^2 J_m \theta_m + S b_m \theta_m + n \theta_L \left[S^2 J_L + S b_L\right]$$

$$T_m = S^2 J_m \theta_m + S b_m \theta_m + n^2 \theta_m \left[S^2 J_L + S b_L\right]$$

$$\therefore T_m = \left[S^2 J_m + S b_m + n^2 (S^2 J_L + S b_L)\right] \theta_m$$

$$\therefore \frac{\theta_m}{T_m} = \frac{1}{[S^2 J_m + S b_m + n^2 (S^2 J_L + S b_L)}$$

$$= \frac{1}{S^2 J_m + S b_m + n^2 S^2 J_L + n^2 S b_L}$$

$$\therefore \frac{\theta_m}{T_m} = \frac{1}{S^2 (\underbrace{J_m + n^2 J_L}_{J_e}) + S(\underbrace{b_m + n b_L}_{b_e})}$$

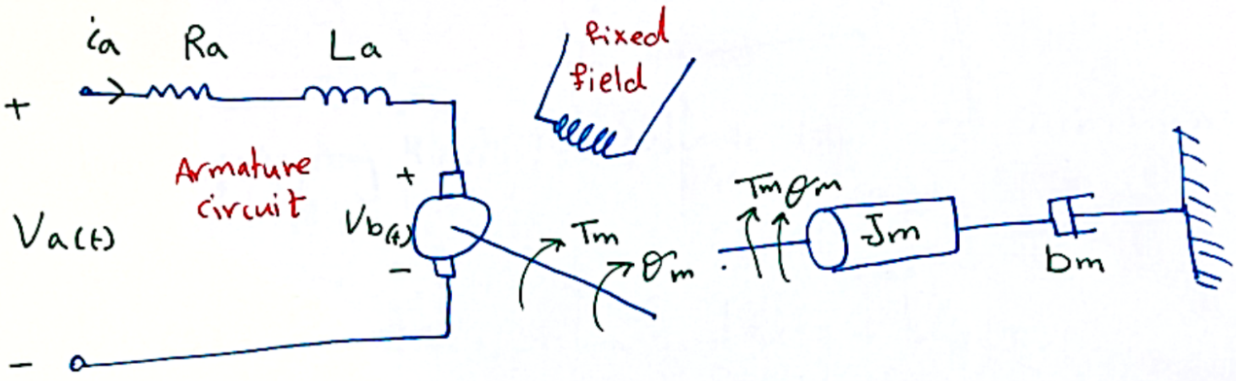
$\Rightarrow$ e = equivalent

$$\boxed{\frac{\theta_L}{T_m} = \frac{n \theta_m}{T_m} = \frac{n}{S^2 J_e + S b_e}}$$

Electromechanical System:~

DC motor:~

electromechanical System have electrical and mechanical parts.



$$\boxed{\begin{aligned} T_m &= K_t \cdot i_a \quad \text{---} (1) \\ V_b &= K_b \cdot \dot{\theta}_m \quad \text{---} (2) \end{aligned}}$$

where:

K_t = torque constant

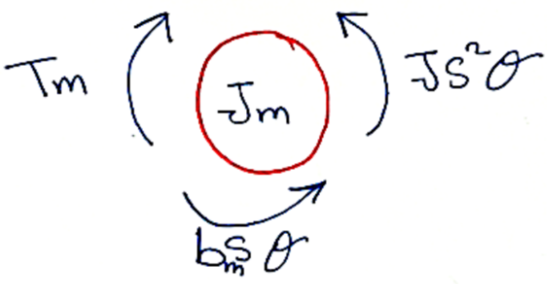
i_a = armature current

K_b = e.m.f constant

V_b = back e.m.f

V_a = input Voltage

mechanical part:~



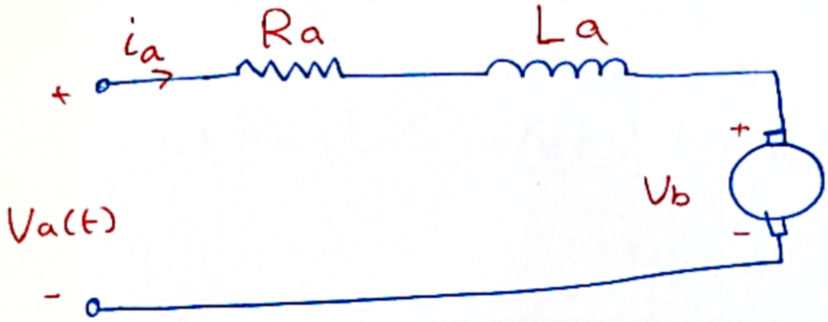
$$T_m = J_m S^2 \theta_m + b_m S \theta_m$$

$$= (J_m S + b_m) S \theta$$

$$= (J_m S + b_m) \omega_m(s)$$

$$\therefore T_m = J_m S \cdot \omega_m(s) + b_m \cdot \omega_m(s) \quad \text{----} (3)$$

electrical part :-



by applying kirchoff voltage law to armature circuit:

$$V_a(s) = R_a \cdot I_a(s) + L_a S \cdot I_a(s) + V_b(s).$$

$$V_a(s) = R_a \cdot I_a(s) + L_a S \cdot I_a(s) + K_b \cdot \omega_m(s)$$

$$V_a(s) - K_b \cdot \omega_m(s) = [R_a + L_a S]\, I_a(s)$$

$$\therefore I_a(s) = \frac{V_a(s) - K_b \cdot \omega_m(s)}{R_a + L_a S}.$$

From mechanical part:

$$T_m = K_t \cdot I_a$$

$$\therefore T_m = K_t * \frac{V_a(s) - K_b \cdot \omega_m(s)}{R_a + L_a S}$$

from eq: 3

$$J_m \cdot S \cdot \omega_m(s) + b_m \cdot \omega_m(s) = K_t * \frac{V_a(s) - K_b \cdot \omega_m(s)}{R_a + L_a s}$$

$$(J_m S + b)(R_a + L_a S) \cdot W_{m(s)} = K_t \cdot V_{a(s)} - K_t \cdot K_b \cdot \omega_{m(s)}$$

$$\left[(J_m S + b)(R_a + L_a S) + K_t K_b\right] \cdot W_{m(s)} = K_t \cdot V_{a(s)}.$$

$$\therefore \frac{W_{m(s)}}{V_{a(s)}} = \frac{K_t}{[J_m S + b][R_a + L_a S] + K_t \cdot K_b}$$

The numerical values of L_a & b_m are typically small relative to other parameters.

$\therefore$ A simplified model will be:

$L_a = b_m = 0$

$$\therefore \frac{W_{m(s)}}{V_{a(S)}} = \frac{K_t}{[J_m S \cdot R_a] + K_t \cdot K_b}$$

$$= \frac{1/K_b}{\frac{J_m \cdot R_a}{K_t \cdot K_b} \cdot S + 1}$$

$$= \frac{K_m}{\tau_m S + 1}$$

where:

$$K_m = \frac{1}{K_b}$$

$$\tau_m = \frac{J_m \cdot R_a}{K_t \cdot K_b}$$

Dept. of Communication Tech. Engineering

Fourth Stage

Control System

Al- Farahidi University

2023-2024

Lec.4

Block Diagram

Assistant lecturer

Yasameen Hameed Al-aarajy

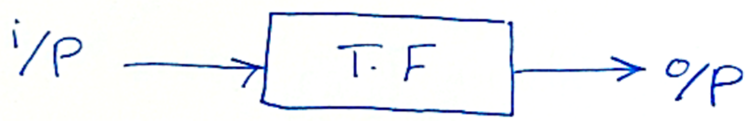
~ Block Diagram models & Reduction rules ~

Block diagram illustrate the operation of different system components, since the block diagram gives the relation ship between the i/p and o/p of the components.

Symbols used in block diagram :

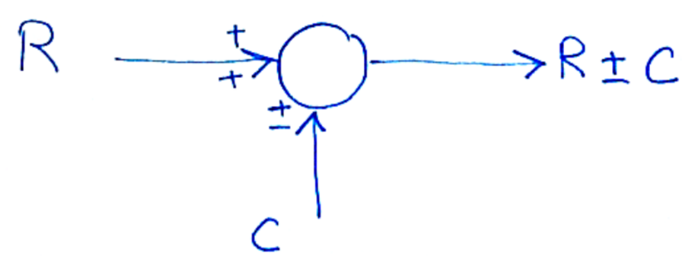
1- Block :~

The T.F of the system element is placed in the block symbolized by :



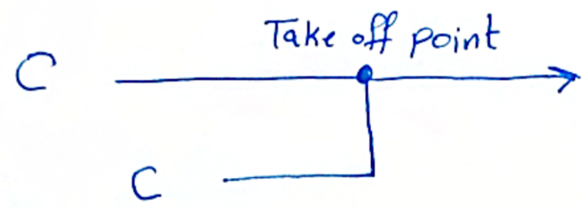
2- Summing Point :~

The operation of addition or subtraction is performed by this system element.



3- Take off point :~

This operation is used to provide a dual i/p or o/p to a system element.

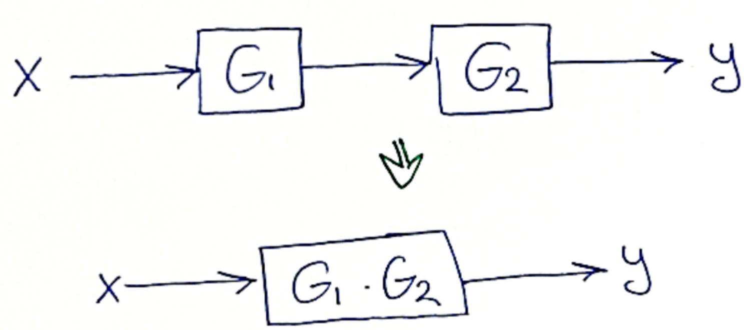


4- Directional arrows :~

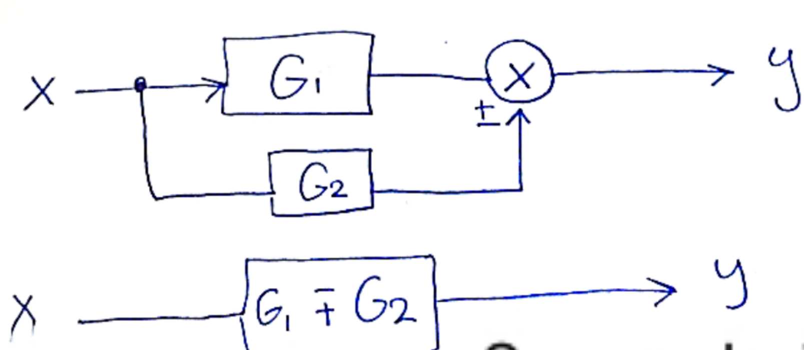
This system defines unidirectional flow of the system.

