Dept. of Communication Tech. Engineering

Fourth Stage

Control System

Al- Farahidi University

2023-2024

Lec.8

Time Response Analysis

Assistant lecturer

Yasameen Hameed Al-aarajy

Introduction

- In time-domain analysis the response of a dynamic system to an input is expressed as a function of time.
- It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.
- Usually, the input signals to control systems are not known fully ahead of time.
- For example, in a radar tracking system, the position and the speed of the target to be tracked may vary in a random fashion.
- It is therefore difficult to express the actual input signals mathematically by simple equations.

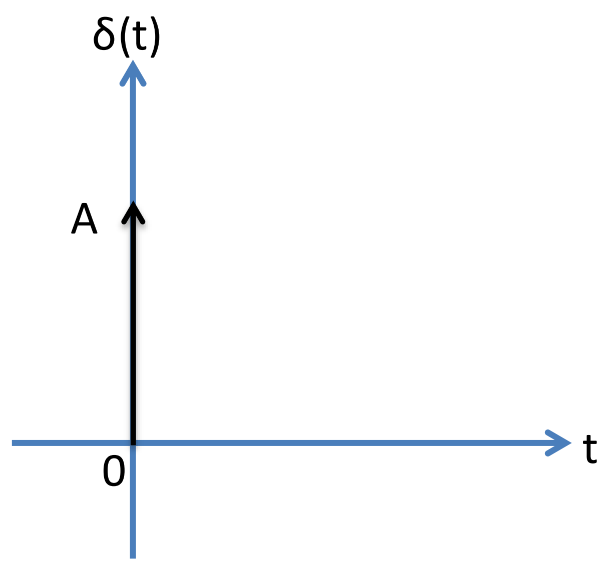
Standard Test Signals
اشارات الفحص

- The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and constant acceleration.
- The dynamic behavior of a system is therefore judged and compared under application of standard test signals – an impulse, a step, a constant velocity, and constant acceleration.
- Another standard signal of great importance is a sinusoidal signal.

Standard Test Signals

- Impulse signal اشارة النبضة
  - The impulse signal imitate the sudden shock characteristic of actual input signal.

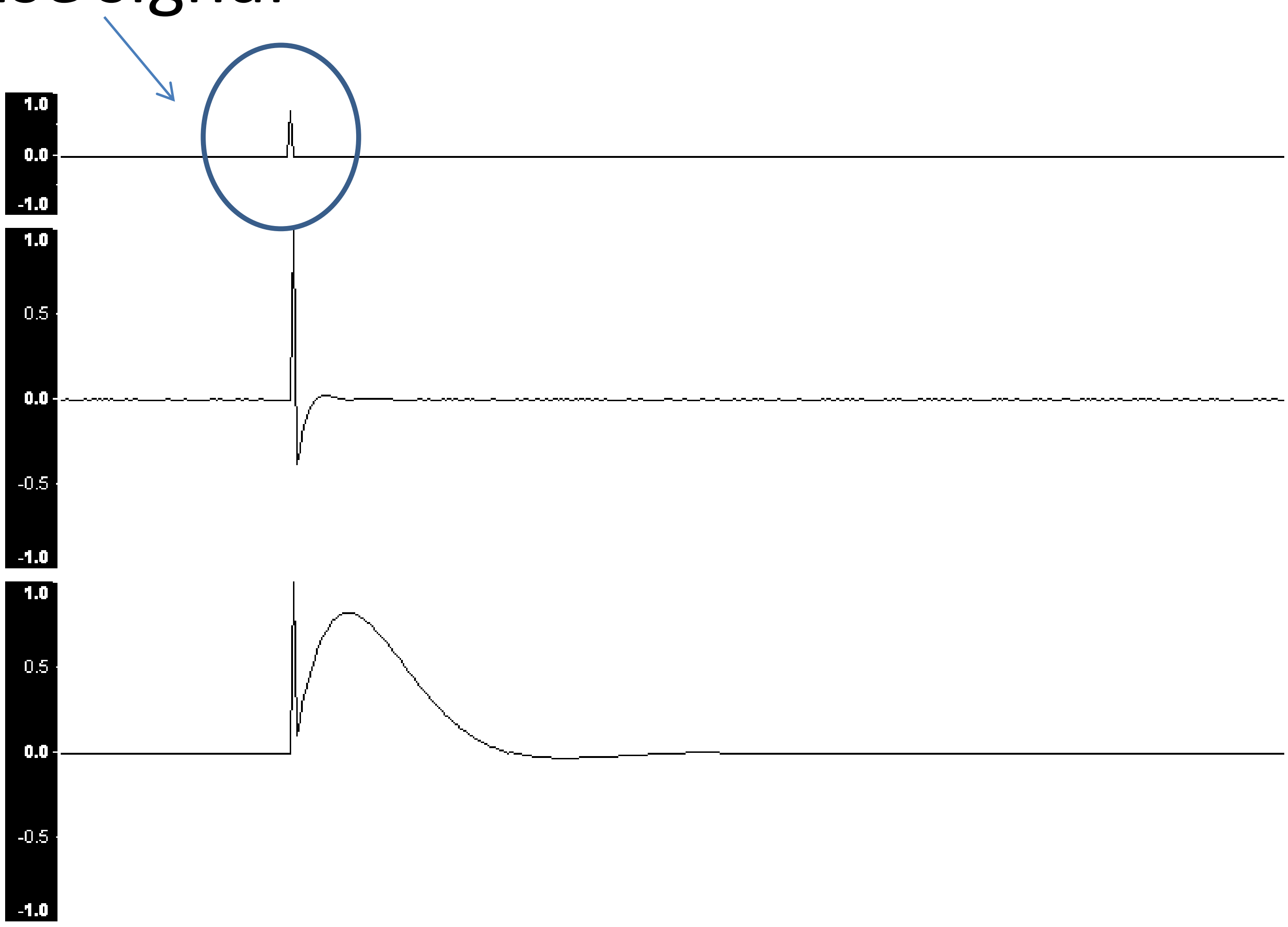
$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$



  - If A=1, the impulse signal is called unit impulse signal.

Standard Test Signals

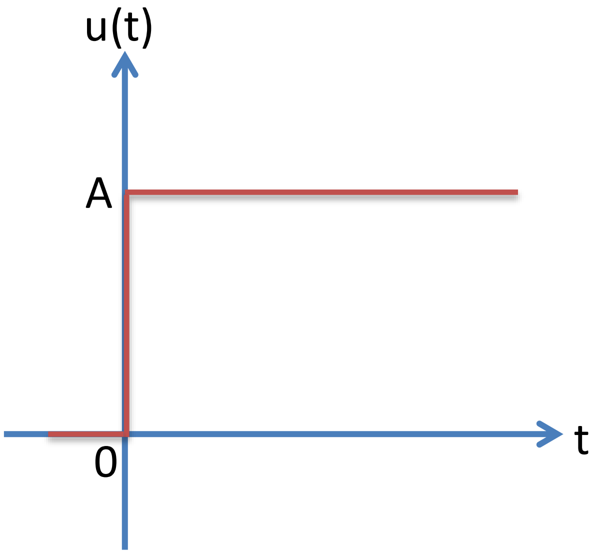
- Impulse signal



Source: English Wikipedia. Iain. Original image: [1]

Standard Test Signals

- Step signal اشارة الخطوة
  - The step signal imitate the sudden change characteristic of actual input signal.

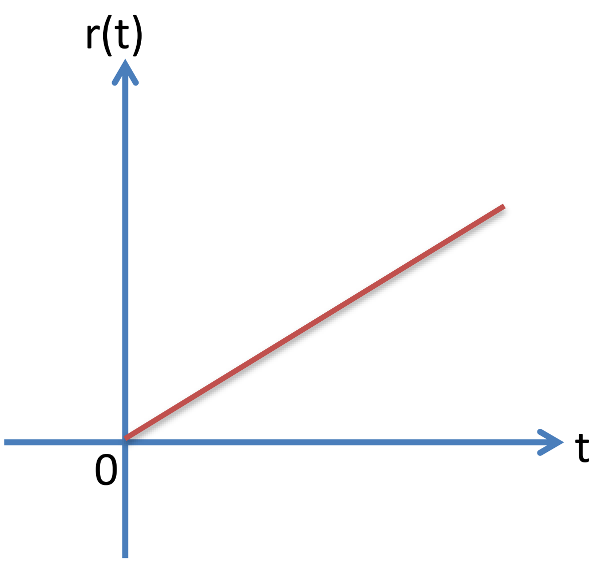


$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

  - If A=1, the step signal is called unit step signal

Standard Test Signals

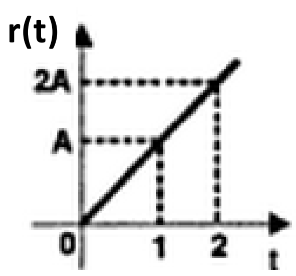
- Ramp signal اشارة المنحدر
  - The ramp signal imitate the constant velocity characteristic of actual input signal.



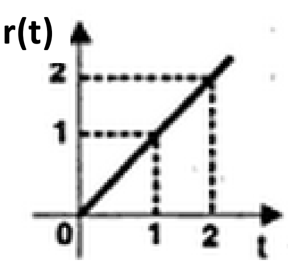
$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

  - If *A=1*, the ramp signal is called unit ramp signal

ramp signal with slope A

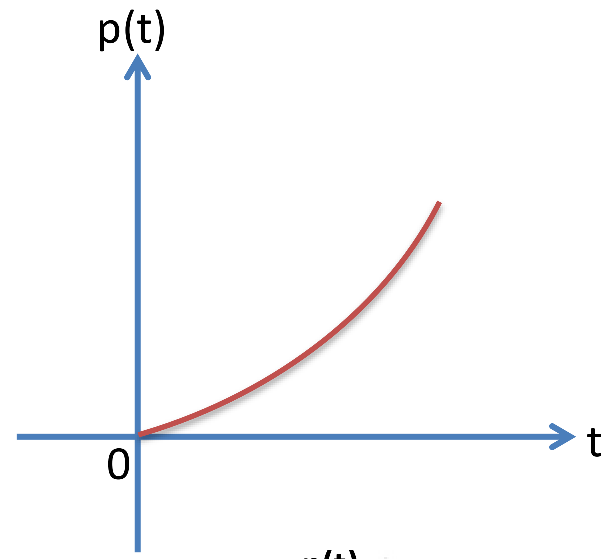


unit ramp signal



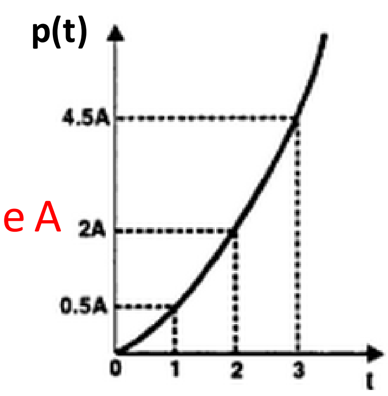
Standard Test Signals

- Parabolic signal اشارة القطع المكافئ
  - The parabolic signal imitate the constant acceleration characteristic of actual input signal.

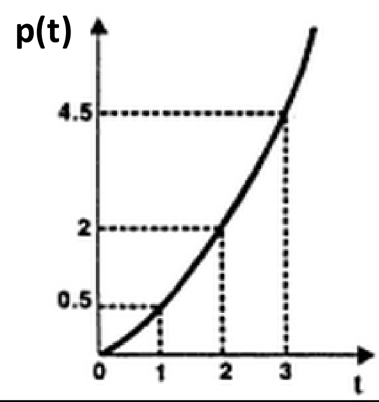


$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

  - If *A=1*, the parabolic signal is called unit parabolic signal.



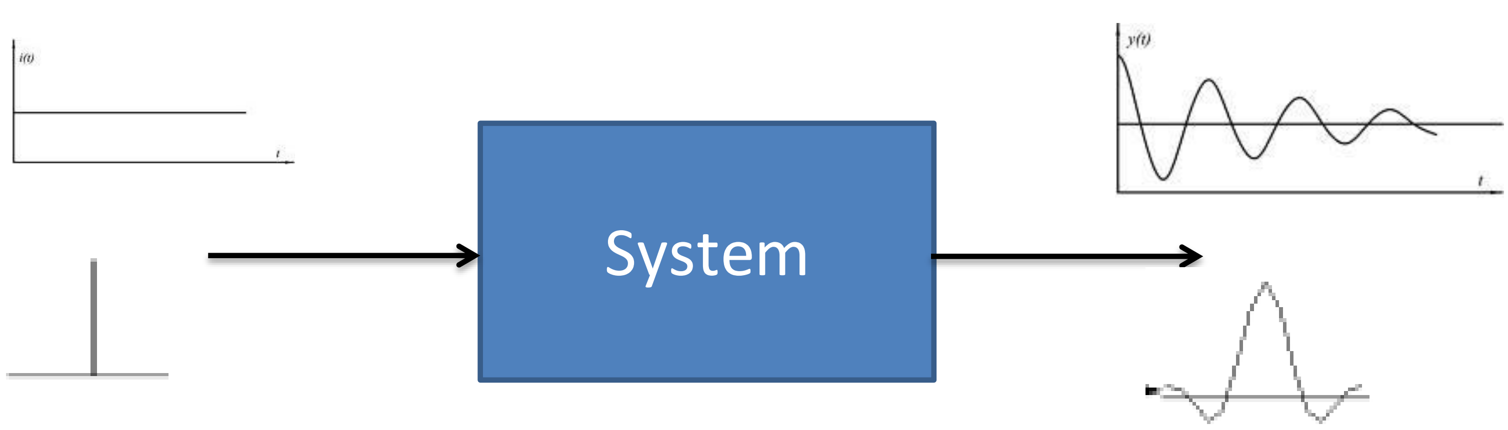
parabolic signal with slope A



Unit parabolic signal

Time Response of Control Systems

- Time response of a dynamic system response to an input expressed as a function of time.



- The time response of any system has two components
  - Transient response
  - Steady-state response.

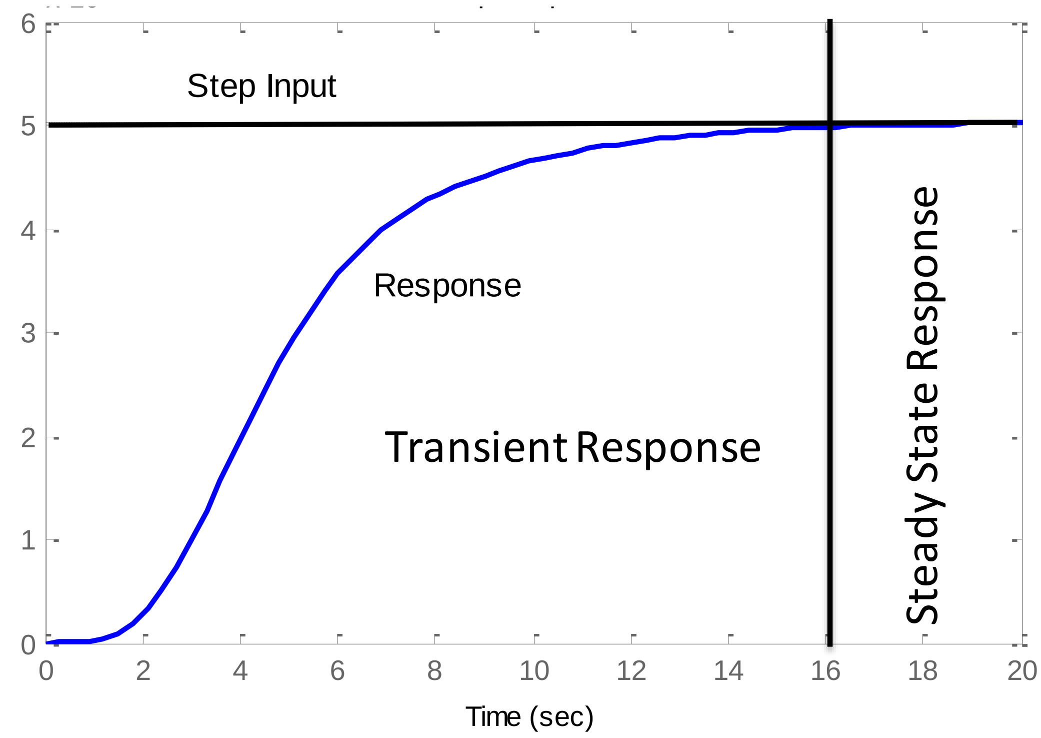
Time Response of Control Systems

الاداء الزمني لمنظومات السيطرة

- When the response of the system is changed form rest or equilibrium it takes some time to settle down.