~ Rules of Block Diagram ~

1- Combine all series [cascade] blocks :~

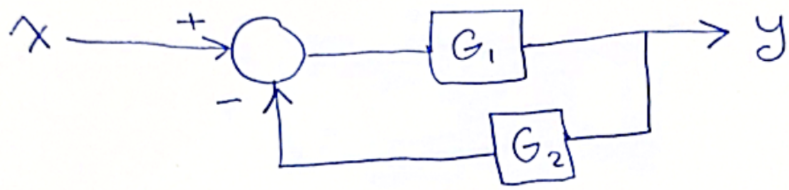


2. Combine all Parallel blocks :~



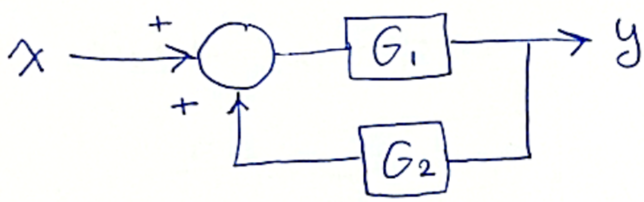
3- Eliminate all feed back loops :

a- negative feed back :~



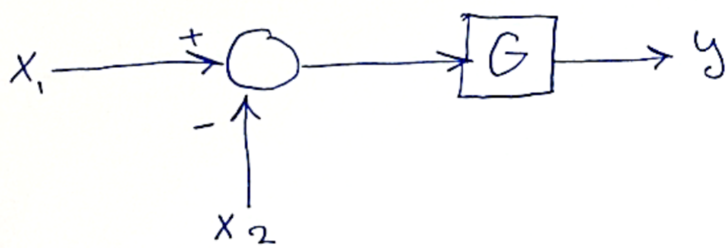
$$y = \frac{G_1}{1 + G_1 G_2}$$

b- Positive feed back :~

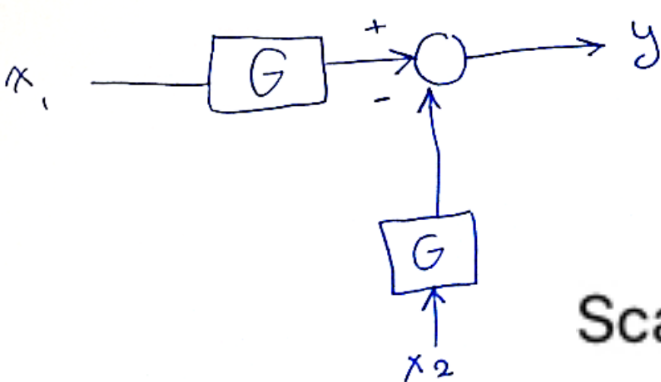


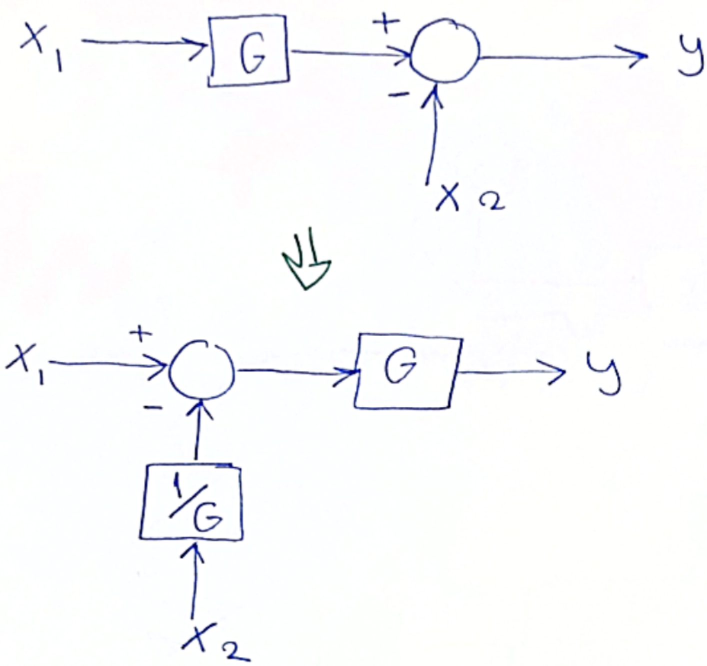
$$y = \frac{G_1}{1 - G_1 G_2}$$

4- Shift Summing Points to left :

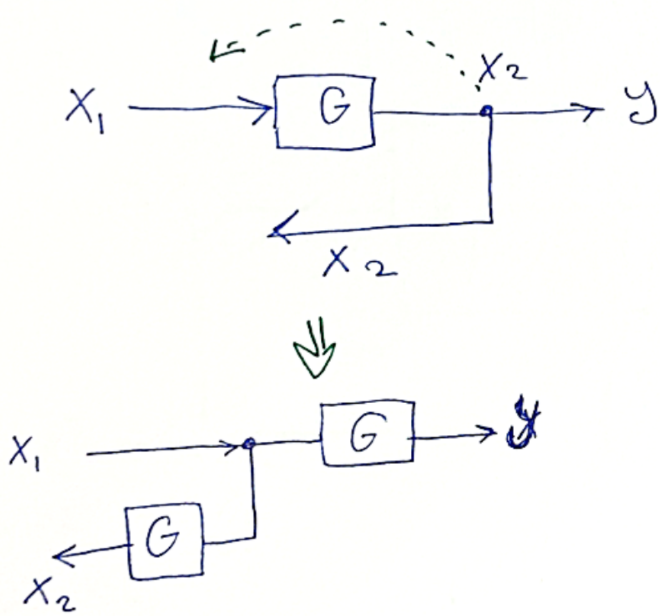


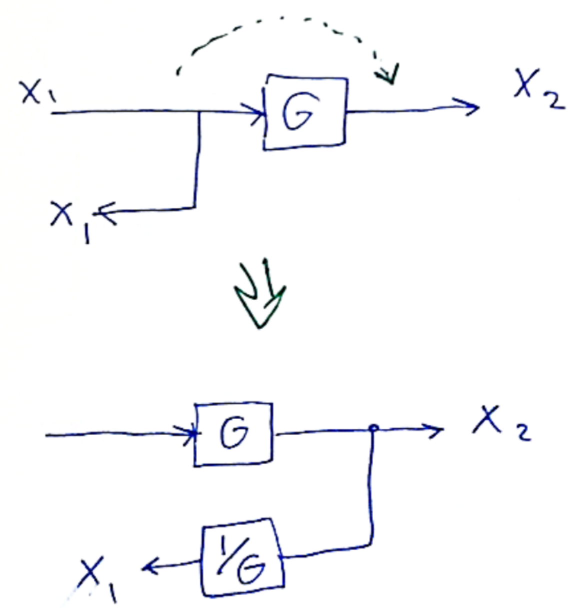
$\Downarrow$



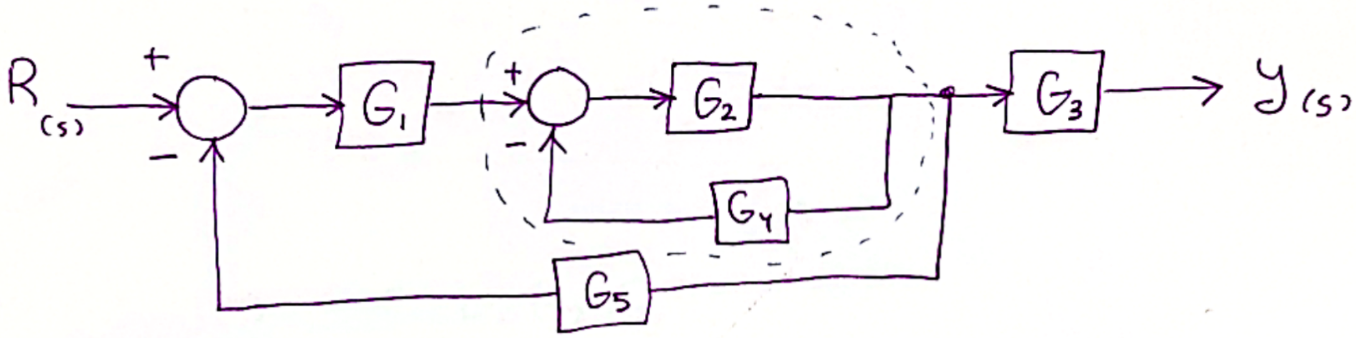


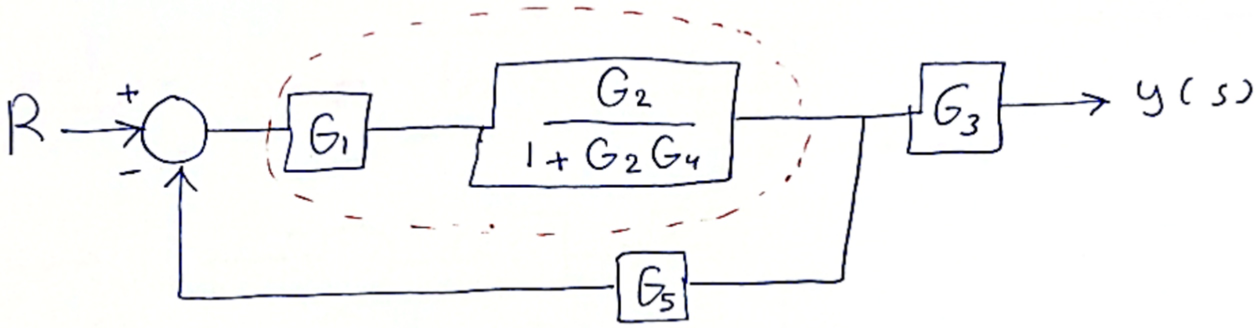
5- Shift take points :~

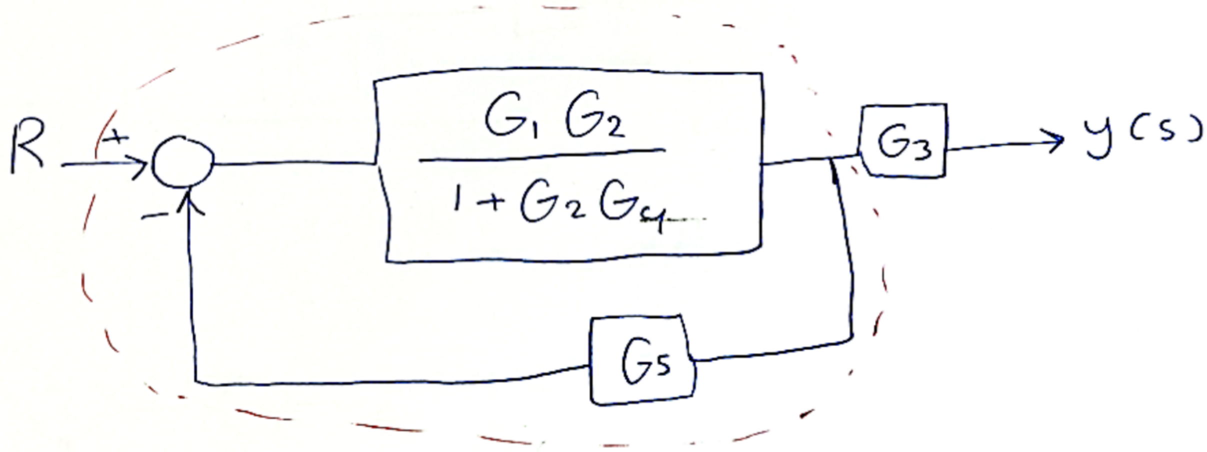


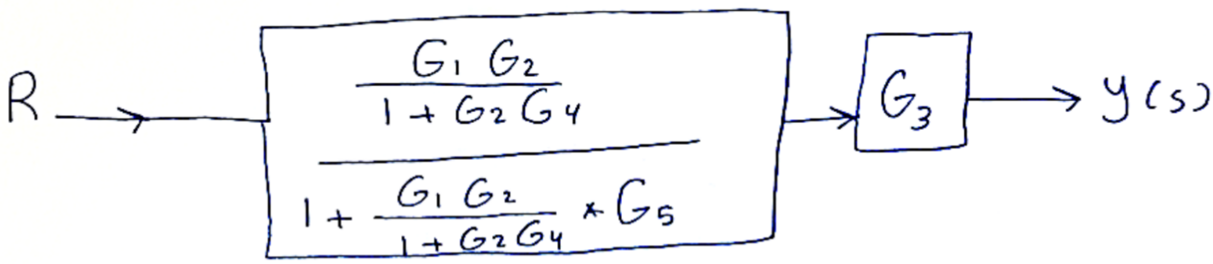


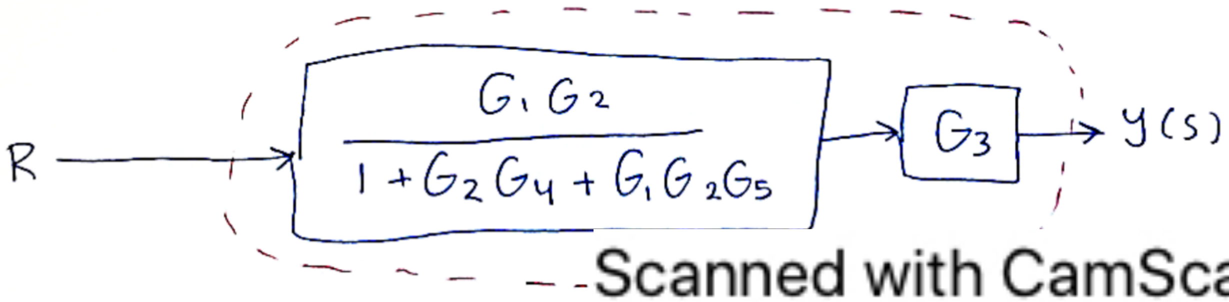
example 1: Simplify to get $\frac{Y(s)}{R(s)}$

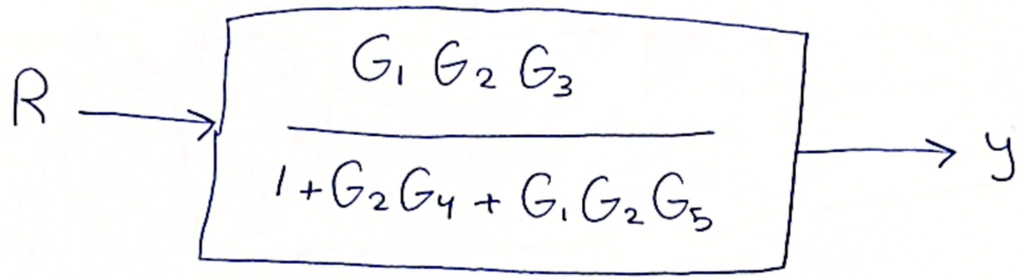








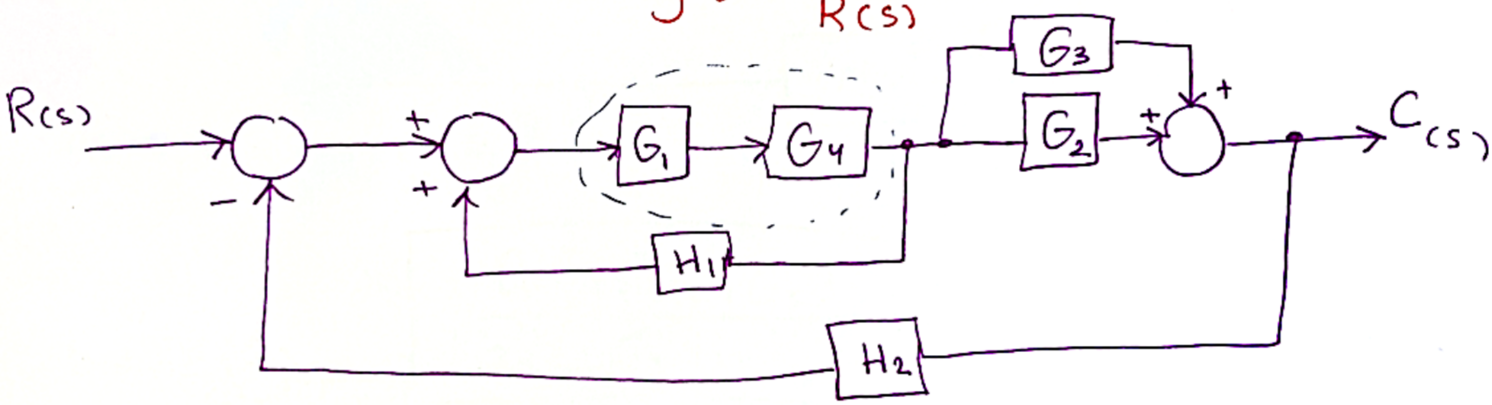


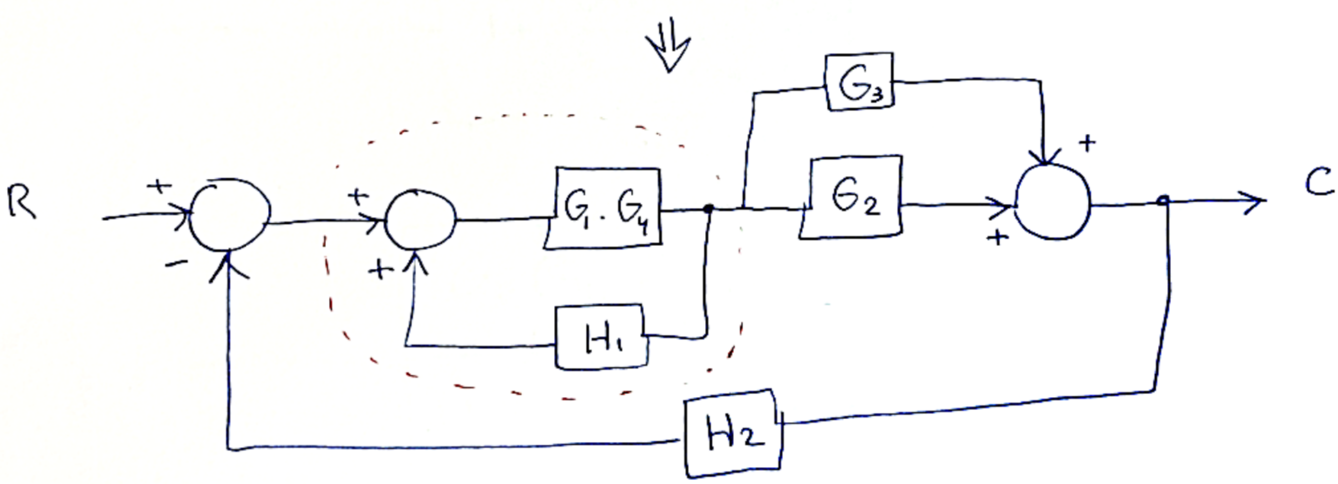


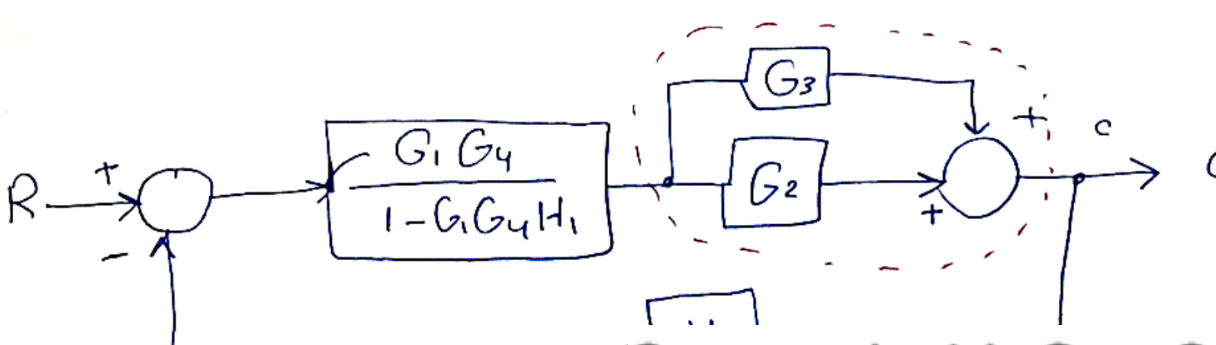
$$\therefore \frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3}{1+G_2G_4+G_1G_2G_5}$$