- Transient response is the response of a system from rest or equilibrium to steady state.

- The response of the system after the transient response is called steady state response.



" Time Response Analysis "

" التحليل الزمني لمنظومات السيطرة "

1- First Order System :~

$$R(s) \longrightarrow \boxed{\frac{K}{TS+1}} \longrightarrow C(s)$$

$$\frac{C(s)}{R(s)} = \frac{K}{TS+1}$$

$R(s)$ = input signal

$R(s)$:~

- Impulse input

  $R(s) = A$

  unit impulse i/p $\rightarrow$ $R(s) = 1$

- Step Input

  $R(s) = \frac{A}{S}$

  unit step i/p $\longrightarrow$ $R(s) = \frac{1}{S}$

- Ramp Input

  $R(s) = \frac{A}{S^2}$

  unit ramp i/p $\longrightarrow$ $R(s) = \frac{1}{S^2}$

ex: first order system with unit step input :~

$$\frac{C(s)}{R(s)} = \frac{K}{TS+1}$$

$$\therefore C(S) = \frac{K}{TS+1} * R(s)$$

$$= \frac{K}{TS+1} * \frac{1}{S}$$

---

ex: first order system with ramp input :~

$$\frac{C(S)}{R(S)} = \frac{K}{TS+1}$$

$$\therefore C(S) = \frac{K}{TS+1} * R(S)$$

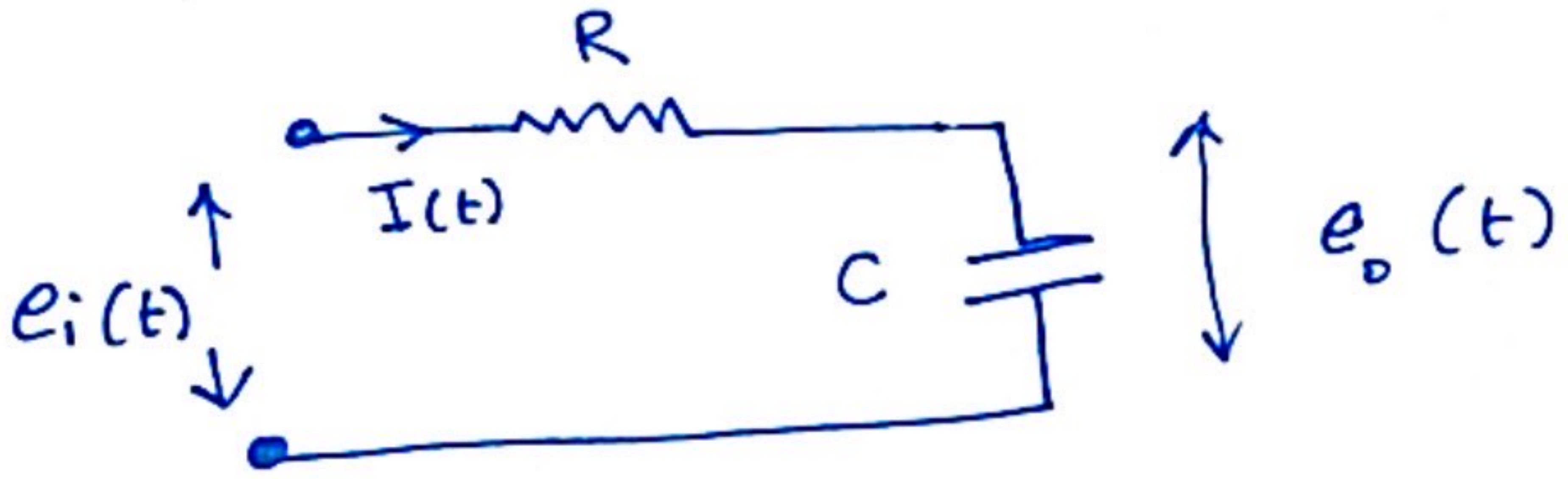
$$= \frac{K}{TS+1} * \frac{A}{S^2}$$

or a

ex :~ Check if the following System represent a first order system, then find [K, T, a] , if :

$e_i(t) = 5\ V$

Sol)



Sol)

$$\frac{e_o(s)}{e_i(s)} = \frac{I(s) * \frac{1}{cs}}{[I(s) * R] + [I(s) * \frac{1}{cs}]}$$

$$= \frac{\cancel{I(s)} * \frac{1}{cs}}{\cancel{I(s)} \left[R + \frac{1}{cs}\right]}$$

$$= \left.\frac{1/cs}{R + 1/cs}\right] * \frac{cs}{cs}$$

$$\therefore \frac{e_o(s)}{e_i(s)} = \frac{1}{Rcs + 1}$$

$$\therefore e_o(s) = \frac{1}{RCS+1} * e_i(s)$$

note :~

$e_i(t) = 5$

↓ taking Laplace

$\therefore e_i(s) = \frac{5}{s}$

⇓

$$\therefore E_o(s) = \frac{1}{Rcs+1} * \frac{5}{s}$$

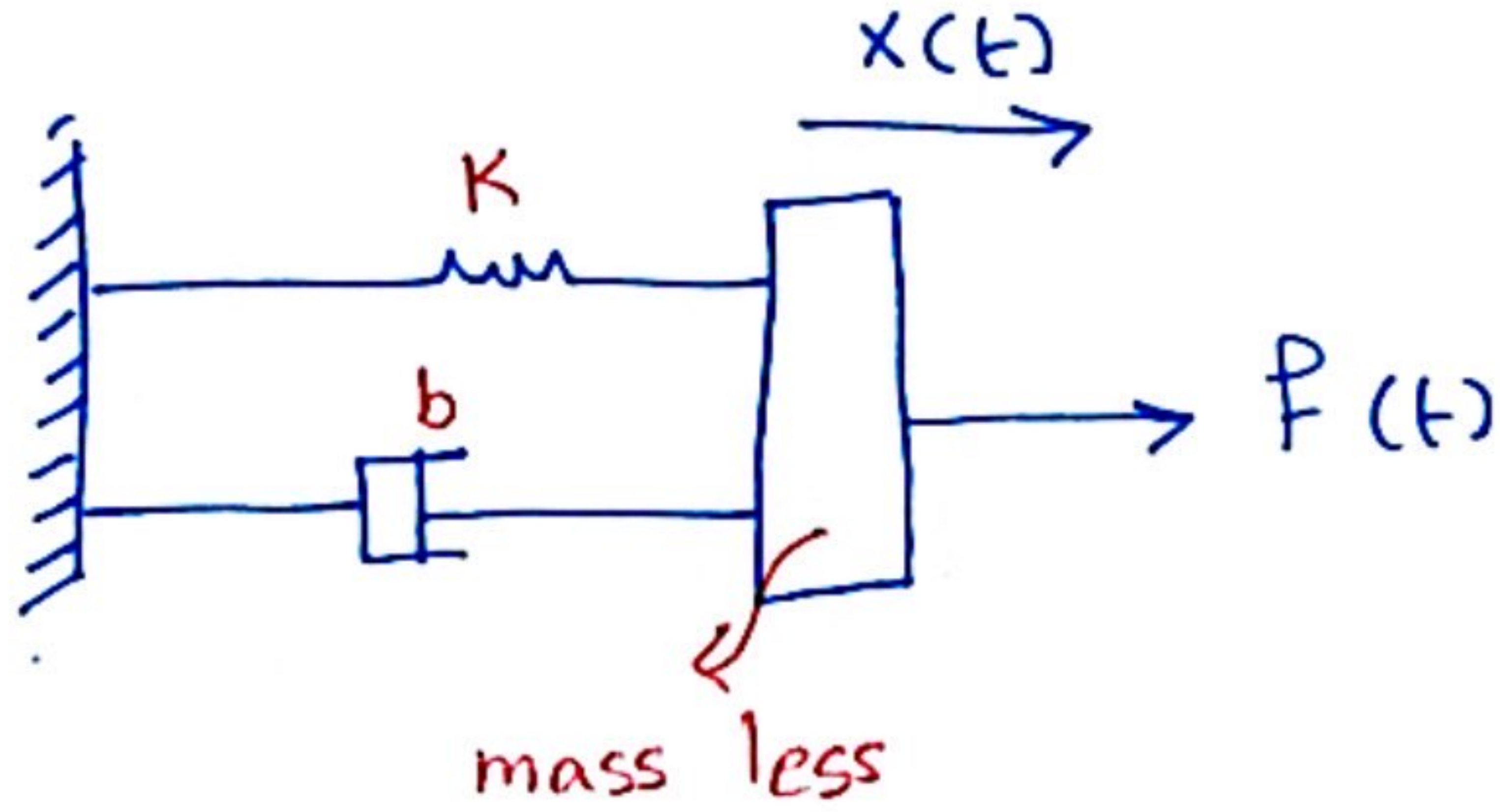
$$\boxed{\therefore K = 1 \ , \ T = RC \ , \ a = 5}$$