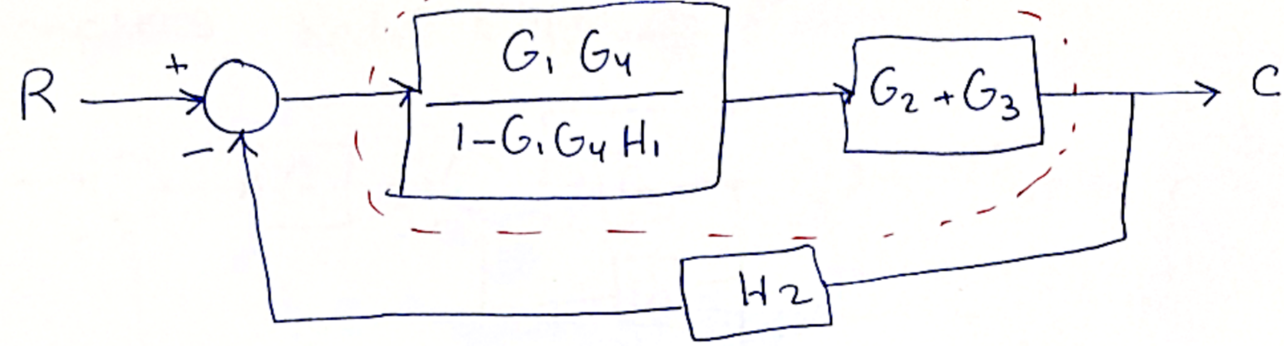
---

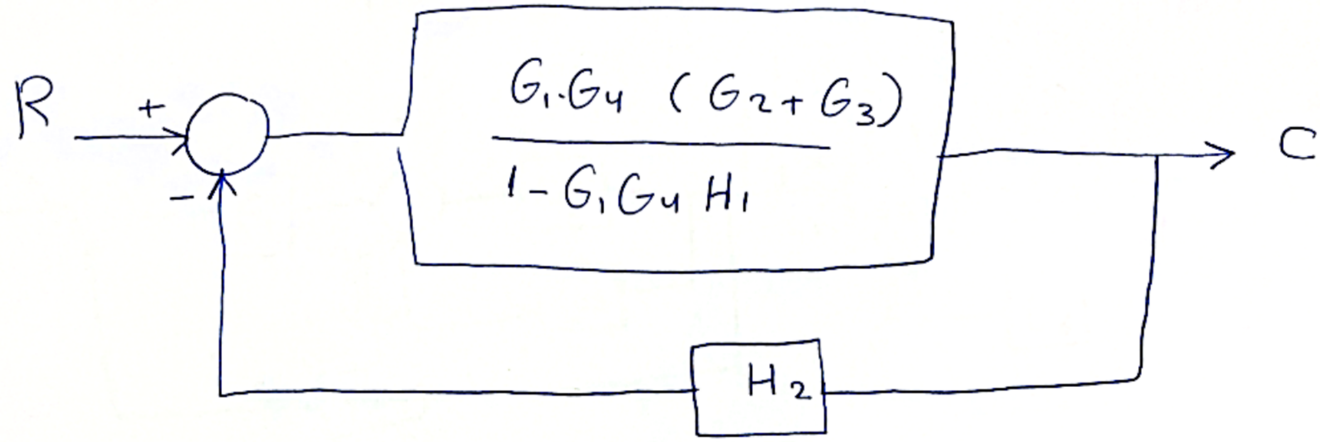
example 2:~ Reduce to get $\frac{C(s)}{R(s)}$