ex:~ find $\frac{X(s)}{F(s)}$, if $f(t) = 50\,t\ N$ ← نيوتن

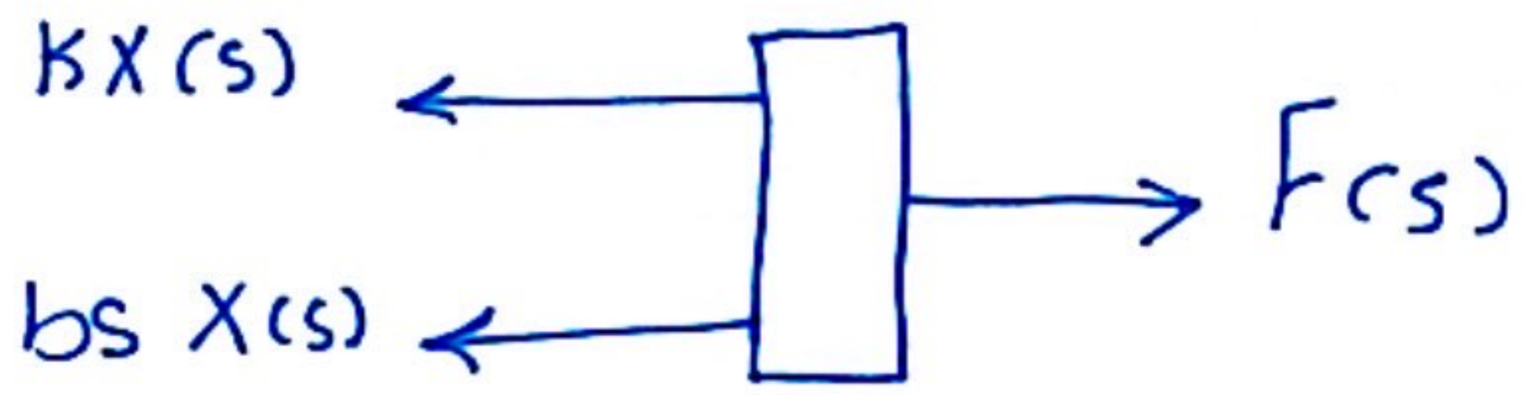
laplace :

$F(s) = \frac{50}{S^2}$

Ramp i/p



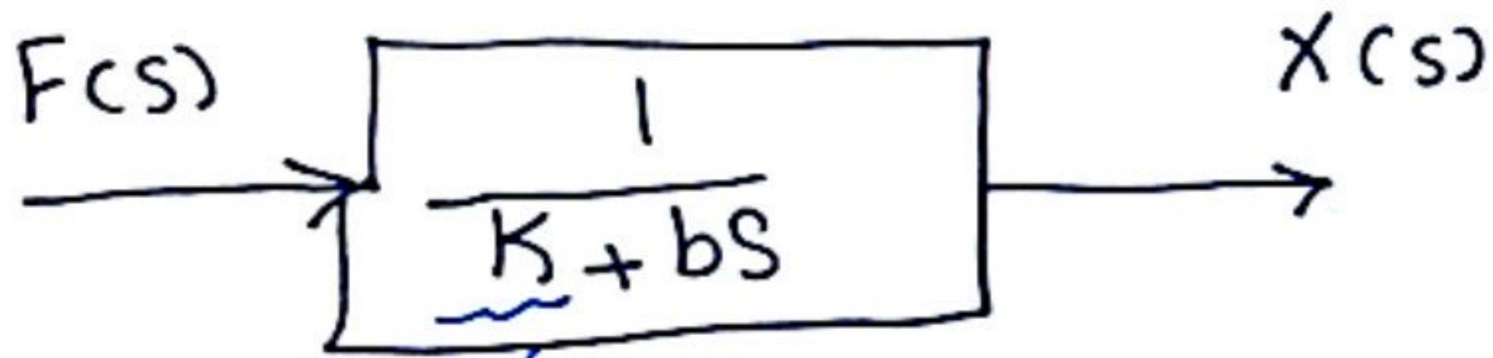
⇓



$$K\,X(s) + bS\,X(s) = F(s)$$

$$X(s)\,[K + bs] = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{1}{K + bS}$$



يجب أن يكون 1

نقسم على K :.

$$\therefore \frac{X(s)}{F(s)} = \frac{1/K}{1 + b/K\, S}$$

$$\therefore X(s) = \frac{1/K}{\frac{b}{K}S+1} * F(s)$$

$$= \boxed{\frac{1/K}{\frac{b}{K}S+1} * \frac{50}{S^2}}$$

Scanned with CamScanner

$$\therefore \; K = \frac{1}{k}$$

$$T = \frac{b}{k}$$

$$a = 50$$

---

ex: First Order System with Step Input Response:

$$R(s) = \frac{a}{S}$$

Sol)

$$R(s) \longrightarrow \boxed{\frac{K}{TS+1}} \longrightarrow C(S)$$

$$\frac{C(S)}{R(S)} = \frac{K}{TS+1}$$

$$\therefore \; C(S) = \frac{K}{TS+1} * R(s)$$

$$\therefore \; C(S) = \frac{K}{TS+1} * \frac{a}{S}$$

الآن : لكي نقوم بعملية التحليل الزمني [Time Response Analysis] يجب أن أحول الدالة الى Time domain

∴ يجب استخدام : laplace inverse

في مثل هذه الدوال تستخدم طريقة الـ partial لايجاد laplace inverse

now, finding $c(t)$:

$$C(S) = \frac{K a}{(TS+1)S} = \frac{A}{TS+1} + \frac{B}{S}$$

$$A = \left[\frac{K a}{S}\right]_{S = \frac{-1}{T}}$$

نعوض المقامات بصفر ونجد الحدود أو الجذور

$$\left[TS+1 = 0 \Rightarrow TS = -1 \Rightarrow S = \frac{-1}{T} \right.$$

$$\therefore \; \boxed{A = -KaT}$$

$\Downarrow$

$$B = \left[\frac{Ka}{TS+1}\right]_{S=0} \Rightarrow \boxed{B = Ka}$$

$$\therefore C(S) = \frac{\overbrace{-KaT}^{A}}{TS+1} + \frac{\overbrace{Ka}^{B}}{S}$$