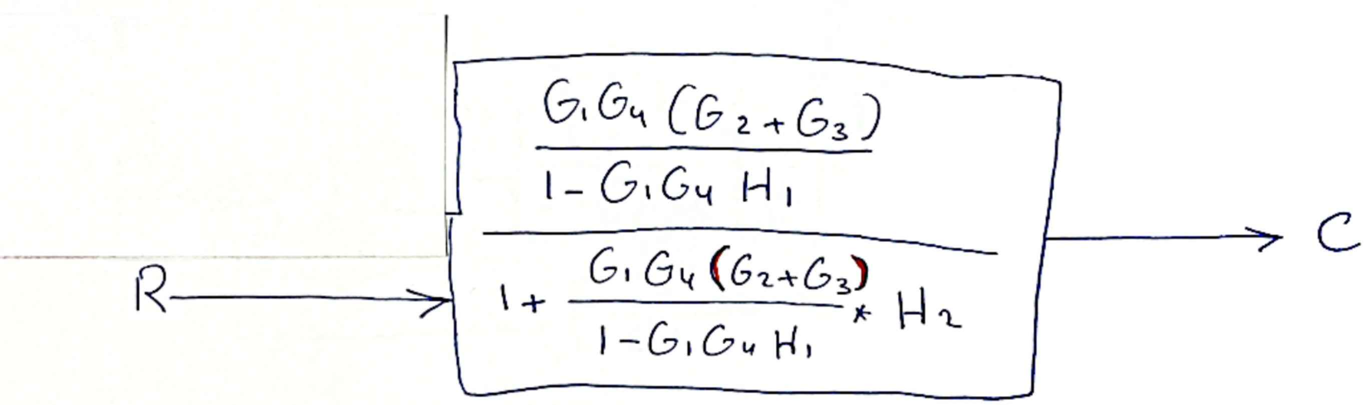


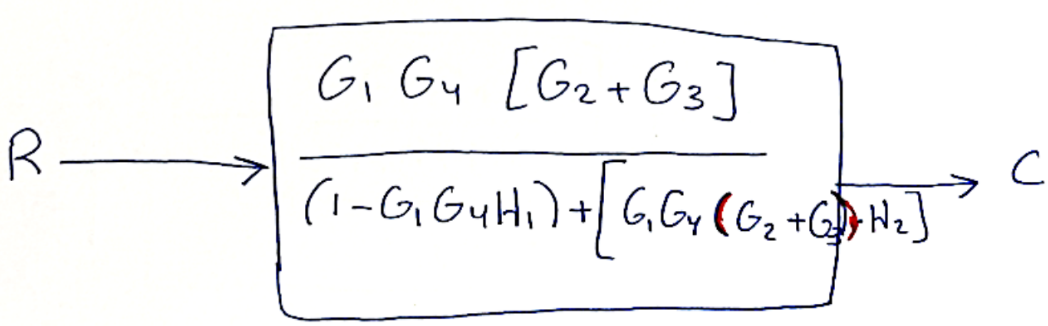






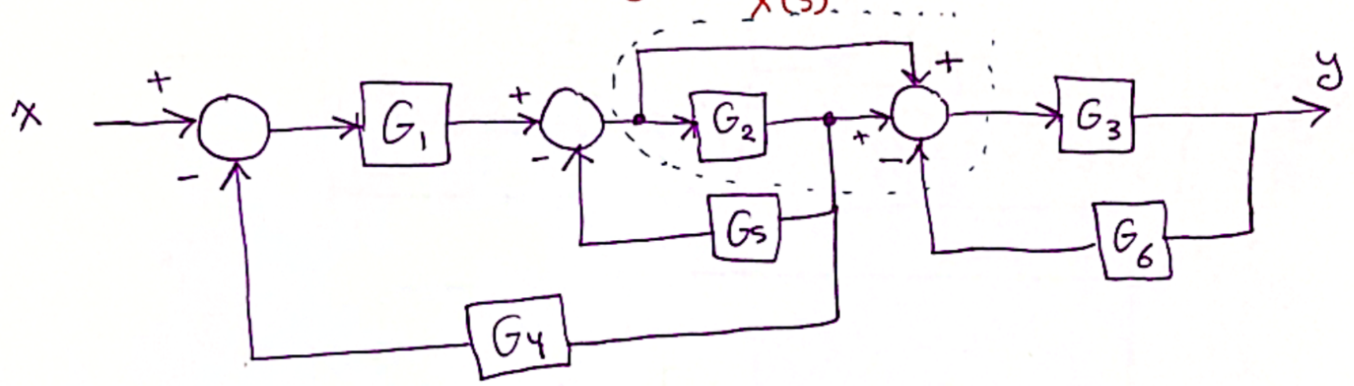


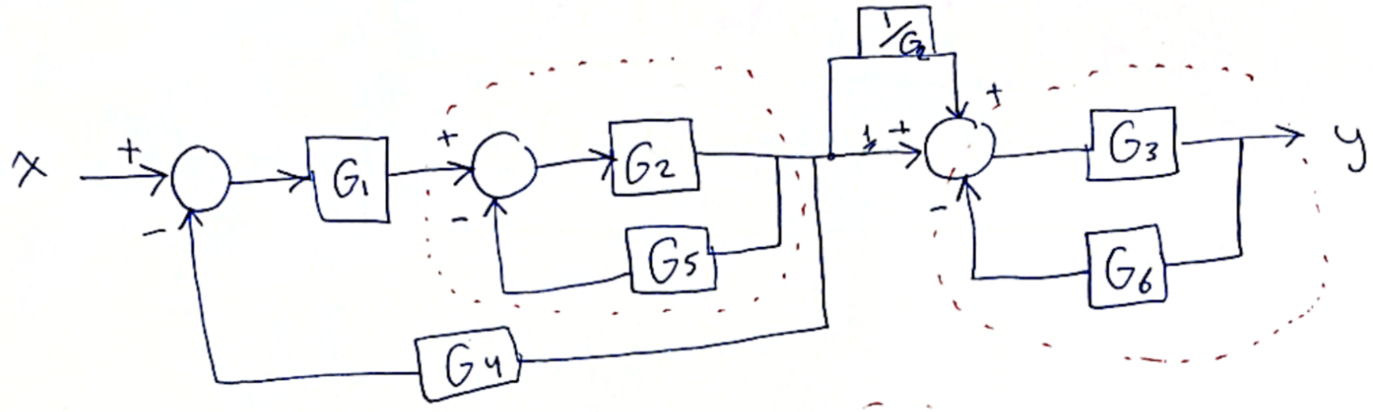


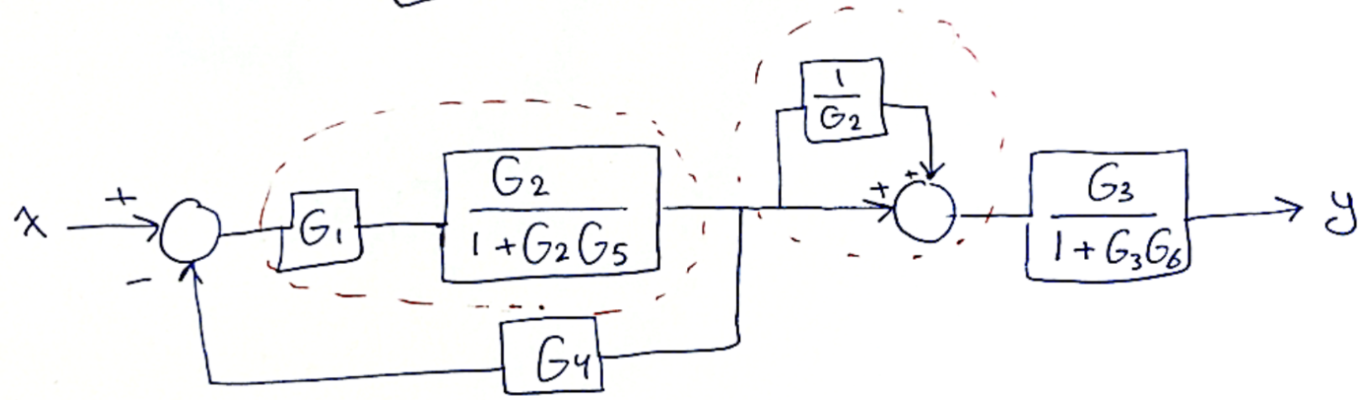


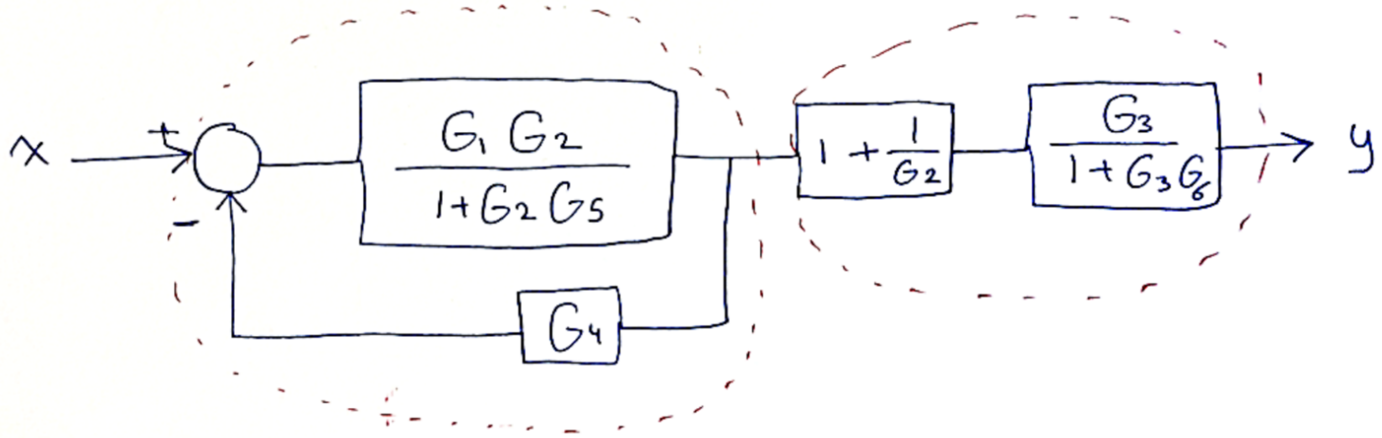
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2+G_3)}{(1-G_1 G_4 H_1)+[G_1 G_4 (G_2+G_3) \cdot H_2]}$$

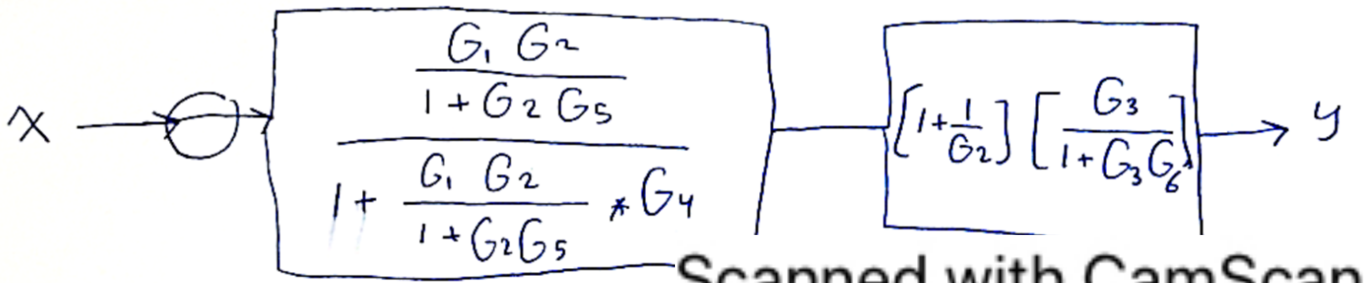
example: 3 Reduce to get $\frac{y(s)}{X(s)}$,

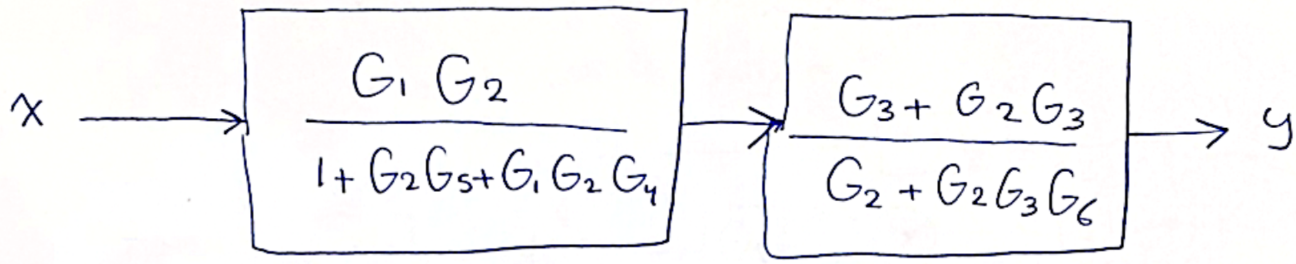


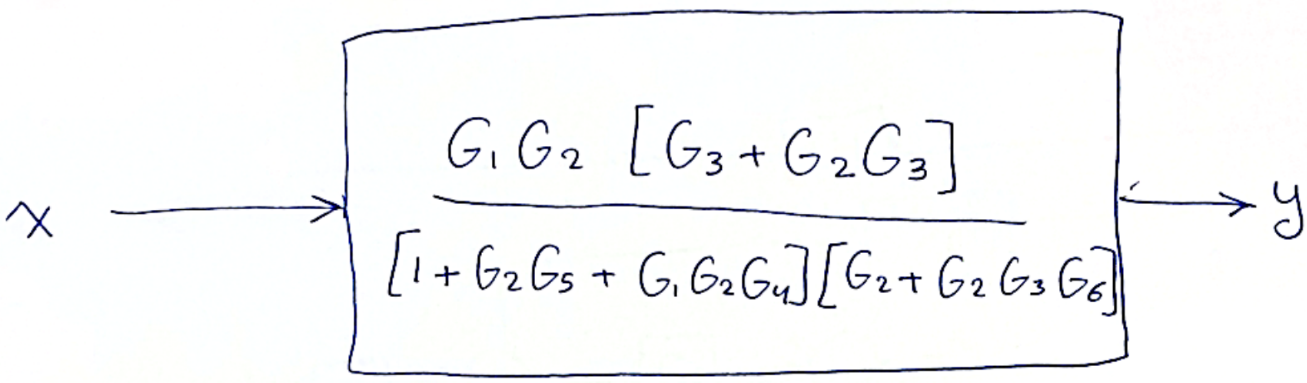




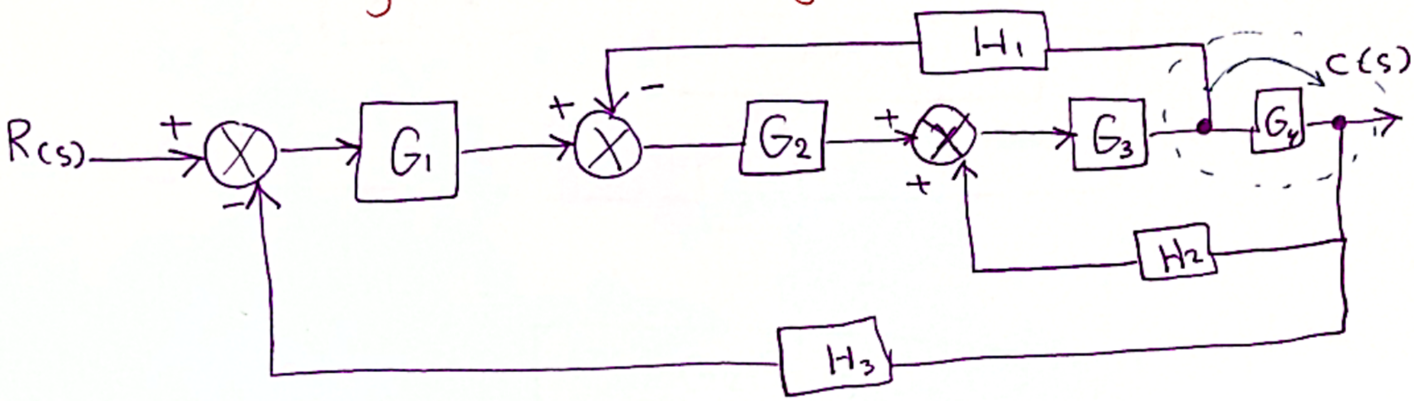


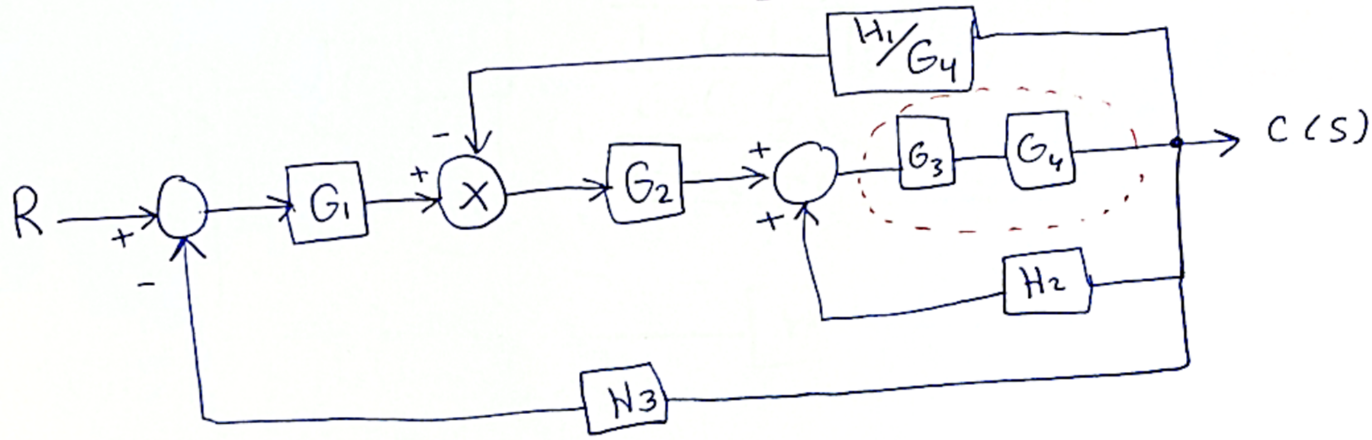


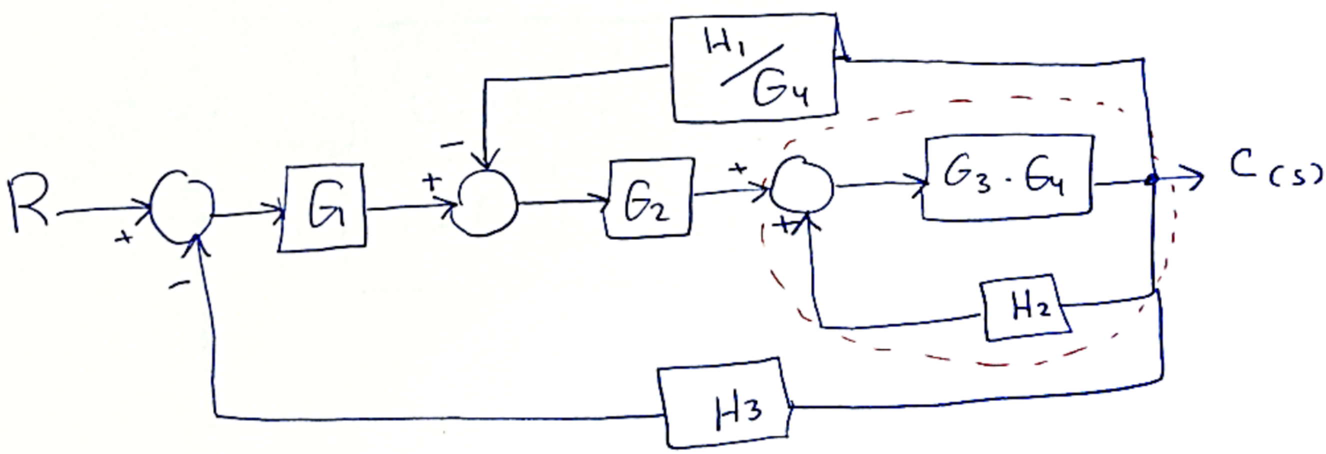

x
$\frac{G_1 G_2}{1+G_2G_5+G_1G_2G_4}$
$\frac{G_3+G_2G_3}{G_2+G_2G_3G_6}$
y

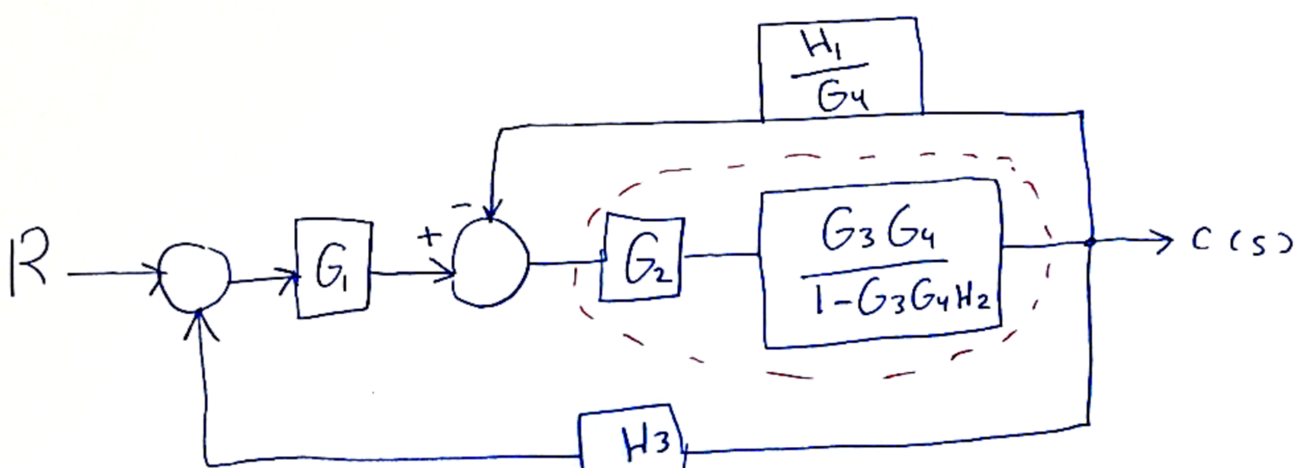

x
$\frac{G_1 G_2 \left[G_3+G_2G_3\right]}{\left[1+G_2G_5+G_1G_2G_4\right]\left[G_2+G_2G_3G_6\right]}$
y

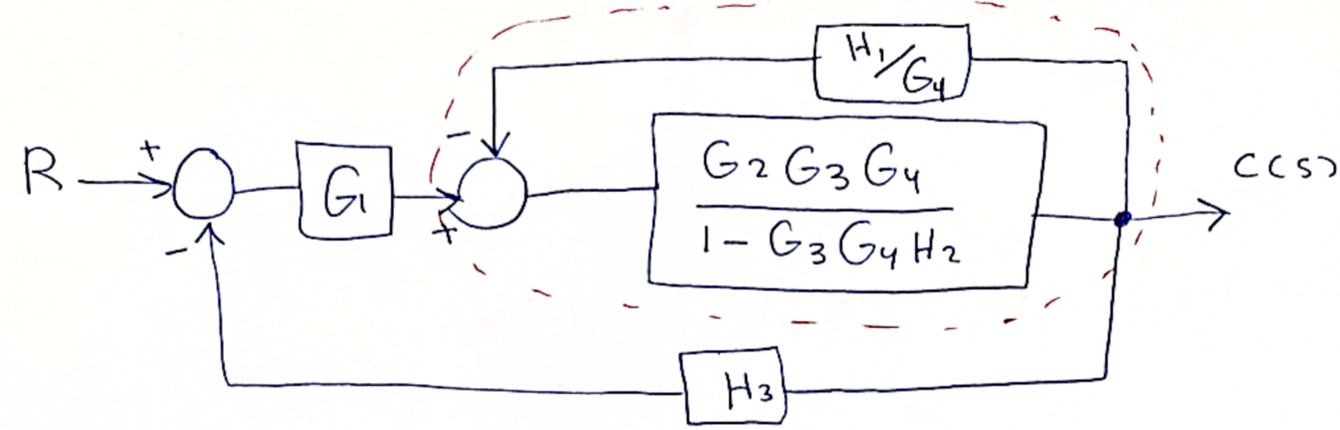
example: 4: Find the over all transfer function of the block diagram shown in figure.