$$\therefore C(S) = \frac{-Ka}{S+\frac{1}{T}} + \frac{Ka}{S}$$

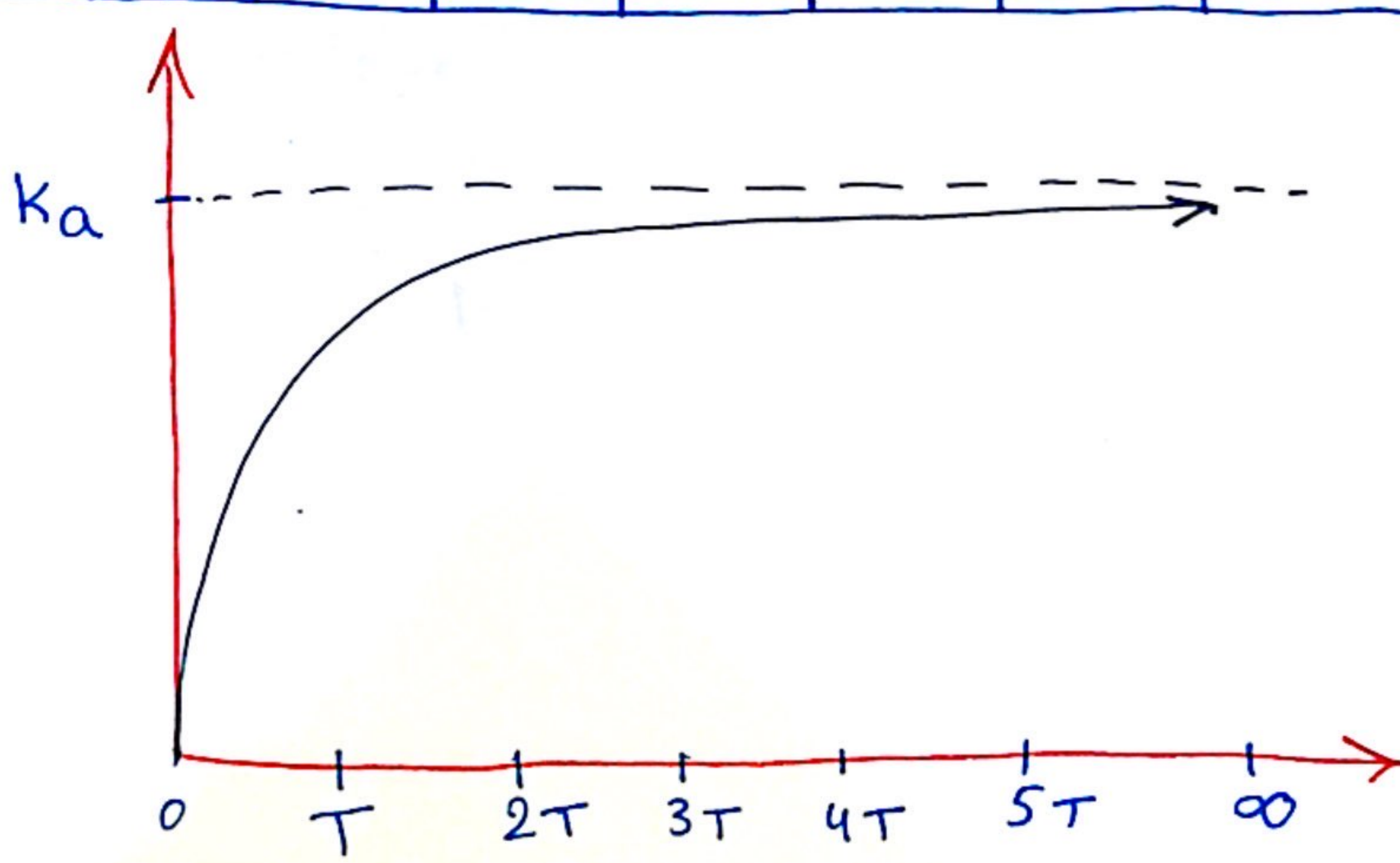
الآن : بعد التبسيط
نحول من S domain
الى Time domain

$$e^{-at} \xrightarrow{\mathcal{L}} \frac{1}{s+a}$$

$$\therefore C(t) = \underbrace{-Ka\, e^{-\frac{1}{T}\cdot t}} + Ka$$

$$\boxed{\therefore C(t) = Ka\left[1 - e^{-\frac{1}{T}\cdot t}\right]}$$

t	0	T	2T	3T	4T	5T	∞
$C(t)$	0	0.63 Ka	0.86 Ka	0.95 Ka	0.98 Ka	0.99 Ka	Ka



ex:) First Order System with unit step input:

$R(s) = \frac{1}{s}$

Sol)

$$R(s) \longrightarrow \boxed{\frac{1}{Ts+1}} \longrightarrow C(s)$$

Sol) from the block diagram we found that:

$K = 1$, $T = T$, $a = 1$, unit step i/p

$$\therefore \quad C(s) = \frac{1}{Ts+1} * R(s)$$

$$= \frac{1}{Ts+1} * \frac{1}{s} = \frac{1}{s(Ts+1)}$$

$$= \frac{A}{Ts+1} + \frac{B}{s}$$

$$\therefore \quad A = \left[\frac{1}{s}\right]_{s=\frac{-1}{T}} \longrightarrow \boxed{A = -T}$$

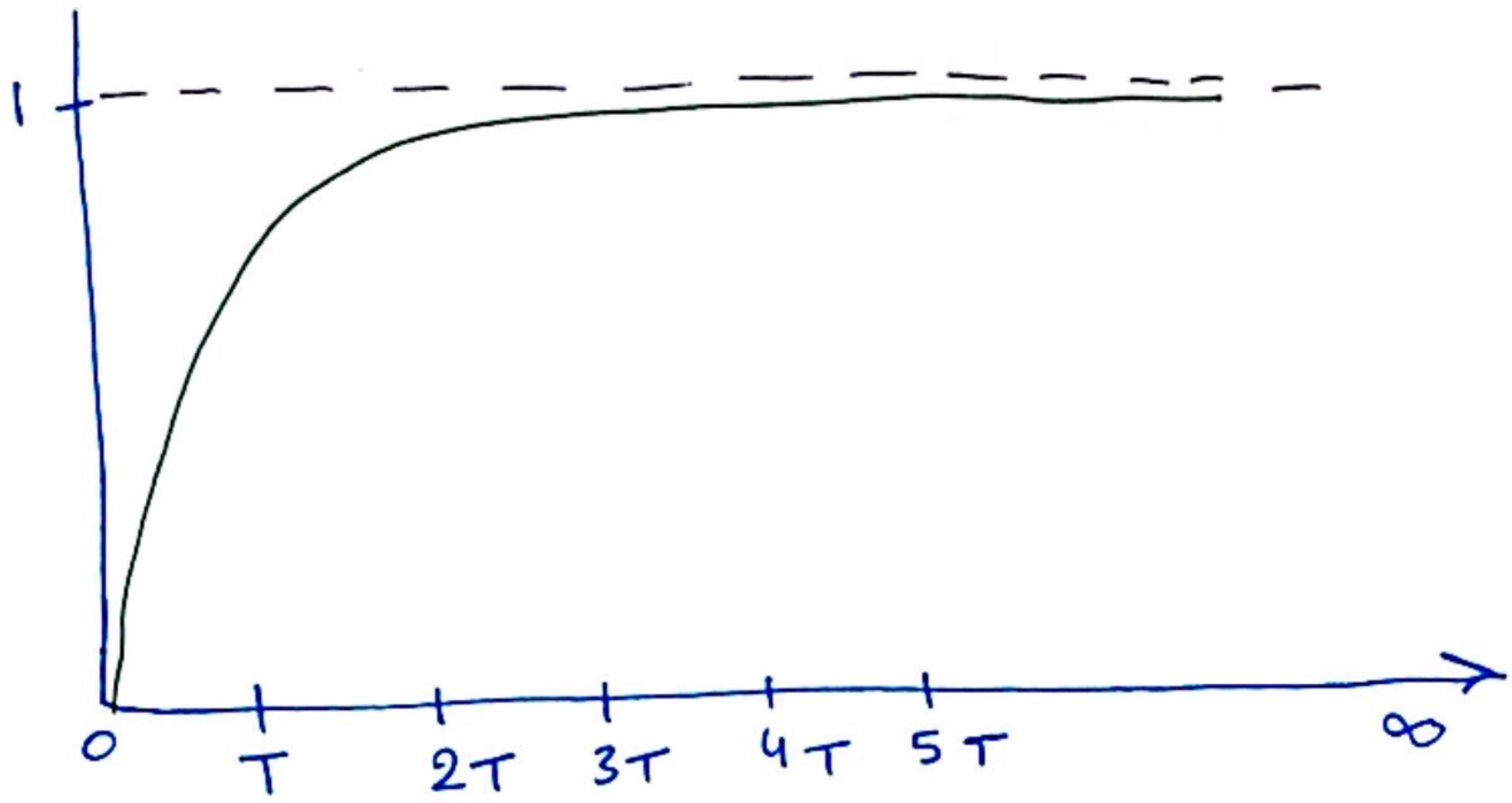
$$B = \left[\frac{1}{Ts+1}\right]_{s=0} \longrightarrow \boxed{B = 1}$$

$$\therefore \quad C(s) = \frac{-T}{Ts+1} + \frac{1}{s}$$

$$= \frac{-1}{s+\frac{1}{T}} + \frac{1}{s}$$

$$\therefore \quad C(t) = -e^{-\frac{1}{T}\cdot t} + 1$$

$$= 1 - e^{\frac{-1}{T}\cdot t}$$



---

ex: First Order System with unit Impulse Response

$R(t) = a$

$\therefore R(s) = 1$

$$R(s) \longrightarrow \boxed{\frac{5}{4S+1}} \longrightarrow C(S)$$

$K = 5$, $T = 4$, $a = 1$, unit impulse input.