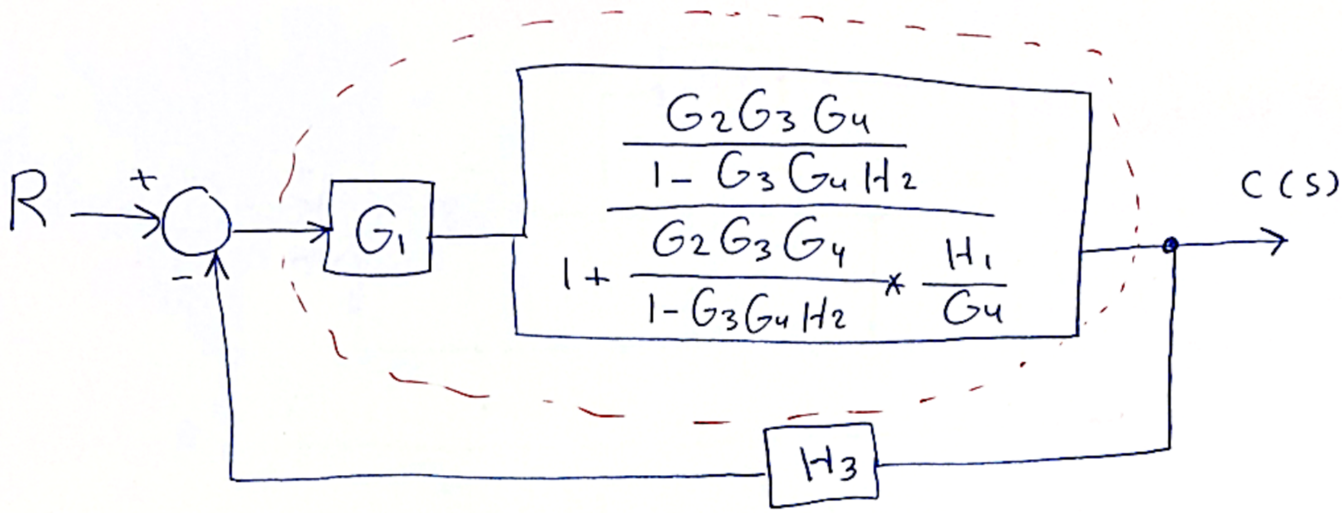


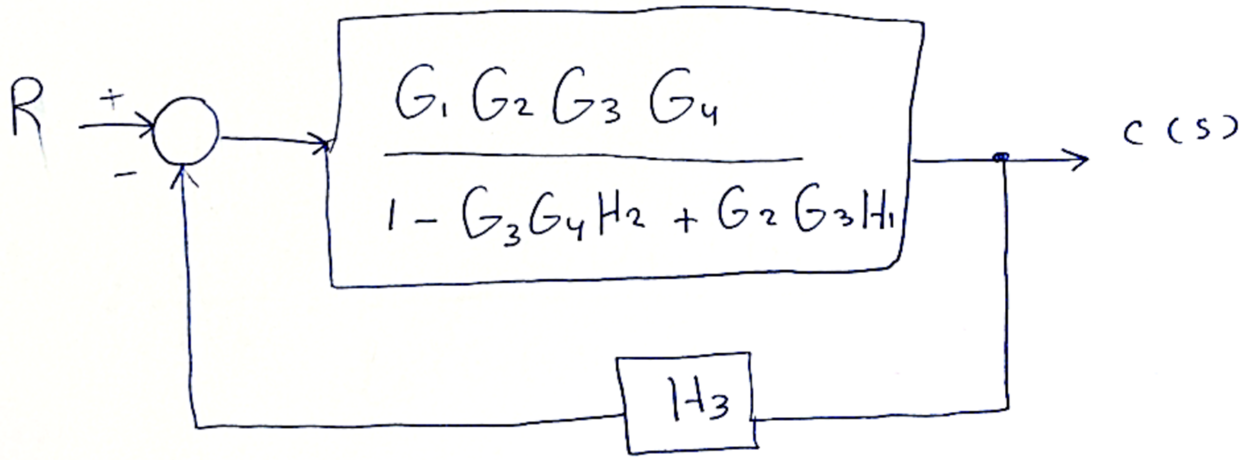


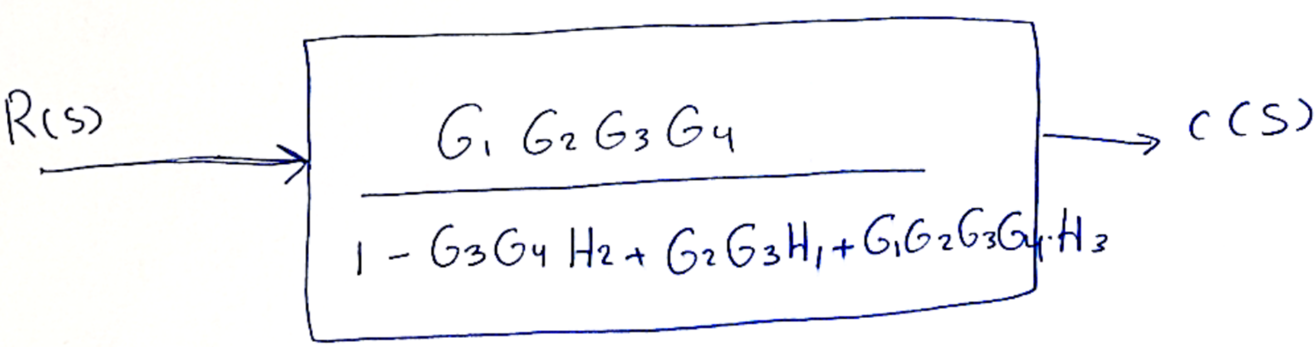





R
+
-
G1
H1/G4
-
+
G2 G3 G4
1 - G3 G4 H2
C(s)
H3

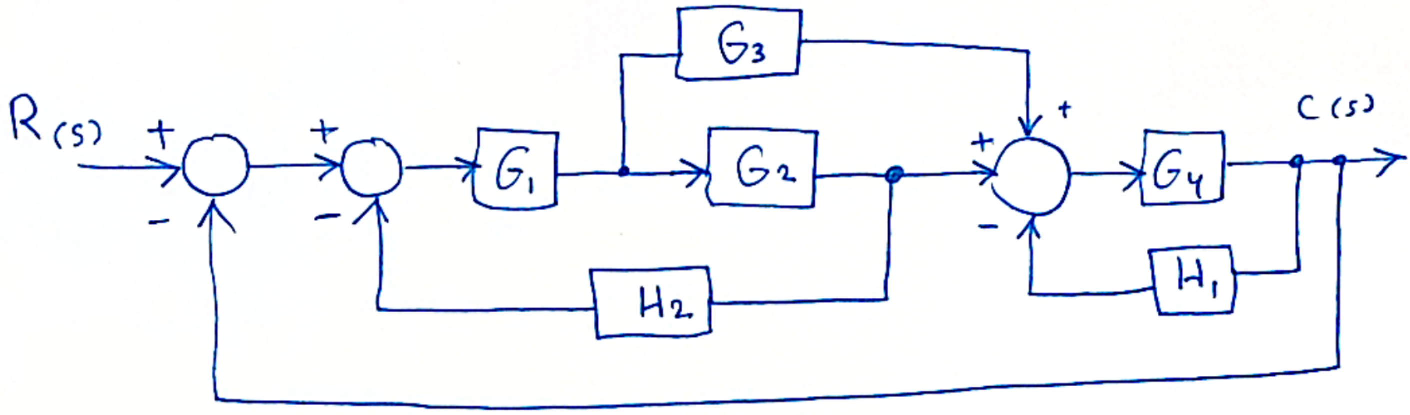

R
+
-
G1
G2G3G4
1 - G3G4H2
1 + G2G3G4 / 1 - G3G4H2 × H1/G4
C(s)
H3


R
+
-
G1 G2 G3 G4
1 - G3G4H2 + G2G3H1
C(s)
H3


R(s)
G1 G2 G3 G4
1 - G3G4 H2 + G2G3H1 + G1G2G3G4·H3
C(S)

H.W)

Simplify the block diagram and find the Transfer function of the system:



Ans: $$\frac{C(s)}{R(s)} = \frac{G_1 G_4 [G_2 + G_3]}{[1 + G_1 G_2 H_2][1 + G_4 H_1] + G_1 G_4 [G_2 + G_3]}$$

Dept. of Communication Tech. Engineering

Fourth Stage

Control System

Al- Farahidi University

2023-2024

Lec.5

Signal Flow Graph

Assistant lecturer

Yasameen Hameed Al-aarajy

"Signal Flow Graph"

- Alternative method to block diagram representation, developed by : Samuel Jefferson Mason.

- A signal flow-graph consists of a network in which nodes are connected by directed branches.

- It depicts the flow of signals from one point of a system to another, and gives the relationships among the signals.

Mason's Gain Formula :-

The relation between the i/p variable & the o/p variable of a signal flow graphs is given by the net gain between the i/p & the o/p nodes and is known as:
Overall gain of the system.
Mason's gain formula for the determination of overall system gain is given by :

$$\boxed{T.F = \frac{1}{\Delta} \sum_{i=1}^{n} P_i \Delta_i}$$

or

$$\boxed{T.F = \sum_{i=1}^{n} \frac{P_i \Delta_i}{\Delta}}$$

where:

T.F = Transfer function, or
Over all gain of the system.

P_i = Forward Path gain قيمة المسار الأمامي

Δ_i = The value of Δ for that part of the graph not touching the forward path. معامل المسار الأمامي

Δ = Determinant of the graph. محدد ماسون

$$\Delta = 1 - \left[\begin{matrix}\text{sum of loop}\\ \text{gains of all}\\ \text{indivisulal}\\ \text{loops}\end{matrix}\right] + \left[\begin{matrix}\text{Sum of gain products}\\ \text{of all possible}\\ \text{combinations of two}\\ \text{non touching loops}\end{matrix}\right] - \left[\begin{matrix}\text{Sum of gain}\\ \text{Products of}\\ \text{all three}\\ \text{non touching}\\ \text{loops}\end{matrix}\right] +$$

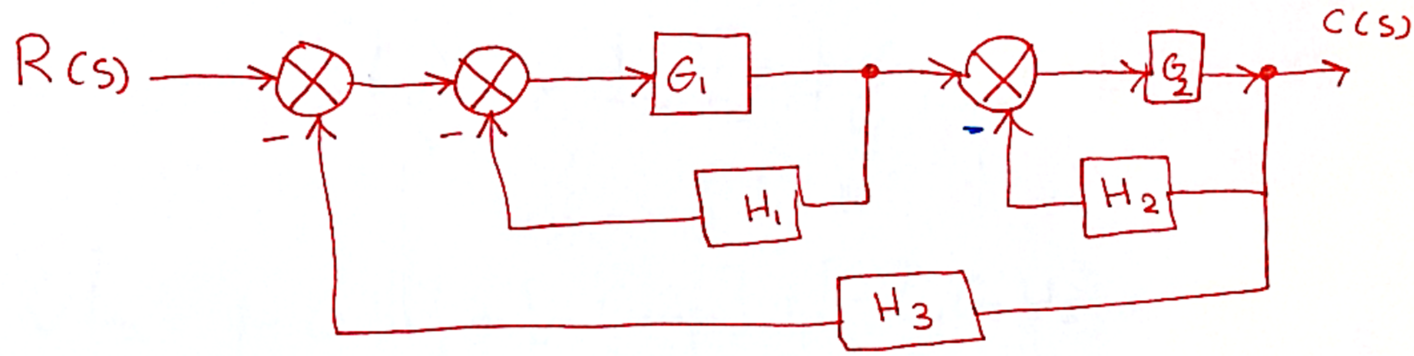
$$= 1 - \sum_a L_a + \sum_{bc} L_b L_c - \sum_{def} L_d L_e L_f + \cdots$$

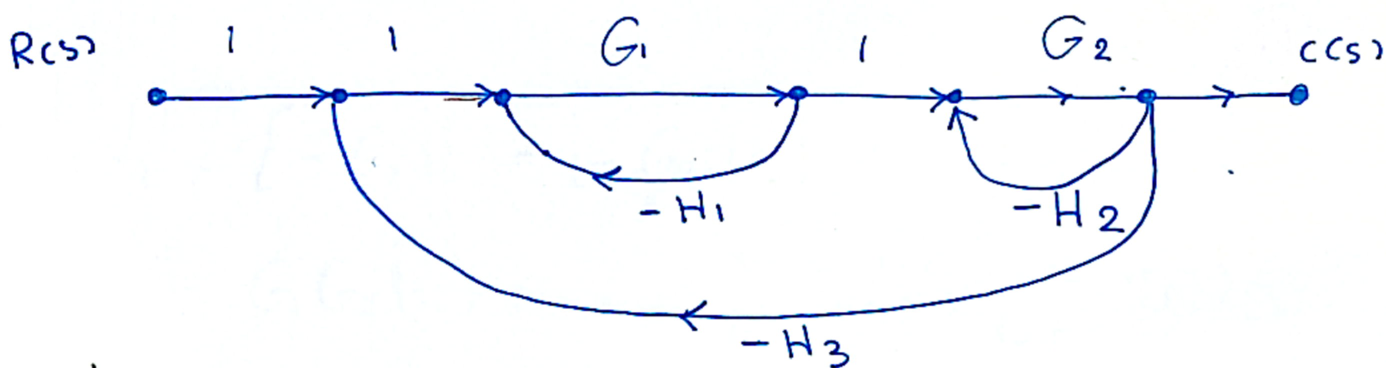
مجموع المسارات المغلقة [Loops]

حاصل جمع كل مسارين مغلقين غير متلامسين

حاصل جمع كل ثلاث مسارات مغلقة غير متلامسة

example : 1 :





Sol:

1- نحدد اللوبات

2- نجد قيمة Δ [محدد ماسون]

3- نحدد المسارات الامامية

4- نجد قيمة Δ_i [معامل المسار الأمامي]

① no. of loops gain:

نجد قيم اللوبات المغلقة

$$L_1 = -G_1 H_1$$

$$L_2 = -G_2 H_2$$

$$L_3 = -G_1 G_2 H_3$$

② نجد قيمة Δ محدد ماسون

$$\Delta = 1 - \sum L_a + \sum L_b L_c - \sum L_d L_e L_f$$

$$\sum L_a = [-G_1H_1] + [-G_2H_2] + [-G_1G_2H_3]$$
$$= -[G_1H_1 + G_2H_2 + G_1G_2H_3]$$

مجموع المسارات المغلقه loops

$$\sum L_b L_c = [-G_1H_1] * [-G_2H_2]$$
$$= G_1G_2H_1H_2$$

حاصل ضرب مسارين مغلقين غير متلامسين

$$\sum L_d L_e L_f = 0$$

$$\therefore \Delta = 1 + G_1H_1 + G_2H_2 + G_1G_2H_3 + G_1G_2H_1H_2$$

---

③ Forward Path Gain :~

نجد عدد و قيم المسارات الأمامية

$n = 1$ عدد المسارات الامامية

$$P_1 = 1 * 1 * G_1 * 1 * G_2 * 1$$
$$= G_1G_2$$

قيمة المسار الامامي الاول

---

④ $\Delta i = 1 - 0 + 0$

$= 1$

نفس قانون Δ لكن مسارات غير متلامسة للمسار الاول

مجموع ال loops الغير متلامسة معه

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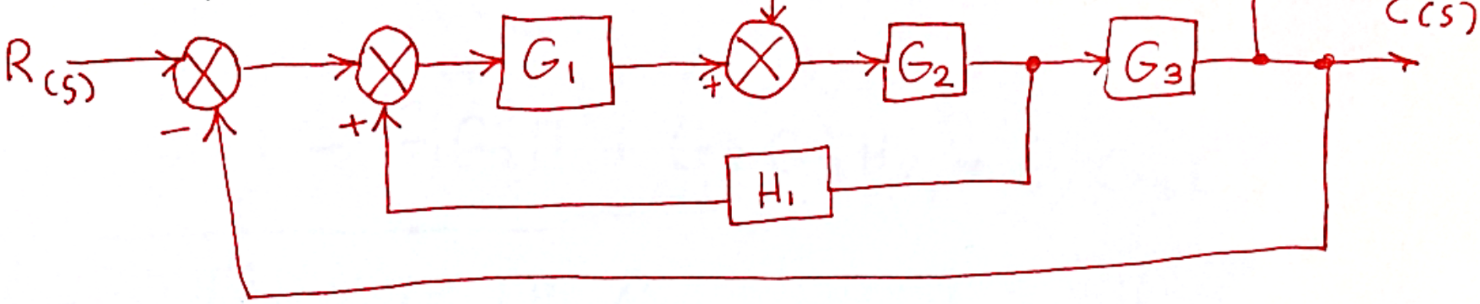
$$\therefore T.F = \frac{1}{\Delta} \sum P_i \Delta_i$$

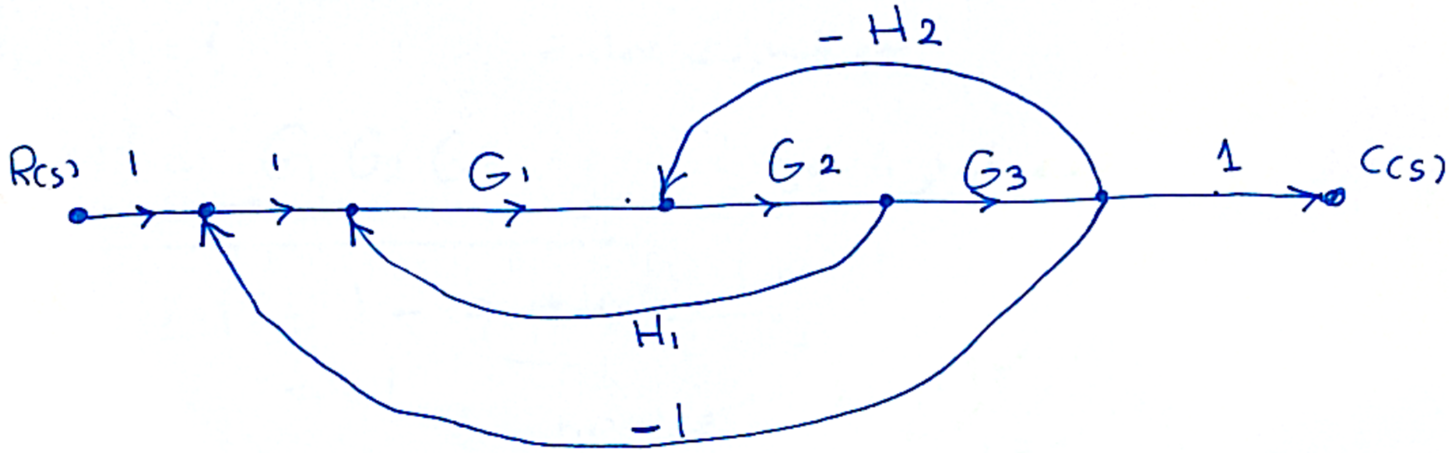
$1/\Delta$ $\quad P_i \quad \Delta_i$

$$= \frac{1}{1 + G_1 H_1 + G_2 H_2 + G_1 G_2 H_3 + G_1 G_2 H_1 H_2} * G_1 G_2 * 1$$

$$= \frac{G_1 G_2}{1 + G_1 H_1 + G_2 H_2 + G_1 G_2 H_3 + G_1 G_2 H_1 H_2}$$

example: 2





Sol:

① no. of loops gain:

$$L_1 = G_1 G_2 H_1$$

$$L_2 = -G_2 G_3 H_2$$

$$L_3 = -G_1 G_2 G_3$$

⇒ اولاً: نجد قيم اللوبات المغلقة

② $\Delta = 1 - \sum L_a + \sum L_b L_c - \sum L_d L_e L_f$

⇓

$\sum L_a = G_1 G_2 H_1 - G_2 G_3 H_2 - G_1 G_2 G_3$ ← مجموع اللوبات

$\sum L_b L_c = 0$ ← حاصل ضرب مسارين مغلقين غير متلامسين

$\sum L_d L_e L_f = 0$ ← متلامسة

$$\therefore \Delta = 1 - [G_1G_2H_1 - G_2G_3H_2 - G_1G_2G_3] + 0 - 0$$

$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

---

③ Forward Path Gain :-

$n = 1$ عدد المسارات الامامية

$P_1 = G_1 G_2 G_3$ قيمة المسار الامامي

---

④ $\Delta_1 = 1 - 0 + 0$

$= 1$

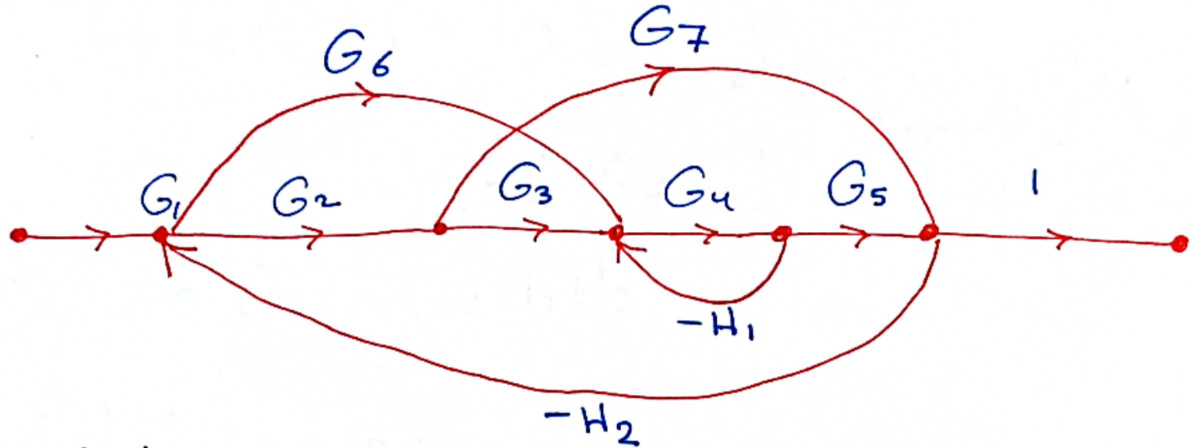
مجموع الـ loops الغير متلامسة معه

مجموع 2 loops غير متلامسة مع هذا المسار

$$\therefore T.F = \frac{C(s)}{R(s)} = \frac{\sum P_i \Delta_i}{\Delta}$$

$$= \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

example : 3



Sol

① no. of loops gain :

$L_1 = -G_4 H_1$

$L_2 = -G_2 G_3 G_4 G_5 H_2$

$L_3 = -G_2 G_7 H_2$

$L_4 = -G_6 G_4 G_5 H_2$

أولاً: نجد قيم ال loops المغلقة

② $\Delta = 1 - \sum L_a + \sum L_b L_c - \sum L_d L_e L_f$

$$\sum L_a = -[G_4 H_1] + [-G_2 G_3 G_4 G_5 H_2] + [-G_2 G_7 H_2] + [-G_6 G_4 G_5 H_2]$$

مجموع ال loops

$$= -[G_4 H_1 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2]$$

$\Downarrow$

$$\sum L_b L_c = L_1 * L_3$$ ← 2 loops حاصل ضرب غير متلامسين

$$= [-G_4 H_1] * [-G_2 G_7 H_2]$$

$$= G_2 G_4 G_7 H_1 H_2$$

$$\sum L_d L_e L_f = 0$$

$$\therefore \Delta = 1 + [G_4 H_1 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2]$$
$$+ [G_2 G_4 G_7 H_1 H_2]$$

---

③ Forward Path Gain :~

$n = 3$ → عدد المسارات الامامية

⇓

$$P_1 = G_1 G_2 G_3 G_4 G_5$$
$$P_2 = G_1 G_6 G_4 G_5$$
$$P_3 = G_1 G_2 G_7$$

⇒ قيم المسارات الامامية
Forward Paths

---

④ Δi

مجموع الـ loops الغير متلامسة معه

$$\Delta_1 = 1 - 0$$

$$\Delta_2 = 1 - 0$$

$$\Delta_3 = 1 - [-G_4 H_1] = 1 + G_4 H_1$$

واحد غير متلامس مع المسار P_3

$$\therefore \; T.F = \frac{\sum P_i \Delta_i}{\Delta}$$

$$= \frac{[G_1 G_2 G_3 G_4 G_5 * 1] + [G_1 G_6 G_4 G_5 * 1] + [G_1 G_2 G_7 * (1 + G_4 H_1)]}{1 + [G_4 H_1 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_6 G_4 G_5 H_2 + H_1 H_2 G_2 G_4 G_7]}$$

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