$$\frac{C(S)}{R(S)} = \frac{5}{4S+1} \longrightarrow C(S) = \frac{5}{4S+1} * R(s)$$

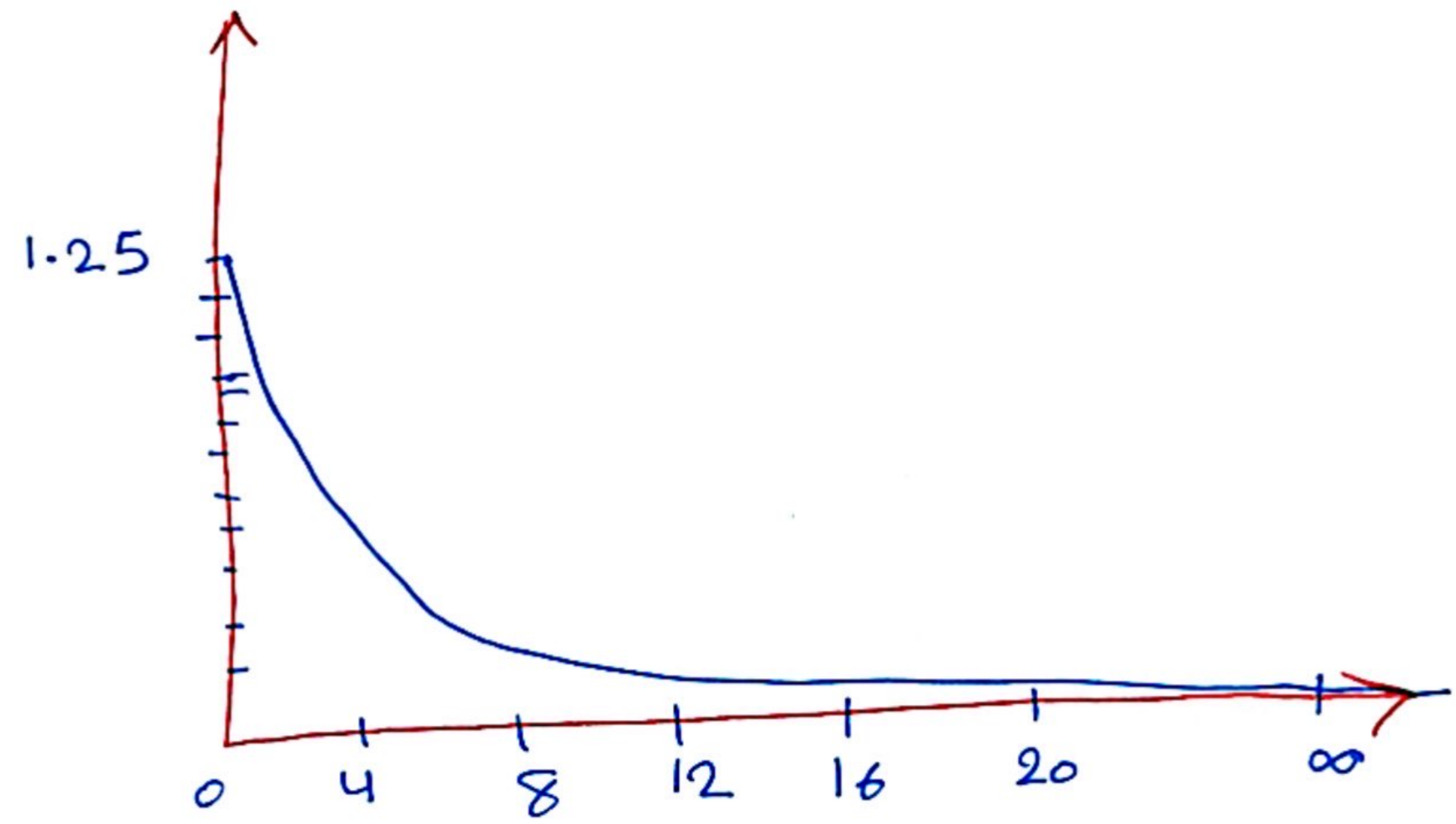
$$\therefore C(S) = \frac{5}{4S+1} * 1 \leftarrow \text{i/p}$$

$$= \frac{5/4}{S + 1/4}$$

$$\therefore C(t) = \frac{5}{4} e^{-1/4\, t}$$

$\Downarrow$

	0	T	2T	3T	4T	5T	
t	0	4	8	12	16	20	∞
$c(t)$	5/4	0.46	0.17	0.06	0.02	8.4×10^{-3}	0



H.W

First Order System with Ramp input response :

$$R(s) = \frac{a}{s^2}$$

Dept. of Communication Tech. Engineering

Fourth Stage

Control System

Al- Farahidi University

2023-2024

Lec.9

Second Order System

Assistant lecturer

Yasameen Hameed Al-aarajy

~ Time Response Analysis ~

2- Second - Order System :~

A general second - order system is characterized by the following transfer function :~

$$R(s) \longrightarrow \boxed{\frac{K\,\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}} \longrightarrow C(S)$$

$$\frac{C(s)}{R(s)} = \frac{K.\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

$$\therefore\ C(s) = \frac{K.\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2} * R(s)$$

The characteristic equation for the system is :

$$S^2+2\xi\omega_n S+\omega_n^2 = 0$$

$\therefore$ its roots [poles] are given by :~

$$\boxed{P_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}}$$

where :~

ω_n = Un - damped natural frequency.

ξ = Damping Ratio.

ω_n = un-damped natural frequency of the second order system. [which is the frequency of oscillation of the system without damping].

ξ = damping ratio of the second order system, which is a measure of the degree of resistance to change in the system output.

ex: Determined the un-damped natural frequency and damping ratio of the following system :~

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 2s + 4}$$

Sol)

$$\frac{C(s)}{R(s)} = \frac{K\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

by comparing:

* $\omega_n^2 = 4 \longrightarrow \omega_n = 2$ rad/sec

* $K\omega_n^2 = 4 \longrightarrow K = 1$

* $2\xi\omega_n = 2 \longrightarrow \xi = \frac{2}{2\omega_n} = \frac{1}{\omega_n} = \frac{1}{2} = 0.5$

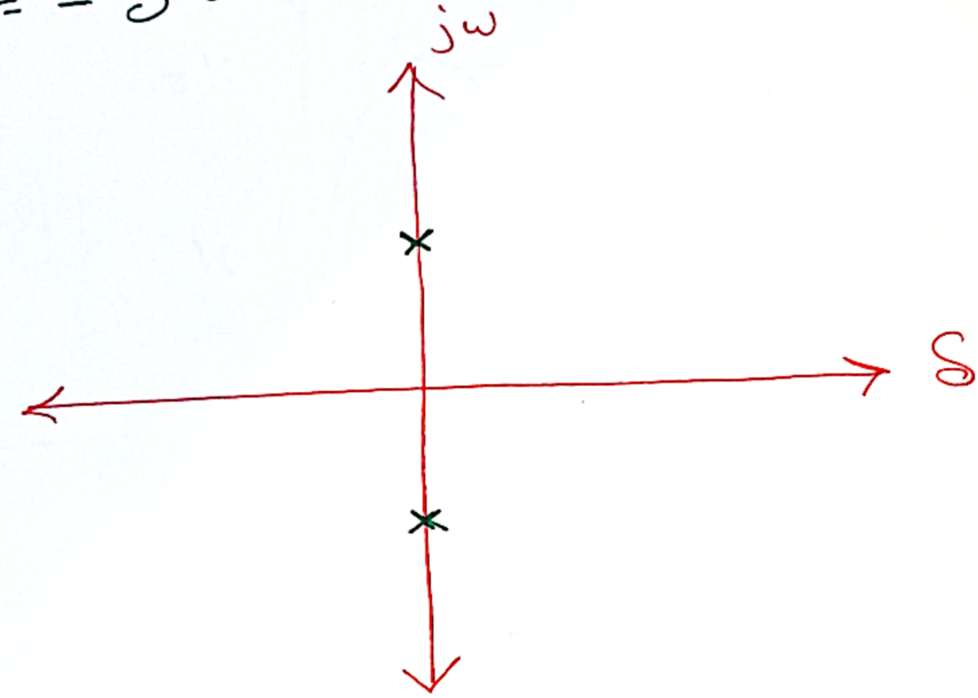
According to the value of ξ, a second order system can be set into one of the four categories:

1- Un-Damped :~

when the system has two imaginary poles.

$[\xi = 0]$

$P_{1,2} = \pm j\omega$



$$G(s) = \frac{K\omega_n^2}{S^2 + \omega_n^2}$$

* for step response :

$$C(S) = G(s) * R(s)$$

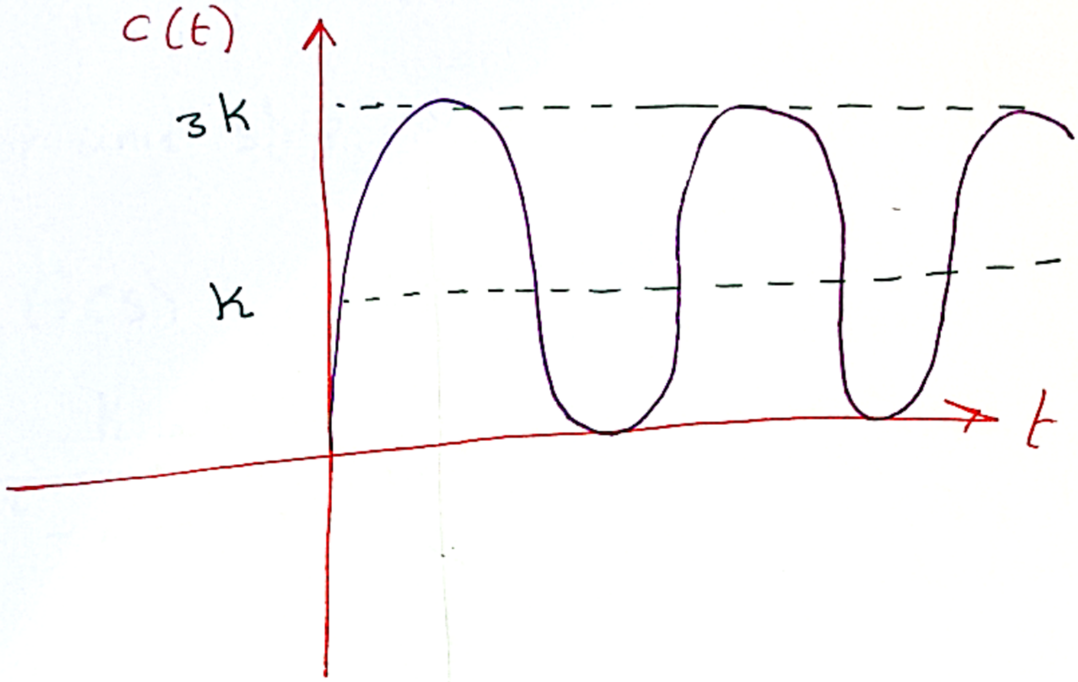
$$= \frac{K\omega_n^2}{S^2 + \omega_n^2} * \frac{1}{S}$$ ← by Partial fraction

⇓

$$\therefore \quad C(S) = K\left[\frac{1}{S} - \frac{S}{S^2+\omega_n^2}\right]$$

by taking Laplace Inverse :

$$\therefore \quad c(t) = K\left[1 - \cos \omega_n t\right]$$



2- Critically Damped :-

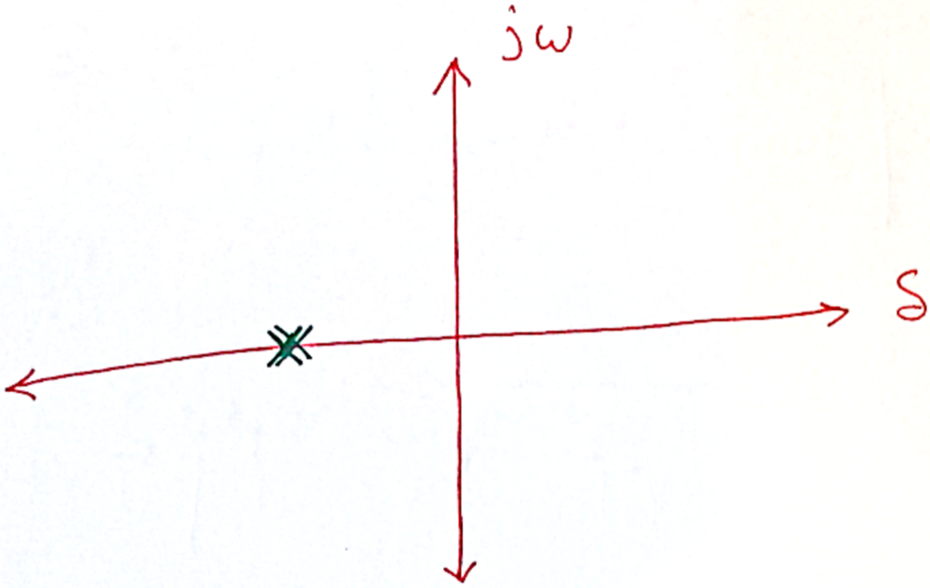
when the system has two real but equal poles.

$[\xi = 1]$

$$P_{1,2} = -\xi\omega_n \pm \sqrt{\xi^2-1}\cdot\omega_n$$

$$= -\omega_n$$

$\Downarrow$



* for unit step response:

$$C(S) = G(S) * R(S)$$

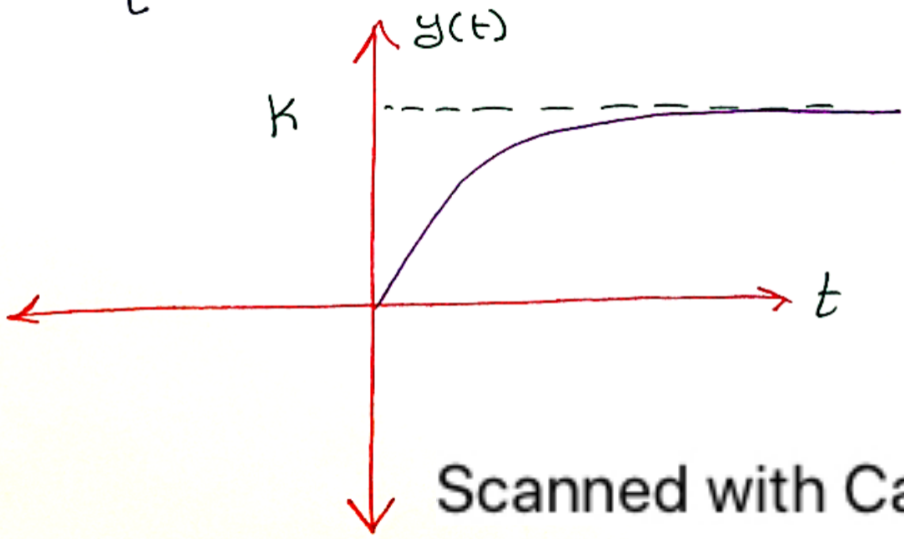
$$= \frac{K.\omega_n^2}{S^2 + 2\omega_n S + \omega_n^2} * \frac{1}{S}$$

by Partial fraction

$$= K\left[\frac{1}{S} - \frac{\omega_n}{(S+\omega_n)^2} - \frac{1}{S+\omega_n}\right]$$

by Taking Laplace inverse:

$$\therefore c(t) = K\left[1 - e^{-\omega_n t} - \omega_n t \cdot e^{-\omega_n t}\right]$$



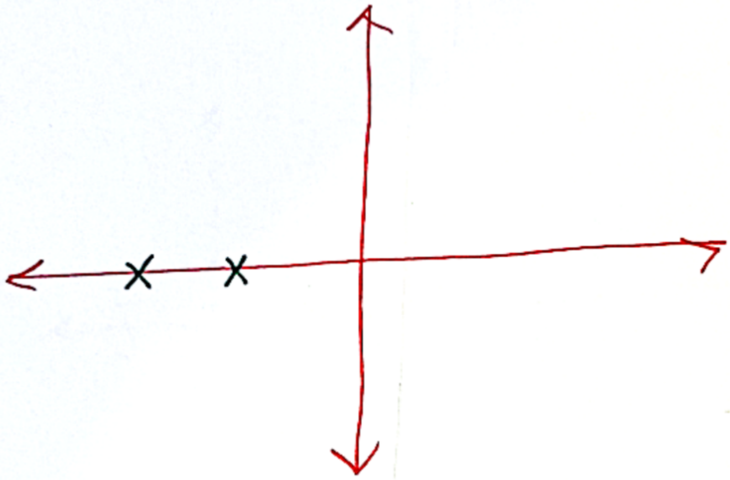
3- Over Damped :-

مميز

when the system has two real distinct poles.

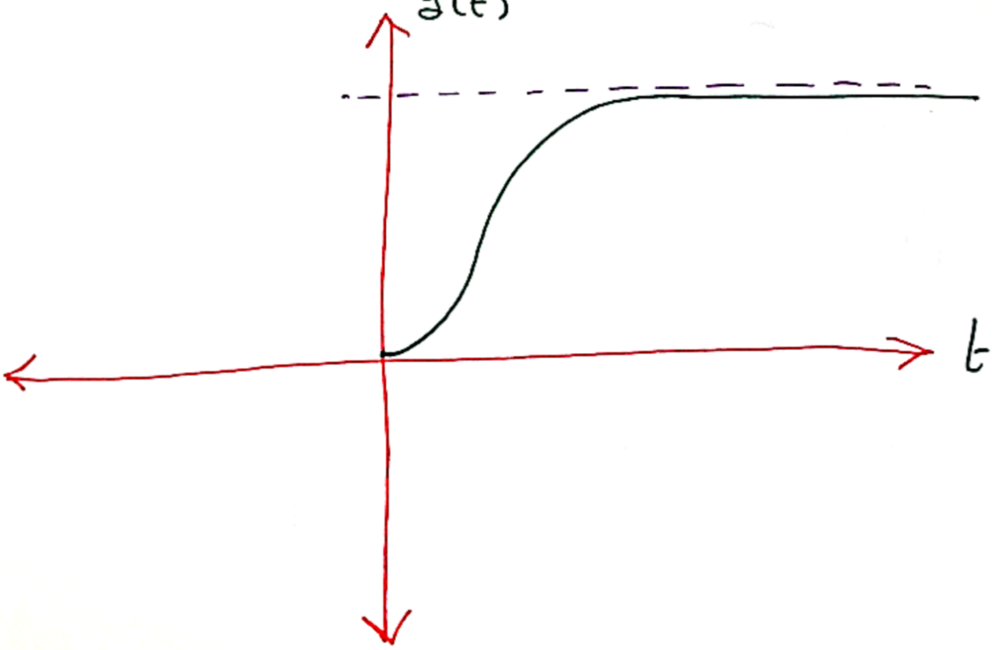
$[\xi > 1]$

$$P_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$



for unit step input :

$$c(t) = K[1 - K_1 e^{(-\xi\omega_n+\omega_n\sqrt{\xi^2-1})t} - K_2 e^{(-\xi\omega_n-\omega_n\sqrt{\xi^2-1})t}$$

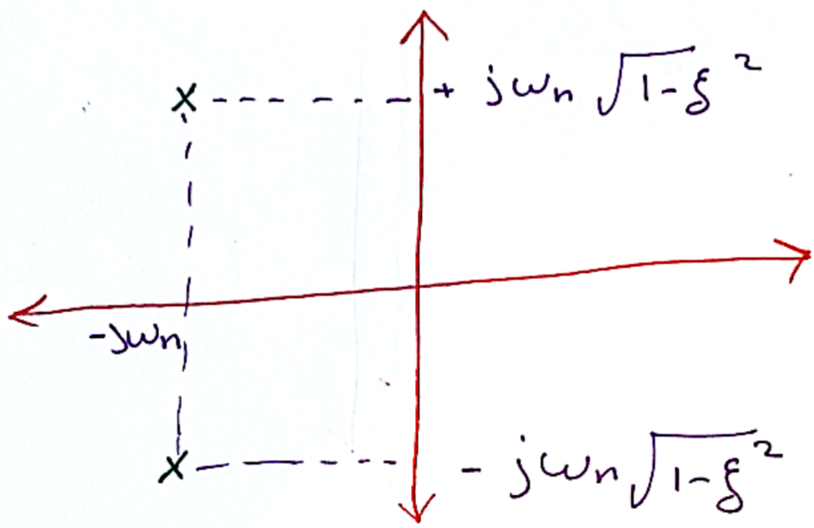


4- Under - Damped :-

when the system has two complex conjugate poles

$[0 < \xi < 1]$

$$P_{1/2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



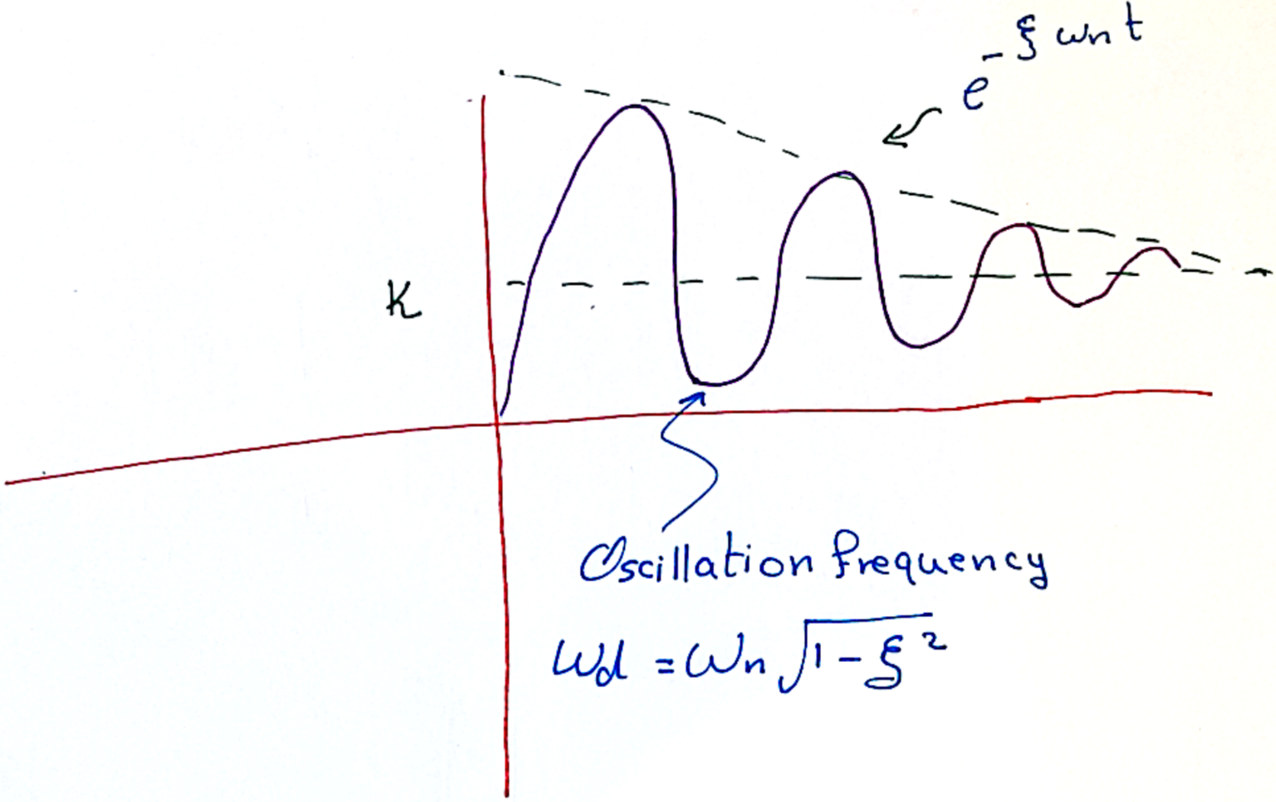
* For unit step input..

$$C(S) = \frac{K\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2} * \frac{1}{S}$$

$$\therefore C(t) = K\left[1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \cdot \cos\left[\omega_n\sqrt{1-\xi^2}\, t - \phi\right]\right]$$

where :

$$\phi = \tan^{-1}\left[\frac{\xi}{\sqrt{1-\xi^2}}\right]$$


$e^{-\xi \omega_n t}$
K
Oscillation frequency
$\omega_d = \omega_n \sqrt{1-\xi^2}$